FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Princeton University Princeton gravity Initiative

DAAD Deutscher Akademischer Austausch Dienst German Academic Exchange Service

Black-hole dynamics in quadratic gravity

w/ **Jun Zhang**
2209.01867

w/ **Hyun Lim**
PRD 104 (2021) 8
& work in progress

Aaron Held

DAAD PRIME Fellow at Jena University & The Princeton Gravity Initiative, Princeton University

12th December 2022,
2022 Winter CAS-JSPS Workshop in Cosmology, Gravitation and Particle Physics, FZU, Prague

Black-hole phenomenology in **theories beyond GR**

ghost-free theories

Lovelock's theorem → field equations of GR

+ **no other DOF**
+ four dimensions
+ diffeo symmetry
+ local action

Black-hole phenomenology in **theories beyond GR**

leading-order curvature corrections

$$\mathcal{L} = \frac{1}{16\pi G}R + \alpha R_{ab}R^{ab} - \beta R^2$$

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classical objections against ghosts

ghosts → runaways

ghosts → ill-posedness

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ghosts $\not\Rightarrow$ runaways — Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4

ghosts $\not\Rightarrow$ ill-posedness — see Part II of today's talk

Black-hole phenomenology in **theories beyond GR**

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ghosts $\not\to$ ill-posedness

ghosts $\not\to$ runaways

see Part II of today's talk

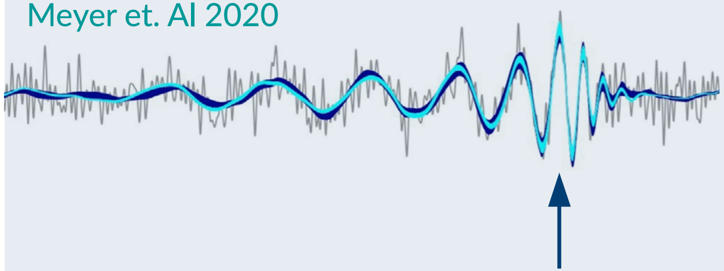
Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4

...

Black-hole phenomenology in **theories** **beyond GR**

stationary solutions

adapted from
Meyer et. Al 2020



Part I:
linear dynamics

Part II:
nonlinear dynamics

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Black-hole phenomenology in **theories beyond GR**

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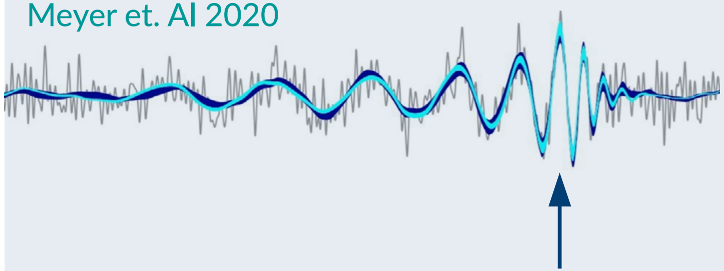

Part I: linear dynamics

Part II: nonlinear dynamics

leading-order curvature corrections

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ghost-free theories

Lovelock's theorem → field equations of GR

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w/ A. Cárdenas-Avendano (ongoing)

w/ H. Delaporte & A. Eichhorn
CQG 39 (2022) 13

w/ Astrid Eichhorn
JCAP 05 (2021) 073
EPJC 81 (2021)

w/ Astrid Eichhorn & Roman Gold
2205.14883

w/ Jun Zhang
2209.01867

w/ Hyun Lim
PRD 104 (2021) 8

w/ Sebastian Garcia-Saenz & Jun Zhang
PRL 127 (2021) 13
JHEP 05 (2022) 139

Quadratic Gravity

Stelle, PRD 16 (1977) 953-969

$$S = \int d^4x \sqrt{|g|} \left[\frac{1}{16\pi G} R + \alpha R_{ab} R^{ab} - \beta R^2 + (\text{curvature})^3 + \ldots \right]$$

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as the EFT of gravity

[modulo field redefinitions]

after field redefinitions see:
Burgess, Living Rev. Rel. 7:5,2004
Endlich et. Al, JHEP 09 (2017) 122
Cayuso, Lehner, PRD 102 (2020) 8

Quadratic Gravity

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$$S = \int d^4x \sqrt{|g|}\, M^2_{\mathrm{Planck}} \left[\frac{1}{2} R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2 massive spin-0 massive spin-2

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massless spin-2 | massive spin-0 | massive spin-2

as a fundamental theory

[perturbatively renormalizable; asymtotically free; ghost]

