Metric-affine gravity and Black holes

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CAS-JSPS workshop, Prague and Castle Třešť, 13/Dec/2022
JCAP 01 (2022) no.01, 011; JCAP 04 (2022) no.04, 011; and 2210.05998
Jointly with Jorge Gigante Valcarcel.
Overview of the Talk

1. Introduction to Metric-affine gravity
   - Why modified gravity?
   - Basic geometrical quantities

2. Metric-Affine gravity
   - Curvature, torsion and nonmetricity
   - Dynamics

3. MAG models with dynamical torsion and nonmetricity
   - Weyl part of nonmetricity
   - Axial symmetry in Weyl-Cartan geometry
   - Spherical symmetry with Shears and Weyl (complete nonmetricity)
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- **4-dimension**
- **2nd order derivatives**: gravitational action contains only second derivatives.
- **Locality**
- **Riemannian geometry**: The connection is the Levi-Civita one.
Why modified gravity?

- GR is not compatible with quantum field theory;

- The cosmological constant $\Lambda$ problem; Dark energy, dark matter.

- The $H_0$ tension: $5\sigma$ tension between current expansion rate $H_0$ using Planck data and direct model-independent measurements in the local universe;

- Big Bang singularity;

- What is really the inflaton?

- Strong gravity regime needs to be tested;

- A good way to understand GR is to modify it;
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How to modify it?

**Non-Riemannian geometry**
- Metric-affine gravity
- Non-commutative geometry
- Einstein-Cartan
- Poincaré gauge gravity
- Teleparallel theories

**Higher-order theories**
- $f(R, \Theta)$ theories
- Conformal Weyl
- Lovelock theories

**Other approaches**
- Padmanabhan thermod.
- Holography
- Analogue gravity
- Entropic gravity
- Other approaches
- Quantization

**Quantum gravity theories**
- Horava-Lifschitz
- String theory
- Loop quantum gravity
- Asymptotic safety
- Supergravity
- Rainbow gravity

**D-dimensional theories**
- Braneworld
- Randall Sundrum
- DGP
- Kaluza-Klein
- EdGB gravity
- M-theory

**Tensor-vector-scalar theories**
- Einstein-Æther
- Proca theories
- Bimetric gravity
- Horndeski
- Beyond Horndeski
- Massive gravity

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**Figure:** Classification of theories of gravity. (S. Bahamonde et.al., “Teleparallel Gravity: From Theory to Cosmology,” [arXiv:2106.13793 [gr-qc]].)
In the most general metric-affine setting, the fundamental variables are a metric $g_{\mu\nu}$ (10 comp.) as well as the coefficients $\tilde{\Gamma}^\rho_{\mu\nu}$ (64 comp.) of an affine connection.
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**Connection decomposition**

\[
\tilde{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + \text{Torsion part} + \text{Nonmetricity part}
\]

\[
\tilde{R}^{\mu\nu\rho\sigma} = \partial_{\rho} \tilde{\Gamma}^{\lambda}_{\mu\nu\sigma} - \partial_{\sigma} \tilde{\Gamma}^{\lambda}_{\mu\nu\rho} + \tilde{\Gamma}^{\lambda}_{\mu\tau\rho} \tilde{\Gamma}^{\tau}_{\lambda\nu\sigma} - \tilde{\Gamma}^{\lambda}_{\mu\tau\sigma} \tilde{\Gamma}^{\tau}_{\lambda\nu\rho}
\]

\[
\tilde{T}^{\mu\nu\rho} = \tilde{\Gamma}^{\mu}_{\rho\nu} - \tilde{\Gamma}^{\mu}_{\nu\rho}
\]

\[
\tilde{Q}^{\mu\nu\rho} = \tilde{\nabla}^{\mu} g_{\nu\rho} = \partial^{\mu} g_{\nu\rho} - \tilde{\Gamma}^{\sigma}_{\nu\mu} g^{\rho}_{\sigma} - \tilde{\Gamma}^{\sigma}_{\rho\mu} g^{\nu}_{\sigma}
\]
Fundamental variables and characteristic tensors

In the most general metric-affine setting, the fundamental variables are a metric $g_{\mu\nu}$ (10 comp.) as well as the coefficients $\tilde{\Gamma}^\rho_{\mu\nu}$ (64 comp.) of an affine connection.

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**Connection decomposition**

$$\tilde{\Gamma}^\lambda_{\mu\nu} = \left\{ \begin{array}{c} \text{Levi-Civita} \\ \Gamma^\lambda_{\mu\nu} \end{array} \right.$$
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The most general connection can be written as

\[
\tilde{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + \frac{1}{2} T^\lambda_{\mu\nu} - T_{(\mu}^\lambda_{\nu)}
\]
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\[ \tilde{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + \frac{1}{2} T^{\lambda}_{\mu\nu} - T(\mu \lambda \nu) + \frac{1}{2} Q^{\lambda}_{\mu\nu} - Q(\mu \lambda \nu), \]

