Even photon breaks de Sitter invariance

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2022 CAS-JSPS Winter Workshop, 12-17 Dec

Based on publications to appear soon:

Glavan *Photon quantization in cosmological spaces*

Glavan, Prokopec *Photon propagator in de Sitter in a general covariant gauge*

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Some background

Quantum loop corrections in inflation:

gravitational particle production of IR light scalars and gravitons
— effects mediated to conformally coupled fields (e.g. photon)

$\Longrightarrow$ fully dimensionally regulated and renormalized computations

$\longrightarrow$ propagators basic blocks of QFT perturbation theory

Idealization of slow-roll inflation: de Sitter space
— maximally symmetric space: $D(D+1)/2$ isometries

cosmology: Poincaré patch of de Sitter (dS)

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^{\,2} = a^2(\eta)[-d\eta^2 + d\vec{x}^{\,2}] \qquad a(\eta) = -1/(H\eta) \tag{1}$$

Reminder: massless minimally coupled scalar (MMCS)

$$S[\phi] = \int d^Dx\,\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \frac{m^2}{2}\phi^2\right] \qquad m^2 \to 0 \quad (2)$$

Scalar propagator, $i\big[{}^+\Delta^+\big](x;x') = \langle\Omega|\mathcal{T}\hat{\phi}(x)\hat{\phi}(x')|\Omega\rangle\,,$

does not admit a dS invariant solution for $m^2 \to 0$.

Allen, Folacci, *The Massless Minimally Coupled Scalar Field in De Sitter Space*, Phys. Rev. D **35** (1987), 3771

Assume ansatz dependent on dS invariant distance

$$i\big[{}^+\Delta^+\big](x;x') = \mathcal{F}_\nu(y)\,, \qquad y = aa'H^2\Big[\|\Delta\vec{x}\|^2 - (|\Delta\eta| - i\varepsilon)^2\Big] \quad (3)$$

so that equation of motion becomes,

$$\bigg[(4y-y^2)\frac{\partial^2}{\partial y^2} + D(2-y)\frac{\partial}{\partial y} + \nu^2 - \big(\tfrac{D-1}{2}\big)^2\bigg]\mathcal{F}_\nu(y) = 0 \qquad (4)$$

where

$$\nu^2 = \big(\tfrac{D-1}{2}\big)^2 - \tfrac{m^2}{H^2} \qquad (5)$$

MMCS propagator

$$\mathcal{F}_\nu(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D-1}{2}+\nu)\,\Gamma(\frac{D-1}{2}-\nu)}{\Gamma(\frac{D}{2})} \times {}_2F_1\Big(\{\tfrac{D-1}{2}+\nu\},\{\tfrac{D-1}{2}-\nu\},1-\tfrac{y}{4}\Big) \tag{6}$$

Problem: $\mathcal{F}_\nu(y) \xrightarrow{m\to 0} \infty$

Solution: MMCS propagator that preserves cosmo. symmetries

$$i\big[{}^+\Delta^+\big](x;x') = \mathcal{F}_\nu(y) - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(2\nu)\,\Gamma(\nu)}{\Gamma(\nu+\frac{1}{2})\,\Gamma(\frac{D-1}{2})} \frac{\Big(\frac{k_0^2}{aa'H^2}\Big)^{\frac{D-1}{2}-\nu}}{\frac{D-1}{2}-\nu}\Bigg|_{\nu\to\frac{D-1}{2}} \tag{7}$$

dS breaking visible also for derivative

$$\partial_\mu i\big[{}^+\Delta^+\big](x;x') = \partial_\mu \mathcal{F}_\nu(y) + aH\delta_\mu^0 \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \tag{8}$$

Alternatively: mode function analysis, symmetry generators, etc.