Stelle, PRD 16 (1977) 953-969

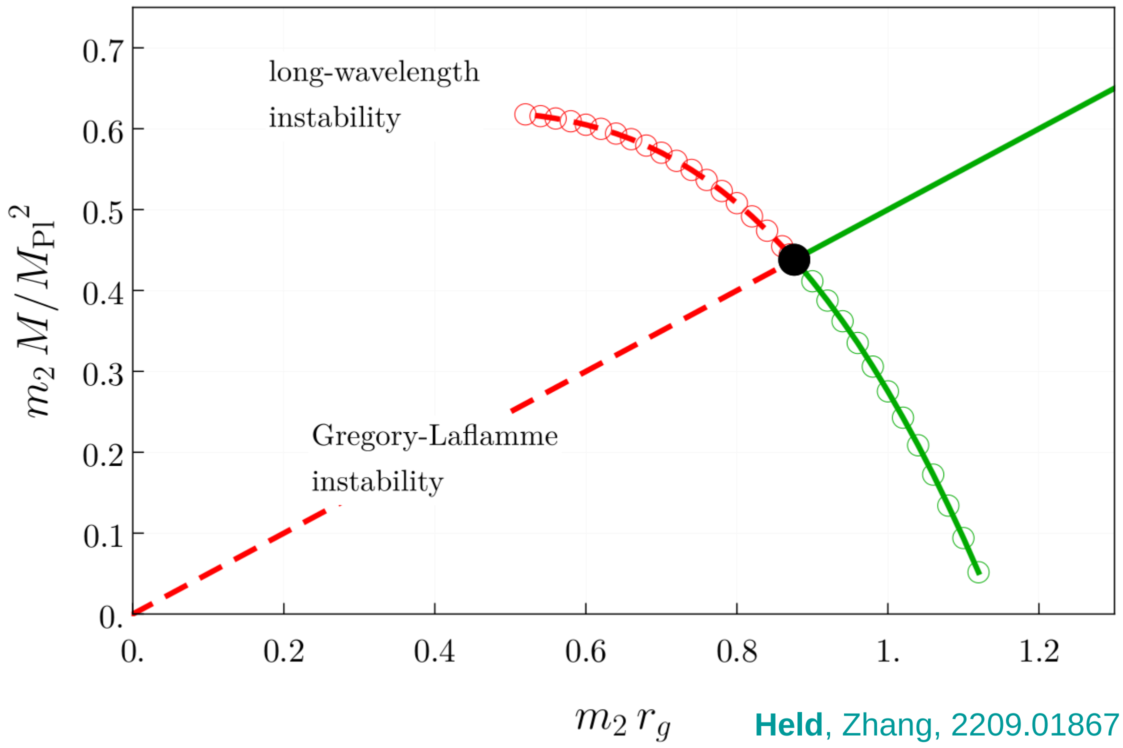
Avramidi, Barvinsky, PLB 159 (1985) 269-274

Bender, Mannheim, PRL 100 (2008)