### Connection decomposition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Levi-Civita</strong></td>
<td>$\Gamma^{\lambda}_{\mu\nu}$</td>
</tr>
<tr>
<td><strong>Torsion part</strong></td>
<td>$\frac{1}{2} T^{\lambda}_{\mu\nu} - T(\mu \lambda \nu)$</td>
</tr>
<tr>
<td><strong>Nonmetricity part</strong></td>
<td>$\frac{1}{2} Q^{\lambda}_{\mu\nu} - Q(\mu \lambda \nu)$</td>
</tr>
<tr>
<td><strong>Curvature</strong></td>
<td>$\tilde{R}^{\mu}<em>{\nu\rho\sigma} = \partial</em>{\rho} \tilde{\Gamma}^{\mu}<em>{\nu\sigma} - \partial</em>{\sigma} \tilde{\Gamma}^{\mu}<em>{\nu\rho} + \tilde{\Gamma}^{\mu}</em>{\tau\rho} \tilde{\Gamma}^{\tau}<em>{\nu\sigma} - \tilde{\Gamma}^{\mu}</em>{\tau\sigma} \tilde{\Gamma}^{\tau}_{\nu\rho}$</td>
</tr>
<tr>
<td><strong>Torsion</strong></td>
<td>$\tilde{T}^{\mu}<em>{\nu\rho} = \tilde{\Gamma}^{\mu}</em>{\rho\nu} - \tilde{\Gamma}^{\mu}_{\nu\rho}$</td>
</tr>
<tr>
<td><strong>Nonmetricity</strong></td>
<td>$\tilde{Q}<em>{\mu\nu\rho} = \tilde{\nabla}</em>{\mu} g_{\nu\rho} = \partial_{\mu} g_{\nu\rho} - \tilde{\Gamma}^{\sigma}<em>{\nu\mu} g</em>{\sigma\rho} - \tilde{\Gamma}^{\sigma}<em>{\rho\mu} g</em>{\nu\sigma}$</td>
</tr>
</tbody>
</table>
What does curvature geometrically represent?

**Curvature tensor** $\tilde{R}^\alpha_{\beta\mu\nu}$

Rotation experienced by a vector when it is parallel transported along a closed curve
What does torsion geometrically represent?

**Torsion tensor** $\tilde{T}^\alpha_{\mu\nu}$

non-closure of the parallelogram formed when two infinitesimal vectors are parallel transported along each other.
What does non-metricity geometrically represent?

Non-metricity tensor $\tilde{Q}_{\alpha\mu\nu}$ measures how much the length and angle of vectors change as we parallel transport them, so in metric spaces the length of vectors is conserve.
Figure: Classification of metric-affine geometries - Cube
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The curvature tensor of an affinely connected metric space-time contains corrections provided by the presence of torsion and nonmetricity: ($\nabla_\nu$ just Levi-Civita)

\[
\tilde{R}^\lambda_{\rho\mu\nu} = R^\lambda_{\rho\mu\nu} + \nabla_\mu N^\lambda_{\rho\nu} - \nabla_\nu N^\lambda_{\rho\mu} + N^\lambda_{\sigma\mu} N^\sigma_{\rho\nu} - N^\lambda_{\sigma\nu} N^\sigma_{\rho\mu},
\]

where

\[
N^\lambda_{\mu\nu} = \frac{1}{2} \left( T^\lambda_{\mu\nu} - T^\lambda_{\mu\nu} - T^\lambda_{\nu\mu} \right) + \frac{1}{2} \left( Q^\lambda_{\mu\nu} - Q^\lambda_{\mu\nu} - Q^\lambda_{\nu\mu} \right),
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Furthermore, the latter also leads to the definition of three independent traces of this tensor, namely the Ricci and co-Ricci tensors:

\[ \tilde{R}_{\mu\nu} = \tilde{R}^\lambda_{\mu\lambda\nu}, \quad \hat{R}_{\mu\nu} = \tilde{R}^\lambda_{\mu\nu\lambda}, \]

as well as the so-called homothetic curvature tensor \( \tilde{R}^\lambda_{\chi\mu\nu} \), which encodes the change of lengths of vectors provided by the trace part of nonmetricity under their parallel transport along closed loops.
Due to torsion, this connection introduces modifications in the covariant derivative which indeed involves a change on its commutation relations when considering an arbitrary vector $v^\lambda$:

$$[\tilde{\nabla}_\mu, \tilde{\nabla}_\nu] v^\lambda = \tilde{R}^\lambda_{\rho\mu\nu} v^\rho + T^\rho_{\mu\nu} \tilde{\nabla}_\rho v^\lambda.$$
Due to torsion, this connection introduces modifications in the covariant derivative which indeed involves a change on its commutation relations when considering an arbitrary vector $v^\lambda$:

$$[\tilde{\nabla}_\mu, \tilde{\nabla}_\nu] \, v^\lambda = \tilde{R}^\lambda{}_{\rho\mu\nu} \, v^\rho + T^\rho{}_{\mu\nu} \, \tilde{\nabla}_\rho v^\lambda.$$ 

The change of lengths of a given vector $k^\mu$ as well as the change of angles between two unit timelike vectors $\hat{m}^\mu$ and $\hat{n}^\mu$, under a general parallel transport defined by a tangent vector $V^\mu$, is proportional to the nonmetricity tensor:

$$V^\lambda \tilde{\nabla}_\lambda \|k\|^2 = V^\lambda Q_{\lambda\mu\nu} k^\mu k^\nu,$$

$$V^\lambda \tilde{\nabla}_\lambda \left( g_{\mu\nu} \hat{m}^\mu \hat{n}^\nu \right) = V^\lambda Q_{\lambda\mu\nu} \hat{m}^\mu \hat{n}^\nu - \frac{1}{2} V^\lambda Q_{\lambda\mu\nu} \left( \hat{m}^\mu \hat{m}^\nu + \hat{n}^\mu \hat{n}^\nu \right) \hat{m}^\rho \hat{n}_\rho.$$
Dynamics of metric-affine geometry