Photon in dS

What about photons (massless vector field/EM field)?

$$S[A_\mu] = \int d^Dx \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\rho} F_{\nu\sigma} \right]. \tag{9}$$

In $D = 4$ conformally coupled to gravity

$\implies$ *they do not see cosmological expansion*

What about its propagator?

$$i\left[{}^{+}_{\mu}\Delta^{+}_{\nu}\right](x;x') = \langle \Omega | \mathcal{T} \hat{A}_\mu(x) \hat{A}_\nu(x') | \Omega \rangle \tag{10}$$

$$i\left[{}^{-}_{\mu}\Delta^{+}_{\nu}\right](x;x') = \langle \Omega | \hat{A}_\mu(x) \hat{A}_\nu(x') | \Omega \rangle \tag{11}$$

— requires fixing the gauge

Photon propagator

Most convenient for computation:
average gauges (sometimes called covariant gauges)

— defined by the gauge-fixing term to the action

$$S_\star = S + S_{\rm gf} \tag{12}$$

dS maximally symmetric space $\longrightarrow$ generally covariant gauge-fixing term,

$$S_{\rm gf}[A_\mu] = \int d^D x \, \sqrt{-g} \left[-\frac{1}{2\xi} \left(\nabla^\mu A_\mu \right)^2 \right] . \tag{13}$$

Propagator for $\boxed{\xi = 1 \text{ in arbitrary } D}$ first worked out by Allen & Jacobson

Vector Two Point Functions in Maximally Symmetric Spaces, Commun. Math. Phys. **103** (1986), 669

Covariant gauge photon propagator

Field operator satisfies equation of motion descending from the gauge-fixed action $\mathcal{D}_\mu{}^\nu \hat{A}_\nu = 0$

$\Longrightarrow$ propagator satisfies e.o.m.

$$\mathcal{D}_\mu{}^\rho i\big[{}_\rho\Delta_\nu\big](x;x') = g_{\mu\nu}\frac{i\delta^D(x-x')}{\sqrt{-g}}\,, \tag{14}$$

where,

$$\mathcal{D}_{\mu\nu} = g_{\mu\nu}\Box - \Big(1-\frac{1}{\xi}\Big)\nabla_\mu\nabla_\nu - R_{\mu\nu}\,. \tag{15}$$

Allen and Jacobson use the simple looking choice $\xi=1$.

Solving the equation? — make dS invariant ansatz!

Covariant gauge photon propagator

de Sitter invariant distance $y = H^2 a a' \Delta x^2, \quad \Delta x^2 = \|\Delta\vec{x}\|^2 - \Delta\eta^2$.

$\Longrightarrow$ build propagator out of y,

$$i\big[{}_\mu^+\Delta_\nu^+\big](x;x') = (\partial_\mu \partial'_\nu y)\mathcal{C}_1(y) + (\partial_\mu y)(\partial'_\nu y)\mathcal{C}_2(y)\,. \qquad (16)$$

Solve resulting equations for scalar structure functions $\mathcal{C}_1$ and $\mathcal{C}_2$ and impose correct singularity structure to get unique solution,

$$i\big[{}_\mu^+\Delta_\nu^+\big](x;x') = \frac{(\partial_\mu\partial'_\nu y)}{2\nu H^2}\bigg[-(\nu+\tfrac{1}{2})\mathcal{F}_\nu(y) - \Big(1-\frac{\xi}{\xi_s}\Big)\frac{\partial}{\partial y}\frac{\partial}{\partial \nu}\mathcal{F}_{\nu+1}(y)\bigg]$$
$$+ \frac{(\partial_\mu y)(\partial'_\nu y)}{2\nu H^2}\bigg[-\tfrac{1}{2}\mathcal{F}_\nu(y) - \Big(1-\frac{\xi}{\xi_s}\Big)\frac{\partial^2}{\partial y^2}\frac{\partial}{\partial \nu}\mathcal{F}_{\nu+1}(y)\bigg]. \qquad (17)$$

where parameters are,

$$\nu = \tfrac{D-3}{2} \xrightarrow{D\to 4} \tfrac{1}{2}\,, \qquad \xi_s = \tfrac{D-1}{D-3} \xrightarrow{D\to 4} 3\,. \qquad (18)$$

Subsequent dS photon propagators

Tsamis & Woodard $\boxed{D \text{ and } \xi \to 0}$ same approach as A-J

A Maximally symmetric vector propagator, J. Math. Phys. **48** (2007), 052306 [arXiv:gr-qc/0608069 [gr-qc]].

Youssef $\boxed{D = 4 \text{ and } \xi}$ same approach as A-J

Infrared behavior and gauge artifacts in de Sitter spacetime: The photon field, Phys. Rev. Lett. **107** (2011), 021101 [arXiv:1011.3755 [gr-qc]].