Donoghue, Menezes, PRD 104 (2021) 4

Part I:
Linear dynamics & stability

Linear stability: Spherically-symmetric BHs **Held**, Zhang, 2209.01867



Held, Zhang, 2209.01867

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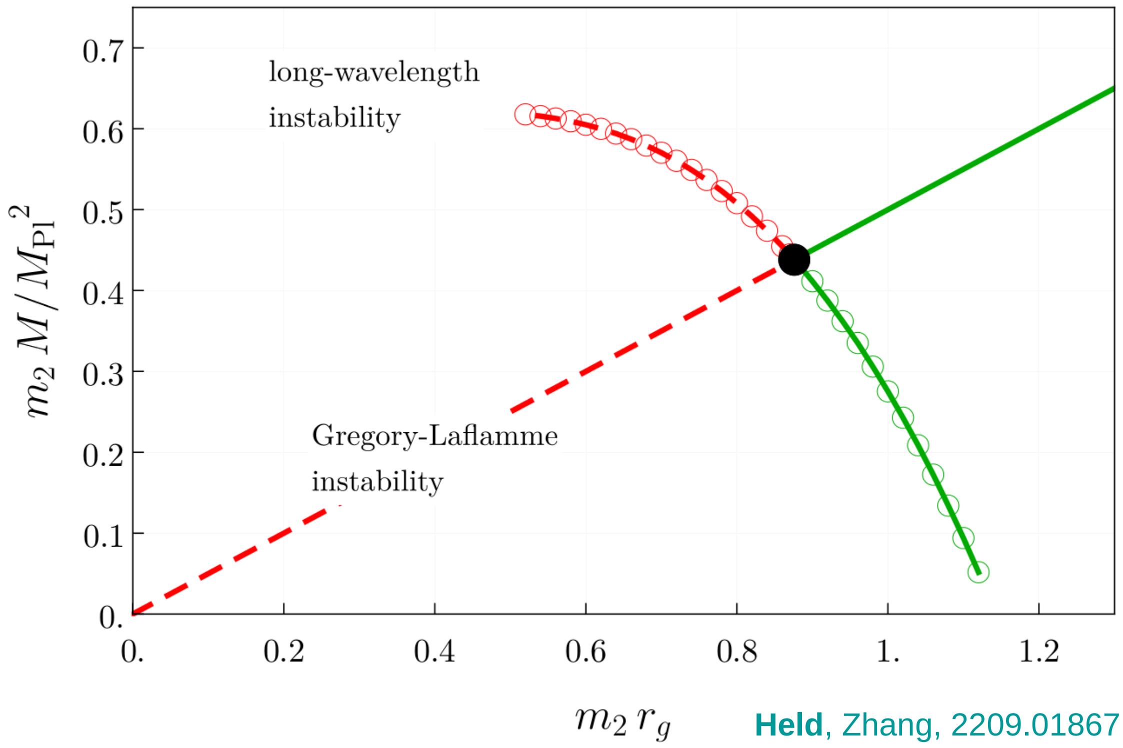

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  Svarc et. Al, 2209.15089

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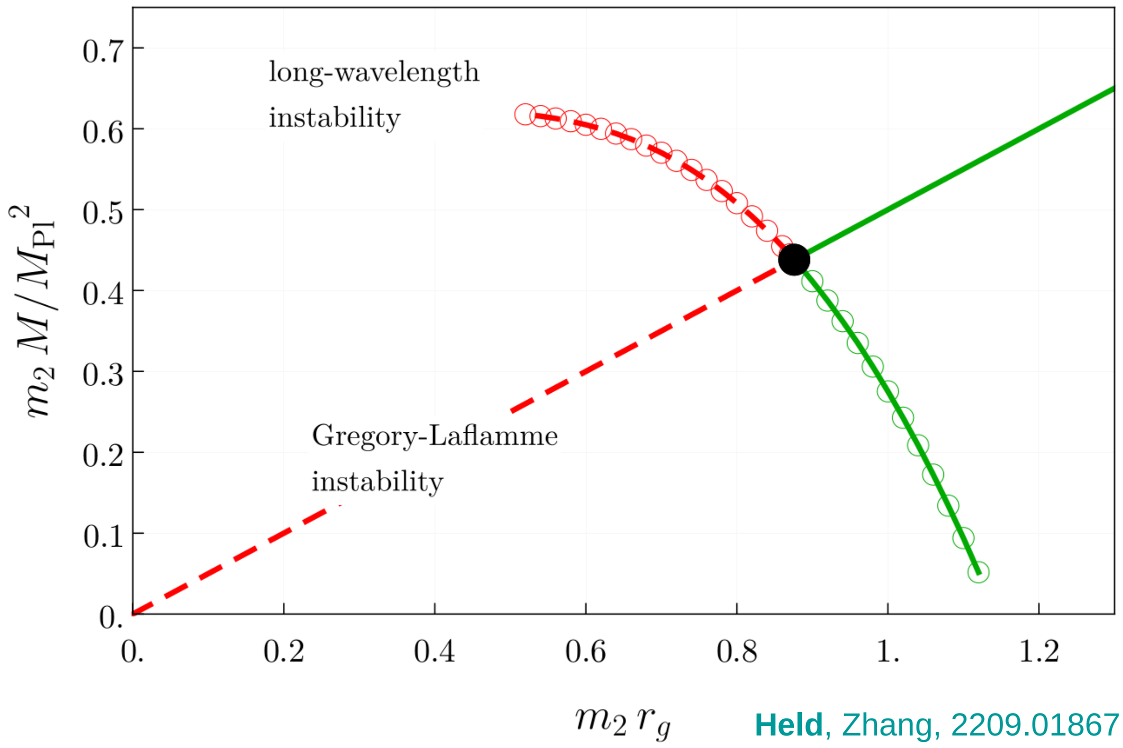

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- **Large BHs are stable**
- **Small BHs are unstable**