Gravitational action with dynamical torsion and nonmetricity:

\[ S = \int d^4x \sqrt{-g} \left[ \mathcal{L}_m - \frac{1}{16\pi} \mathcal{L}_g(\tilde{\mathcal{R}}, \mathcal{T}, Q) \right]. \]
Dynamics of metric-affine geometry

- Gravitational action with dynamical torsion and nonmetricity:

\[
S = \int d^4 x \sqrt{-g} \left[ \mathcal{L}_m - \frac{1}{16\pi} L_g(\tilde{R}, T, Q) \right].
\]

- Correspondence between geometry and matter:

\[
\begin{align*}
\frac{\delta S_g}{\delta e^a_{\ \nu}} &= 16\pi \theta^\nu_a, \\
\frac{\delta S_g}{\delta \omega^a_{\ b\nu}} &= 16\pi \Delta_a^{\ b\nu}.
\end{align*}
\]

Here $\theta^\nu_a$ is the energy-momentum tensor (canonical) and $\Delta_a^{\ b\nu}$ is the hypermomentum density tensor.
Dynamics of metric-affine geometry

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Here, \( \theta_a^\nu \) is the energy-momentum tensor (canonical) and \( \Delta_a^{b\nu} \) is the hypermomentum density tensor.

- \( GL(4, R) \) group allows the definition of a large number of scalar invariants depending on the aforementioned tensors.
Dynamics of metric-affine geometry

General quadratic gravitational action with dynamical torsion and nonmetricity:

\[ S = \int d^4 x \sqrt{-g} \left\{ \mathcal{L}_m + \frac{1}{16\pi} \left[ -\tilde{R} + a_1 \tilde{R}^2 + a_2 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho\mu\nu} + a_3 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\rho\lambda\mu\nu} \\
+ a_4 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\nu\rho} + a_5 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\mu\rho\nu} + a_6 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\lambda\rho\nu} + a_7 \tilde{R}_{\rho\lambda\mu\nu} \tilde{R}^{\mu\lambda\rho\nu} \\
+ a_8 \tilde{R}_{\mu\nu\lambda} \tilde{R}^{\mu\nu\lambda} + a_9 \tilde{R}_{\mu\nu\lambda} \tilde{R}^{\nu\mu\lambda} + a_{10} \tilde{R}_{\mu\nu\lambda} \tilde{R}^{\mu\nu\lambda} + a_{11} \tilde{R}_{\mu\nu\lambda} \tilde{R}^{\nu\mu\lambda} + a_{12} \tilde{R}_{\mu\nu\lambda} \tilde{R}^{\mu\nu\lambda} \\
+ a_{13} \tilde{R}_{\mu\nu\lambda} \tilde{R}^{\nu\mu\lambda} + a_{14} \tilde{R}_{\lambda\mu\nu\rho} \tilde{R}^{\rho \lambda\mu\nu} + a_{15} \tilde{R}_{\lambda\mu\nu\rho} \tilde{R}^{\mu\rho\lambda\nu} + a_{16} \tilde{R}_{\lambda\mu\nu\rho} \tilde{R}^{\nu\mu\rho\lambda} + a_{17} \tilde{R}_{\lambda\mu\nu\rho} \tilde{R}^{\rho\lambda\mu\nu} \\
+ b_1 T_{\lambda\mu\nu} T^{\lambda\mu\nu} + b_2 T_{\lambda\mu\nu} T^{\mu\lambda\nu} + b_3 T_{\lambda\mu\nu} T^{\nu\mu\lambda} + c_1 T_{\lambda\mu\nu} Q^{\mu\lambda\nu} \\
+ c_2 T^{\lambda\mu\nu} Q_{\lambda\nu} t^{\mu} + c_3 T^{\lambda\mu\nu} Q_{\lambda\nu} t^{\mu} + d_1 Q_{\lambda\mu\nu} Q^{\lambda\mu\nu} + d_2 Q_{\lambda\mu\nu} Q^{\mu\lambda\nu} \\
+ d_3 Q^{\lambda\mu\nu} Q_{\lambda\mu\nu} + d_4 Q_{\lambda\mu\nu} Q^{\mu\lambda\nu} + d_5 Q^{\lambda\mu\nu} Q_{\lambda\mu\nu} \right\} \right\} . \]
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   - Spherical symmetry with Shears and Weyl (complete nonmetricity)
Nonmetricity can be decomposed in the Weyl part plus a "traceless" part:

\[ Q_{\lambda \mu \nu} = g_{\mu \nu} W_{\lambda} + \mathcal{Q}_{\lambda \mu \nu}. \]

where \( W_{\mu} = \frac{1}{4} Q_{\mu \nu} \nu. \)
MAG models with dynamical torsion and nonmetricity (Weyl only)

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where \( W_{\mu} = \frac{1}{4} Q_{\mu\nu} \nu. \)