Fröb & Higuchi $\boxed{D \text{ and } \xi}$ using canonical quantization

Mode-sum construction of the two-point functions for the Stueckelberg vector fields in the Poincaré patch of de Sitter space, J. Math. Phys. **55** (2014), 062301 [arXiv:1305.3421 [gr-qc]].

The problem

Photon field is a gauge field

propagator satisfies the equation of motion ✓

AND

auxiliary conditions ?

de Sitter propagator	$\langle \hat{\Pi}_0 \, \hat{\Pi}_0 \rangle$	$\langle \hat{\Pi}_0 \, \partial_i \hat{\Pi}_i \rangle$	$\langle \partial_i \hat{\Pi}_i \, \partial_j \hat{\Pi}_j \rangle$
$D, \xi = 1$ (Allen & Jacobson)	×	✓	✓
$D, \xi = 0$ (Tsamis & Woodard)	✓	✓	✓
$D = 4, \xi$ (Youssef)	×	✓	✓
D, ξ (Fröb & Higuchi)	×	✓	✓

What are these auxiliary conditions? — correlators of first-class constraints.

The problem

Photon field is a gauge field

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AND

auxiliary conditions **?**

What are these auxiliary conditions?

— *correlators of first-class constraints.*

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D, ξ (Fröb & Higuchi)	×	✓	✓

Canonical quantization: classical formulation

What is this gauge-fixing term? and where does it come from?

— appears already in classical physics

Gauge invariant photon (EM field)

$$S[A_\mu] = \int d^D x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\rho} F_{\nu\sigma} \right]. \tag{19}$$

Treat every vector component as dynamical $\rightarrow$ canonical formulation,

$$\mathscr{S}[A_0, \Pi_0, A_i, \Pi_i, \boldsymbol{\lambda}] = \int d^D x \Big\{ \Pi_0 \partial_0 A_0 + \Pi_i \partial_0 A_i - \Big[\frac{a^{4-D}}{2} \Pi_i \Pi_i - A_0 \partial_i \Pi_i + \Pi_0 \partial_i A_i - (D-2) a H \Pi_0 A_0 + \frac{a^{D-4}}{4} F_{ij} F_{ij} \Big] - \boldsymbol{\lambda} \Pi_0 \Big\} \tag{20}$$

Canonical quantization: gauge ambiguities

Equations of motion:

$$\partial_0 A_0 = \partial_i A_i - (D-2)aHA_0 + \boldsymbol{\lambda}\,, \tag{21}$$

$$\partial_0 \Pi_0 = \partial_i \Pi_i + (D-2)aH\Pi_0\,, \tag{22}$$

$$\partial_0 A_i = a^{4-D}\Pi_i + \partial_i A_0\,, \tag{23}$$

$$\partial_0 \Pi_i = \partial_i \Pi_0 + a^{D-4}\partial_j F_{ji}\,, \tag{24}$$

and two first-class constraints,

$$\Pi_0 = 0\,, \qquad \partial_i \Pi_i = 0\,. \tag{25}$$

Gauge symmetry $\Longrightarrow$ system does not fix Lagrange multiplier $\boldsymbol{\lambda}$

Canonical quantization: fixing gauge

Fixing gauge $\equiv$ fixing $\boldsymbol{\lambda}$

Dirac-Bergmann algorithm: write gauge conditions (strong equalities) that fix $\boldsymbol{\lambda}$

More general: *average gauges*: choose $\boldsymbol{\lambda}$ by hand in e.o.m.

$$\boldsymbol{\lambda} \longrightarrow \overline{\boldsymbol{\lambda}}(A_\mu, \Pi_\mu) \,. \tag{26}$$

$$\texttt{dynamical eqs.:} \quad \begin{cases} \partial_0 A_0 = \partial_i A_i - (D-2) a H A_0 + \overline{\boldsymbol{\lambda}} \,, \\ \partial_0 \Pi_0 = \partial_i \Pi_i + (D-2) a H \Pi_0 \,, \\ \partial_0 A_i = a^{4-D} \Pi_i + \partial_i A_0 \,, \\ \partial_0 \Pi_i = \partial_i \Pi_0 + a^{D-4} \partial_j F_{ji} \,, \end{cases} \tag{27}$$

$$\texttt{constraints:} \qquad \Pi_0 = 0 \,, \qquad \partial_i \Pi_i = 0 \,. \tag{28}$$