Dynamics: linear DoF

Background: decomposition

Boundary conditions:

Dynamics: linear DoF

$$\mathcal{L}_{QG} = M_{Pl}^2 \left[\frac{1}{2} R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

- massless spin-2 (graviton) h_{ab}
- massive spin-0 ϕ
- massive spin-2 ψ_{ab}

Background: decomposition

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Background: decomposition

- spherical harmonics $Y_{\ell m}(\theta, \phi)$

$$h_{ab}^{(polar)} = e^{-i\omega t}\, h_{ab}^{(polar)\ell m}(r)\, Y^{\ell m}(\theta, \phi)$$
$$h_{ab}^{(axial)} = e^{-i\omega t}\, h_{ab}^{(axial)\ell m}(r)\, Y^{\ell m}(\theta, \phi)$$
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- axisymmetric perturbations $m = 0$

$$h_{ab}^{(polar)} = e^{-i\omega t} \begin{pmatrix} AH_0 & H_1 & 0 & 0 \\ H_1 & H_2/B & 0 & 0 \\ 0 & 0 & r^2\mathcal{K} & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta\,\mathcal{K} \end{pmatrix} Y^\ell(\theta)$$

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$$\frac{d^2}{dr_*^2} \psi(r) + \psi(r) \left[\omega^2 - V(r) \right] = 0$$

GR-background: Brito, Cardoso, Pani ‘13
non-GR: **Held**, Zhang, 2209.01867

Boundary conditions:

Dynamics: linear DoF

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- purely ingoing waves at the horizon

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Boundary conditions:

- purely ingoing waves at the horizon
- outgoing waves at asymptotic infinity define QNMs
- ingoing waves at asymptotic infinity define bound states

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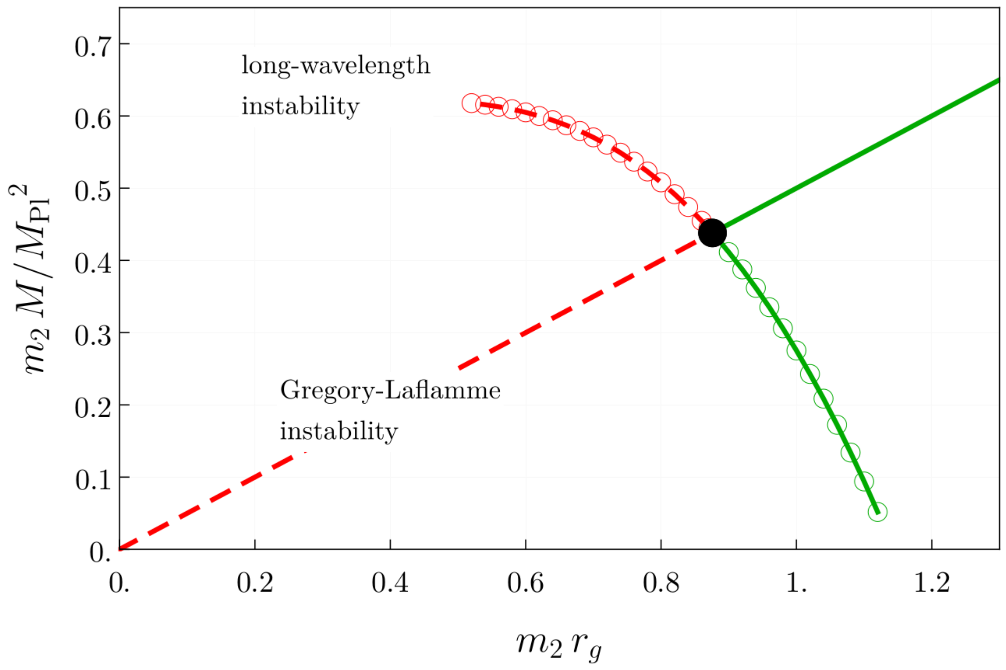
- positive imaginary part signals instability
- negative imaginary part signals stability