- Quadratic gravitational action with dynamical torsion and nonmetricity in Weyl-Cartan geometry (\( Q_{\lambda\mu\nu} = g_{\mu\nu} W_{\lambda} \) and \( \mathcal{Q}_{\lambda\mu\nu} = 0 \))

\[
S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_m + \frac{1}{64\pi} \left[ -4R - 6d_1 \tilde{R}_{\lambda[\rho\mu\nu]} \tilde{R}^{\lambda[\rho\mu\nu]} \\
- 9d_1 \tilde{R}_{\lambda[\rho\mu\nu]} \tilde{R}_{\mu\nu}^{\lambda} + 8d_1 \tilde{R}_{[\mu\nu]} \tilde{R}^{\lambda} + \frac{1}{8} (32e_1 + 8e_2 + 17d_1) \tilde{R}^\lambda_{\lambda\mu\nu} \tilde{R}^\rho_{\rho\mu\nu} \\
- 7d_1 \tilde{R}_{[\mu\nu]} \tilde{R}^\lambda_{\lambda\mu\nu} + 3 (1 - 2a_2) T_{[\lambda\mu\nu]} T^{[\lambda\mu\nu]} \right] \right\}.
\]

1 S. Bahamonde and J. G. Valcarcel, JCAP 09, 057 (2020).
2 S. Bahamonde and J. G. Valcarcel, JCAP 01 (2022) no.01, 011.
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Quadratic gravitational action with dynamical torsion and nonmetricity in Weyl-Cartan geometry (\( Q_{\lambda\mu\nu} = g_{\mu\nu} W_{\lambda} \) and \( \mathcal{Q}_{\lambda\mu\nu} = 0 \))

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- 7d_1 \tilde{R}_{[\mu\nu]} \tilde{R}^{\lambda} \lambda^{\mu\nu} + 3 (1 - 2a_2) T_{[\lambda\mu\nu]} T^{[\lambda\mu\nu]} \right] \right\} .
\]

Absence of a general Birkhoff’s theorem in MAG: new spherically and axially symmetric vacuum solutions with independent dynamical torsion and nonmetricity fields\(^1,2\)

\(^1\) S. Bahamonde and J. G. Valcarcel, JCAP 09, 057 (2020).
\(^2\) S. Bahamonde and J. G. Valcarcel, JCAP 01 (2022) no.01, 011.
Spherical symmetry

- Metric, torsion and nonmetricity in spherically symmetric space-times ($\#2 + \#8 + \#2 = \#12$):

\[
\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda_{\mu\nu} = \mathcal{L}_\xi W_\mu = 0 \implies \mathcal{L}_\xi \tilde{R}_{\lambda\rho\mu\nu} = 0
\]

By solving these equations we find that torsion and nonmetricity behave as:

- Tetrads:
  - $T^t_{\text{tr}} = a(r)$
  - $T^r_{\text{tr}} = b(r)$
  - $T^\theta_{\text{tk}} = f(r)$
  - $T^\theta_{\text{kr}} = g(r)$

- Geometric coefficients:
  - $T^\theta_{\text{tk}} = e_{\alpha \theta}^k e^{\beta \theta \lambda} \epsilon^\beta_{\alpha \lambda} (r)$
  - $T^\theta_{\text{kr}} = e_{\alpha \theta}^k e^{\beta \theta \lambda} \epsilon^\beta_{\alpha \lambda} (r)$

- The metric is in the standard spherically symmetric form:

\[
ds^2 = \Psi_1(r) dt^2 - \Psi_2(r) dr^2 - r^2 d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2
\]

Here, $\epsilon_{kl}$ is the Levi-Civita symbol in two dimensions.
Spherical symmetry

Metric, torsion and nonmetricity in spherically symmetric space-times ($\#2 + \#8 + \#2 = \#12$):

\[ \mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda_{\mu \nu} = \mathcal{L}_\xi W_\mu = 0 \implies \mathcal{L}_\xi \tilde{R}_{\lambda \rho \mu \nu} = 0 \]

By solving these equations we find that torsion and nonmetricity behave as

\[
\begin{align*}
T^t_{\ t r} &= a(r), \quad T^r_{\ t r} = b(r), \quad T^{\theta_k}_{\ t \theta_k} = f(r), \quad T^{\theta_k}_{\ r \theta_k} = g(r) \\
T^{\theta_k}_{\ t \theta_l} &= e^{a \theta_k} e^b_{\ \theta_l} \epsilon_{a b} d(r), \quad T^{\theta_k}_{\ r \theta_l} = e^{a \theta_k} e^b_{\ \theta_l} \epsilon_{a b} h(r), \\
T^t_{\ \theta_k \theta_l} &= \epsilon_{k l} k(r) \sin \theta_1, \quad T^r_{\ \theta_k \theta_l} = \epsilon_{k l} l(r) \sin \theta_1, \\
W_\lambda &= (w_1(r), w_2(r), 0, 0),
\end{align*}
\]

whereas the metric is in the standard spherically symmetric form:

\[
ds^2 = \Psi_1(r) \, dt^2 - \frac{d r^2}{\Psi_2(r)} - r^2 \left( d\theta_1^2 + \sin^2 \theta_1 \, d\theta_2^2 \right).
\]

Here, $\epsilon_{k l}$ is the Levi-Civita symbol in two dimensions.
The solution for the metric behaves as Reissner-Nordström

\[ g_{tt} = -1/g_{rr} \equiv \Psi(r) = 1 - \frac{2m}{r} + \frac{d_1 \kappa_s^2 - 4e_1 \kappa_d^2}{r^2}. \]
Solution with dilations and spin

1. The solution for the metric behaves as Reissner-Nordström

\[ g_{tt} = -1/g_{rr} \equiv \Psi(r) = 1 - \frac{2m}{r} + \frac{d_1 \kappa_s^2 - 4e_1 \kappa_{d,e}^2}{r^2}. \]