Canonical quantization: gauge-fixed action

Gauge-fixed canonical action:

$$\mathscr{S}_\star[A_0, \Pi_0, A_i, \Pi_i] \equiv \mathscr{S}[A_0, \Pi_0, A_i, \Pi_i, \boldsymbol{\lambda} \to \overline{\boldsymbol{\lambda}}]$$
$$= \int d^D x \left\{ \Pi_0 \partial_0 A_0 + \Pi_i \partial_0 A_i - \left[\frac{a^{4-D}}{2} \Pi_i \Pi_i - A_0 \partial_i \Pi_i \right.\right.$$
$$\left.\left. + \Pi_0 \partial_i A_i - (D-2) a H \Pi_0 A_0 + \frac{a^{D-4}}{4} F_{ij} F_{ij} - \overline{\boldsymbol{\lambda}} \Pi_0 \right] \right\} \quad (29)$$

generates dynamical eqs. in (20), but NOT constraints.

$$\boxed{\mathscr{S}[A_0, \Pi_0, A_i, \Pi_i, \lambda] \quad \xrightarrow{g.f.} \quad \left\{ \begin{array}{c} \mathscr{S}_\star[A_0, \Pi_0, A_i, \Pi_i] \\ \textbf{AND} \\ \Pi_0 = 0 \,, \quad \partial_i \Pi_i = 0 \end{array} \right\}} \quad (30)$$

Canonical quantization: average gauges

Particular choice for multiplier defines the average gauge:

$$\boldsymbol{\lambda} \to \overline{\boldsymbol{\lambda}} = -\frac{1}{2}\xi a^{4-D}\Pi_0 \tag{31}$$

Corresponding configuration space (Lagrangian) gauge-fixed action:

$$S_\star[A_\mu] = \int d^D x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\rho} F_{\nu\sigma} - \frac{1}{2\xi} \left(\nabla^\mu A_\mu\right)^2 \right] \tag{32}$$

and auxiliary constraints

$$\Pi_0 = \frac{1}{\xi} a^{D-2} \nabla^\mu A_\mu = 0 \,, \qquad \partial_i \Pi_i = a^{D-4} \partial_i F_{0i} = 0 \tag{33}$$

Canonical quantization: actually quantizing

Quantizing dynamics — same as usual (Heisenberg picture),

$$[\hat{A}_\mu(\eta,\vec{x}),\hat{\Pi}_\nu(\eta,\vec{x}\,')] = i\delta_{\mu\nu}\delta^{D-1}(\vec{x}-\vec{x}\,')\,. \tag{34}$$

Quantizing auxiliary conditions?

- Operator equality $\hat{\Pi}_0 \overset{?}{=} 0$ ✗
- Hermitian operator annihilating state $\hat{\Pi}_0|\Omega\rangle \overset{?}{=} 0$ ✗
- correspondence principle — correlators of constraints vanish $\langle\Omega|\hat{\Pi}_0\hat{\Pi}_0|\Omega\rangle=0$, $\langle\Omega|\hat{\Pi}_0\partial_i\hat{\Pi}_i|\Omega\rangle=0$, $\langle\Omega|\partial_i\hat{\Pi}_i\partial_j\hat{\Pi}_j|\Omega\rangle=0$ ✓

Correct prescription — *non-Hermitian* combination of constraints annihilates the state

$$\boxed{\texttt{schematic}: \qquad \hat{K} = \alpha\hat{\Pi}_0 + \beta\partial_i\hat{\Pi}_i\,, \qquad \hat{K}|\Omega\rangle = 0} \tag{35}$$

This is how to derive Gupta-Bleuler condition!

dS propagator: pedestrian construction

Problematic property $(\nabla^\mu A_\mu = -\xi a^{4-D}\Pi_0)$,

$$\nabla^\mu \nabla'^\nu i\big[{}_\mu^- \Delta_\nu^+\big](x;x') \overset{!}{=} -\xi (D-1) H^2 \mathcal{C} \neq 0 \,. \qquad (36)$$

$\Longrightarrow$ Construct photon propagator from first principles!