2. Nonmetricity sector:

\[ W_\mu = \frac{\kappa_{d,e}}{r} \left( 1, -\frac{1}{\Psi(r)}, 0, 0 \right). \]
The solution for the metric behaves as Reissner-Nordström

\[ g_{tt} = -\frac{1}{g_{rr}} \equiv \Psi(r) = 1 - \frac{2m}{r} + \frac{d_1 \kappa_s^2 - 4 e_1 \kappa_{d,e}^2}{r^2}. \]

Nonmetricity sector:

\[ W_\mu = \frac{\kappa_{d,e}}{r} (1, -1/\Psi(r), 0, 0). \]

Torsion sector:

\[ \bar{S}^a = -\frac{\kappa_s}{r} (1, 1, 0, 0), \]

\[ \bar{T}_{2}^{abc} = \frac{\kappa_s}{3r} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}. \]
What do these charges represent?

- **Torsion part:**

Intrinsic spin generates gravitation. This effect does not exist in GR.

We know that the spin is a fundamental property of particles. Since their masses contribute to gravity, why their spin do not?

The solution is in vacuum and a charge $\kappa_s$ appears (spin charge). Analogue to the case of Schwarzschild where the mass $M$ appears.

We expect that the spin charge might be important in certain astrophysical scenarios such as: highly magnetized neutron stars; supermassive black holes with endowed spin.
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Nonmetricity part - only Weyl:

1. Intrinsic dilations generates gravitation. This effect does not exist in GR.
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4. Do all particles in nature have different dilations? is this property important in particle physics?
Extension to axisymmetric space-times

Metric, torsion and nonmetricity tensors in symmetric space-times:

\[ \mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda_{\phantom{\lambda} \mu\nu} = \mathcal{L}_\xi Q^\lambda_{\phantom{\lambda} \mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda_{\phantom{\lambda} \rho\mu\nu} = 0. \]
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- Stationary and axisymmetric space-times:

\#10 \rightarrow \#4

\[ ds^2 = \Psi_1(r, \vartheta) \, dt^2 - \frac{dr^2}{\Psi_2(r, \vartheta)} - r^2 \Psi_3(r, \vartheta) \left[ d\vartheta^2 + \sin^2 \vartheta (d\varphi - \Psi_4(r, \vartheta) \, dt)^2 \right]; \]

\#24 \begin{cases} T^\lambda_{\mu\nu} = T^\lambda_{\mu\nu}(r, \vartheta) \\ W_\mu = (W_t(r, \vartheta), W_r(r, \vartheta), W_\vartheta(r, \vartheta), W_\varphi(r, \vartheta)) \end{cases}. 

Sebastian Bahamonde (*)
Rotating Kerr-Newman metric structure\(^3\):

\[
\begin{align*}
    ds^2 &= \Psi(r, \vartheta) \, dt^2 - \frac{r^2 + a^2 \cos^2 \vartheta}{(r^2 + a^2 \cos^2 \vartheta) \Psi(r, \vartheta) + a^2 \sin^2 \vartheta} \, dr^2 \\
        &\quad - (r^2 + a^2 \cos^2 \vartheta) \, d\vartheta^2 + 2a \,(1 - \Psi(r, \vartheta)) \sin^2 \vartheta \, dt \, d\varphi \\
        &\quad - \sin^2 \vartheta \,[r^2 + a^2 + a^2 \,(1 - \Psi(r, \vartheta)) \sin^2 \vartheta] \, d\varphi^2,
\end{align*}
\]

\[
\Psi(r, \vartheta) = 1 - \frac{2mr + 4e_1(k_{d,e}^2 + k_{d,m}^2) - d_1 \kappa_s^2}{r^2 + a^2 \cos^2 \vartheta}.
\]

---

\(^3\)S. Bahamonde and J. G. Valcarcel, JCAP 01 (2022) no.01, 011.
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$$- (r^2 + a^2 \cos^2 \vartheta) \, d\vartheta^2 + 2a \, (1 - \Psi(r, \vartheta)) \, \sin^2 \vartheta \, dt \, d\varphi$$

$$- \sin^2 \vartheta \left[ r^2 + a^2 + a^2 \, (1 - \Psi(r, \vartheta)) \, \sin^2 \vartheta \right] \, d\varphi^2,$$

$$\Psi(r, \vartheta) = 1 - \frac{[2mr + 4(\kappa_{d,e}^2 + \kappa_{d,m}^2) - d_1 \kappa_s^2]}{r^2 + a^2 \cos^2 \vartheta}.$$ 

Field strength tensors:

$$\tilde{R}_{[\mu\nu]} = \frac{1}{12} \varepsilon^{\lambda} \sigma_{\mu\nu} \nabla_\lambda \tilde{S}^\sigma + \frac{1}{2} \nabla_\lambda t^\lambda_{\mu\nu}; \quad \tilde{R}^\lambda_{\lambda\mu\nu} = 4 \nabla_{[\nu} W_{\mu]};$$

$$\tilde{R}^\lambda_{[\mu\nu\rho]} = \frac{1}{6} \varepsilon^{\lambda} \sigma_{[\rho\nu} \nabla_{\mu]} \tilde{S}^\sigma + \nabla_{[\mu} t^\lambda_{\rho\nu]} + \frac{1}{4} \varepsilon^{\lambda} \omega_{[\rho} \tilde{t}^\sigma_{1\mu\nu]} \tilde{S}^\omega$$

$$- \frac{1}{18} \varepsilon_{\sigma\mu\nu\rho} \tilde{T}_{1}^\lambda \tilde{S}^\sigma.$$ 

---

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Nonmetricity sector: (no approx.)

\[ w_1(r, \vartheta) = \frac{\kappa_{d,e} r - a \kappa_{d,m} \cos \vartheta}{r^2 + a^2 \cos^2 \vartheta} , \quad w_3(r, \vartheta) = 0, \]

\[ w_2(r, \vartheta) = -\frac{\kappa_{d,e} r}{(r^2 + a^2 \cos^2 \vartheta) \Psi(r, \vartheta) + a^2 \sin^2 \vartheta}, \]

\[ w_4(r, \vartheta) = \kappa_{d,m} \left( \frac{r^2 + a^2}{r^2 + a^2 \cos^2 \vartheta} \cos \vartheta - \gamma \right) - a \frac{\kappa_{d,e} r \sin^2 \vartheta}{r^2 + a^2 \cos^2 \vartheta}. \]
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Torsion sector (decoupling limit between the spin and the orbital angular momentum \( |a\kappa_S| \ll 1 \)):

\[ \bar{S}^a = -\frac{\kappa_S}{r} (1, 1, 0, 0) + \mathcal{O}(a\kappa_S), \]

\[ \bar{T}_{abc}^{2} = \frac{\kappa_S}{3r} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \end{pmatrix} + \mathcal{O}(a\kappa_S). \]
Gravitational spin-orbit interaction

We found a solution in the decoupling limit $a\kappa_s \ll 1$, which ensures that the Maxwell equation and closure conditions are fulfilled by the field strength tensors of torsion

$$\nabla_\lambda \tilde{R}^\lambda_{[\rho\mu\nu]} = \nabla_\mu \tilde{R}^{[\mu\nu]} = 0, \quad \nabla_{[\sigma} \tilde{R}_{\lambda[\rho\mu\nu]} = \nabla_{[\lambda} \tilde{R}_{[\mu\nu]} = 0.$$
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Possible new effects in the decoupling limit

The dynamics of torsion and nonmetricity alters the geometry of the space-time
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- Gravitational spin-orbit interaction:

\[
\mathcal{H}_{LS} = \frac{1}{m^2_e r} \frac{\partial V}{\partial r} \mathbf{L} \cdot \mathbf{S} \approx \frac{d_1}{2r} \frac{\partial g_{tt}}{\partial r} a\kappa_s \cos \vartheta
\]
It is well known that the most general axisymmetric system in vacuum that can describe a BH type D in GR contains:\footnote{J. F. Plebanski and M. Demianski, Annals Phys. 98 (1976), 98-127}

<p>| | |</p>
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Further, one can add a cosmological constant \(\Lambda\) and a electric charge \(q_e\) and magnetic charge \(q_m\).

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Further, one can add a cosmological constant $\Lambda$ and a electric charge $q_e$ and magnetic charge $q_m$.

The solution in GR is called Plebanski-Damianski solution.

---

4 J. F. Plebanski and M. Demianski, Annals Phys. 98 (1976), 98-127
The Plebanski-Damianski metric was recently presented in an improved form with $\Lambda = 0$ in by Podolský and Vrátný (Phys. Rev. D 104 (2021), 084078), and it can be written as

$$ds^2 = \Omega^{-2}(r, \vartheta) \left\{ \Phi_1(r, \vartheta) \left[ dt - (a \sin^2 \vartheta + 2l(\chi - \cos \vartheta)) \, d\varphi \right]^2 - \frac{dr^2}{\Phi_1(r, \vartheta)} - \frac{d\vartheta^2}{\Phi_2(r, \vartheta) \sin^2 \vartheta} \left[ a \, dt - (r^2 + a^2 + l^2 + 2\chi a l) \, d\varphi \right]^2 \right\}.$$ 

where $\Phi_i, \Omega$ are cumbersome functions depending on these parameters.

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\end{align*}$$

where $\Phi_i, \Omega$ are cumbersome functions depending on these parameters.

We just found this new form with the cosmological constant$^5$ with

$$\begin{align*}
\Phi_1(r, \vartheta) &= \frac{Q(r)}{\rho^2(r, \vartheta)}, \quad \Phi_2(r, \vartheta) = \frac{P(\vartheta)}{\rho^2(r, \vartheta)}, \text{ and} \\
\rho^2(r, \vartheta) &= r^2 + (a \cos \vartheta + l)^2. \text{ Here, } Q(r), \Omega(\vartheta) \text{ include the PD quantities.}
\end{align*}$$