- Canonically quantize photon field in dS ✓
- Solve for dynamics of fields (find mode functions) ✓
- Define dS invariant state:
  - find generators of all dS symmetries ✓
  - find dS invariant auxiliary non-Hermitian constraint ✓
  - find a state (mode per mode) that is an eigenstate of dS generators with vanishing eigenvalues ✓
- Compute components of propagator in position space ✓
- Covariantize propagator ✓

dS propagator: result

dS propagator from first principles:
(quantization not relying on symmetries;
imposing symmetries on the state)

$$i\big[{}_{\mu}^{-}\Delta_{\nu}^{+}\big](x;x') = \Big(i\big[{}_{\mu}^{-}\Delta_{\nu}^{+}\big](x;x')\Big)_{\rm dS} + \boxed{\xi \mathcal{C} a a' H^2 \delta_\mu^0 \delta_\nu^0} \qquad (37)$$

dS breaking term appears! preserves dilations but
breaks special spatial conformal transformations.

More familiar form of auxiliary condition,

$$\nabla^\mu i\big[{}_{\mu}^{+}\Delta_{\nu}^{+}\big](x;x') = -i\xi \nabla'_\nu \big[{}^{+}\Delta^{+}\big](x;x') \qquad (38)$$

where MMCS propagator appears,

$$\Box\, i\big[{}_{\mu}^{+}\Delta_{\nu}^{+}\big](x;x') = \frac{i\delta^D(x-x')}{\sqrt{-g}} \,. \qquad (39)$$

IR behaviour

Covariant photon propagators in de Sitter have odd IR behaviour — asymptote to a constant

Youssef, Phys. Rev. Lett. **107** (2011), 021101 [arXiv:1011.3755 [gr-qc]].

Rendell, Int. J. Mod. Phys. D **27** (2018) no.11, 1843005 [arXiv:1802.00687 [gr-qc]].

$$i\bigl[{}_{\mu}\Delta_{\nu}\bigr](x;x') \xrightarrow{y\to\infty} -\xi \mathcal{C} H^2 a a' \delta^0_\mu \delta^0_\nu \,. \tag{40}$$

Our non-covariant propagator exactly cancels this contribution for any gauge-fixing parameter,

$$i\bigl[{}_{\mu}\Delta_{\nu}\bigr](x;x') \xrightarrow{y\to\infty} 0 \,. \tag{41}$$

Energy momentum tensor

Two possible definitions:

from gauge-invariant action,

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \tag{42}$$

and from gauge-fixed action,

$$T^{\star}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_{\star}}{\delta g^{\mu\nu}} = T_{\mu\nu} + T^{\rm gf}_{\mu\nu}\,, \qquad T^{\rm gf}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_{\rm gf}}{\delta g^{\mu\nu}} \tag{43}$$

The extra part has to vanish at the expectation value level

BUT for de Sitter invariant propagator:

$$\langle \hat{T}^{\rm gf}_{\mu\nu} \rangle \overset{!}{=} g_{\mu\nu} \frac{\xi}{2a^4} \langle \hat{\Pi}_0 \hat{\Pi}_0 \rangle = -g_{\mu\nu} \frac{H^4}{32\pi^2} \tag{44}$$

For our dS breaking propagator

$$\langle \hat{T}^{\rm gf}_{\mu\nu} \rangle = 0 \tag{45}$$

Conclusion

- Canonical structure dictates quantization
- Symmetries can sometimes lead you astray
- Photon propagator in de Sitter in a genera covariant gauge is not dS invariant (except for $\xi \to 0$)
- Computations now possible for arbitrary ξ

Energy-momentum tensor: definitions

Two possible definitions:

- from gauge-invariant action

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \ldots \tag{46}$$

- from gauge-fixed action

$$T^{\star}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_{\star}}{\delta g^{\mu\nu}} = T_{\mu\nu} + \ldots \tag{47}$$

Energy-momentum tensor: expectation values

$$\langle \hat{T}_{\mu\nu} \rangle = 0\,. \tag{48}$$

For the formalism to be consistent it has to be

$$\langle \hat{T}^{\mathrm{gf}}_{\mu\nu} \rangle \stackrel{?}{=} 0\,. \tag{49}$$