We found a solution to OUR THEORY in the decoupling limit $|x_i \kappa_s| \ll 1$ with $x = (a, l, \alpha)$ with additional torsion and nonmetricity terms.
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$$w_1(r, \vartheta) = \frac{\kappa_{d,e} r - \kappa_{d,m} (a \cos \vartheta + l)}{r^2 + (a \cos \vartheta + l)^2}, \quad w_2(r, \vartheta) = -\frac{\kappa_{d,e} r - \kappa_{d,m} (a \gamma + l)}{Q(r)},$$

$$w_3(r, \vartheta) = -\kappa_{d,m} \sqrt{K(\vartheta) - \left(\frac{\cot \vartheta - \gamma \csc \vartheta}{P(\vartheta)}\right)^2},$$

$$w_4(r, \vartheta) = \kappa_{d,m} \left[ \frac{(r^2 + a^2 - l^2) \cos \vartheta + al \sin^2 \vartheta + 2\chi l (a \cos \vartheta + l)}{r^2 + (a \cos \vartheta + l)^2} - \gamma \right] - \frac{\kappa_{d,e} r [a \sin^2 \vartheta + 2l (\chi - \cos \vartheta)]}{r^2 + (a \cos \vartheta + l)^2},$$

$$\bar{T}^\vartheta \varphi_t = -\bar{T}^\varphi \varphi_t \sin^2 \vartheta = -\bar{T}^\vartheta \varphi_r \frac{Q(r)}{\rho^2(r, \vartheta)} = \bar{T}^\varphi \varphi_r \frac{Q(r)}{\rho^2(r, \vartheta)} \sin^2 \vartheta = \frac{\kappa_s \sin \vartheta}{r} + O(x_i \kappa_s).$$

Similarly as electromagnetism, the torsion behaves as a Coulomb-like quantity depending on a spin charge $\kappa_s$ and the non-metricity on the dilation charge $\kappa_d$. 
Nonmetricity can be decomposed in the Weyl part plus a "traceless" part:

\[ Q_{\lambda\mu\nu} = g_{\mu\nu} W_{\lambda} + \mathcal{Q}_{\lambda\mu\nu} . \]

where

\[ W_\mu = \frac{1}{4} Q_{\mu\nu} \nu , \]

\[ \mathcal{Q}_{\lambda\mu\nu} = g_{\lambda(\mu} \Lambda_{\nu)} - \frac{1}{4} g_{\mu\nu} \Lambda_{\lambda} + \frac{1}{3} \varepsilon_{\lambda\rho\sigma(\mu} \Omega_{\nu)} \rho\sigma + q_{\lambda\mu\nu} , \]
Nonmetricity decomposition

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\]

We defined a vector, and two traceless and pseudotraceless tensors

\[
\Lambda_{\mu} = \frac{4}{9} (Q_{\nu}^{\mu \nu} - W_{\mu}), \\
\Omega_{\lambda}^{\mu \nu} = - \left[ \varepsilon^{\mu \nu \rho \sigma} Q_{\rho \sigma \lambda} + \varepsilon^{\mu \nu \rho} \lambda \left( \frac{3}{4} \Lambda_{\rho} - W_{\rho} \right) \right], \\
q_{\lambda \mu \nu} = Q_{(\lambda \mu \nu)} - g_{(\mu \nu} W_{\lambda)} - \frac{3}{4} g_{(\mu \nu \Lambda \lambda)},
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Shears: Deformations without changing the volume.
The traceless part of nonmetricity and shears

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- Shears: Deformations without changing the volume.

- Up to now, there are not exact solutions with shears in MAG.
MAG theory with shears

Let us first consider a simple model where torsion is not propagating and the traceless part of nonmetricity is dynamical:

\[
S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ -R + 2f_1 \tilde{R}_{(\lambda\rho)\mu\nu} \tilde{R}^{(\lambda\rho)\mu\nu} \\
+ 2f_2 \left( \tilde{R}_{(\mu\nu)} - \hat{R}_{(\mu\nu)} \right) \left( \tilde{R}^{(\mu\nu)} - \hat{R}^{(\mu\nu)} \right) \right],
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\]

As can be seen, the propagation of the nonmetricity field described in the action is carried out by the symmetric part of the curvature tensor and its symmetric contraction:

\[
\tilde{R}^{(\lambda\rho)}_{\mu\nu} = \tilde{\nabla}_{[\nu} Q_{\mu]} \lambda^\rho + \frac{1}{2} T^\sigma_{\mu\nu} Q_\sigma \lambda^\rho,
\]

\[
\tilde{R}(\mu\nu) - \hat{R}(\mu\nu) = \tilde{\nabla}_{(\mu} Q^{\lambda}_{\nu)} \lambda - \tilde{\nabla}_{\lambda} Q_{(\mu\nu)} \lambda - Q^{\lambda\rho}_{\mu\nu} Q^{\rho}_{(\mu\nu)} + Q_{\lambda\rho(\mu} Q^{\lambda\rho}_{\nu)} + T_{\lambda\rho(\mu} Q^{\lambda\rho}_{\nu)},
\]

which in turn constitute deviations from the third Bianchi of GR.
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Spherical symmetry with nonmetricity and torsion

Metric, torsion and nonmetricity in spherically symmetric space-times ($\#2 + \#8 + \#12 = \#22$):

\[ \mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda_{\mu\nu} = \mathcal{L}_\xi Q_\alpha_{\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}_\lambda^\rho_{\mu\nu} = 0 \]
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- Nonmetricity now contains all the 12 dof:

$$Q_{ttt} = q_1(r), \quad Q_{trr} = q_2(r), \quad Q_{ttr} = q_3(r),$$
$$Q_{tv\vartheta} = Q_{t\varphi\varphi} \csc^2 \vartheta = q_4(r), \quad Q_{rtt} = q_5(r), \quad Q_{rrr} = q_6(r),$$
$$Q_{rtr} = q_7(r), \quad Q_{r\vartheta\vartheta} = Q_{r\varphi\varphi} \csc^2 \vartheta = q_8(r),$$
$$Q_{\vartheta t\vartheta} = Q_{\varphi t\varphi} \csc^2 \vartheta = q_9(r), \quad Q_{\vartheta r\vartheta} = Q_{\varphi r\varphi} \csc^2 \vartheta = q_{10}(r),$$
$$Q_{\vartheta t\varphi} = -Q_{\varphi t\vartheta} = q_{11}(r) \sin \vartheta, \quad Q_{\vartheta r\varphi} = -Q_{\varphi r\vartheta} = q_{12}(r) \sin \vartheta,$$

whereas the metric and torsion are the same as before.
How to find a solution with all of these dof?

- We are only interested in the traceless part of $Q_{\alpha\mu\nu}$ (containing shears), so that:

We eliminate the Weyl part of nonmetricity $W_{\mu} = \frac{1}{4} Q_{\mu\nu\nu} = 0$ by setting

$$q_1(r) = \Psi_1(r)$$

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We imposed $N[\lambda\rho]_{\mu} = 0$ which is equivalent to $T_{\lambda\mu\nu} = Q[\mu\nu]_{\lambda}$:

Shear transformations in the general linear group involves the part of the anholonomic connection describing a shear current or charge to take values in the symmetric traceless part of the Lie algebra.

- We demand the corresponding torsion and nonmetricity scalars of the solution to be regular.

After following these three steps we end up with 2 dof (metric) + 5 dof (torsion/nonmetricity) which is only 7 dof.
How to find a solution with all of these dof?

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New solution only with shears

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The metric behaves as

$$ds^2 = \Psi_1(r) \, dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 \left( d\theta_1^2 + \sin^2 \theta_1 \, d\theta_2^2 \right).$$

with

$$\Psi_1(r) = \Psi_2(r) = 1 - \frac{2m}{r} - \frac{2f_1 \kappa_{\text{sh}}^2}{r^2},$$

where $\kappa_{\text{sh}}$ is interpreted as a new charge, ”shear charge”.
After finding the shear part alone, we found a theory having the first solution (with spin+dilation) plus the second (with only shears).
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The action of the full model is

\[
S = \frac{1}{64\pi} \int \left[ -4R - 6d_1 \tilde{R}_\lambda^{[\rho\mu\nu]} \tilde{R}^{\lambda[\rho\mu\nu]} - 9d_1 \tilde{R}_\lambda^{[\rho\mu\nu]} \tilde{R}^{\mu[\lambda\nu\rho]} \\
+ 2d_1 \left( \tilde{R}_{[\mu\nu]} + \hat{R}_{[\mu\nu]} \right) \left( \tilde{R}^{[\mu\nu]} + \hat{R}^{[\mu\nu]} \right) + 18d_1 \tilde{R}_\lambda^{[\rho\mu\nu]} \tilde{R}^{(\lambda\rho)\mu\nu} \\
- 3d_1 \tilde{R}^{(\lambda\rho)\mu\nu} \tilde{R}^{(\lambda\rho)\mu\nu} + 6d_1 \tilde{R}^{(\lambda\rho)\mu\nu} \tilde{R}^{(\lambda\mu)\rho\nu} + 2(2e_1 - f_1) \tilde{R}^{\lambda\mu\nu} \tilde{R}^{\rho\mu\nu} \\
+ 8f_1 \tilde{R}^{(\lambda\rho)\mu\nu} \tilde{R}^{(\lambda\rho)\mu\nu} - 2f_1 \left( \tilde{R}^{(\mu\nu)} - \hat{R}^{(\mu\nu)} \right) \left( \tilde{R}^{(\mu\nu)} - \hat{R}^{(\mu\nu)} \right) \\
+ 3 \left( 1 - 2a_2 \right) T^{[\lambda\mu\nu]} T^{[\lambda\mu\nu]} \right] d^4 x \sqrt{-g}.
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When traceless part of nonmetricity is zero, the above action coincides with the first one.
Since we already found the solution for each model independently, it is not so difficult to find that the solution for the full model.

\[
ds^2 = \Psi_1(r) dt^2 - \Psi_2(r) - r^2 d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2.
\]

with

\[
\Psi_1(r) = \Psi_2(r) = 1 - \frac{2m}{r} + d_1 \kappa^2 s - 4e_1 \kappa^2 d - 2f_1 \kappa^2 \sinh^2 r,
\]

having the three possible charges: spin, dilation and shear.
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The solution gives us the following metric

\[ ds^2 = \Psi_1(r)\, dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 \left( d\theta_1^2 + \sin^2 \theta_1\, d\theta_2^2 \right) . \]

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On the other hand, the solution can also be trivially generalised to include the cosmological constant and Coulomb electromagnetic fields with electric and magnetic charges $q_e$ and $q_m$, which are decoupled from torsion under the assumption of the minimal coupling principle. This natural extension is then described by a Reissner-Nordström-de Sitter-like geometry

$$
\Psi(r) = 1 - \frac{2m}{r} + \frac{d_1 \kappa_s^2 - 4e_1 \kappa_d^2 - 2f_1 \kappa_{sh}^2 + q_e^2 + q_m^2}{r^2} + \frac{\Lambda}{3} r^2 ,
$$

which turns out to represent the broadest family of static and spherically symmetric black hole solutions obtained in MAG so far.
Conclusions and what do to next

- We found the first solutions with dynamical torsion and nonmetricity. First with the Weyl and then with the traceless part of nonmetricity.

1. Cosmology of the complete model: from inflation to dark energy.
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