Linear stability of BHs in Quadratic Gravity **Held**, Zhang, 2209.01867



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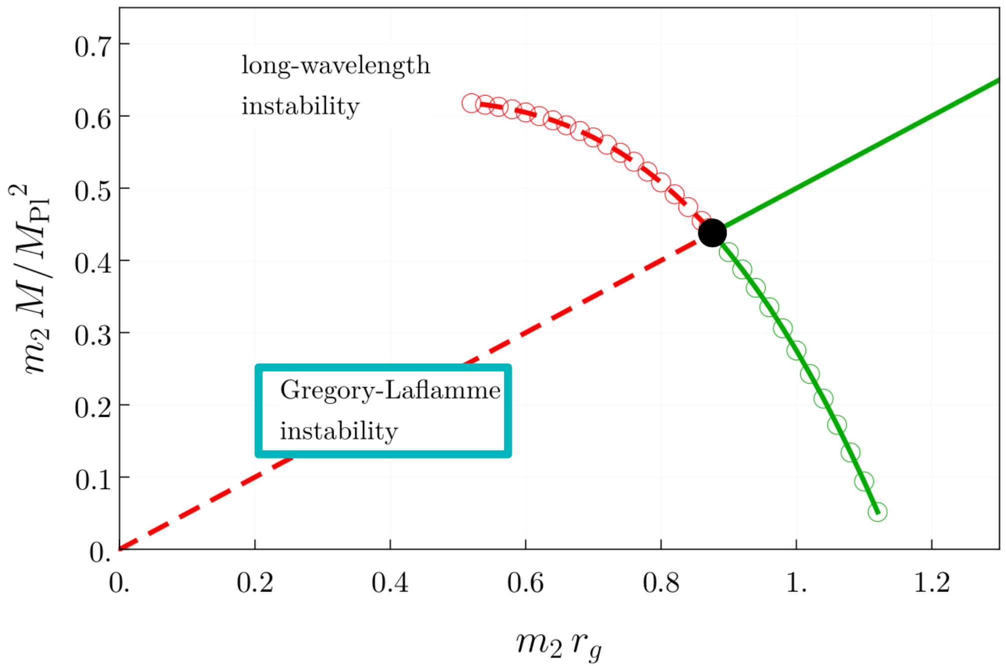

Linear stability of BHs in Quadratic Gravity

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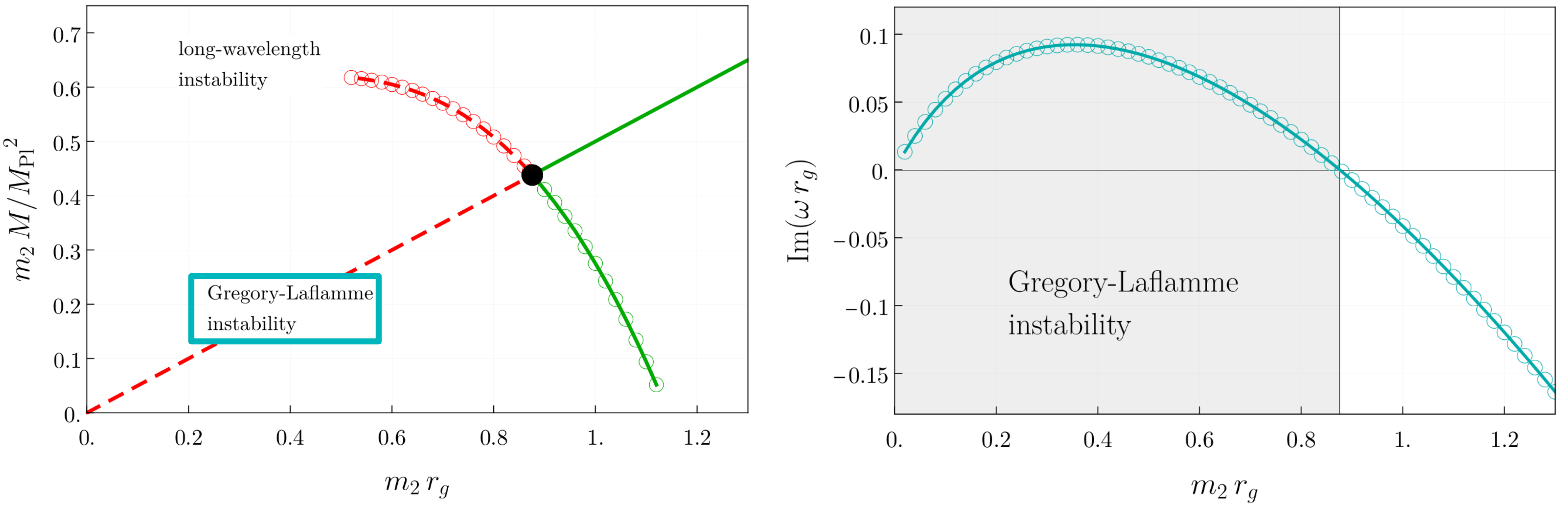

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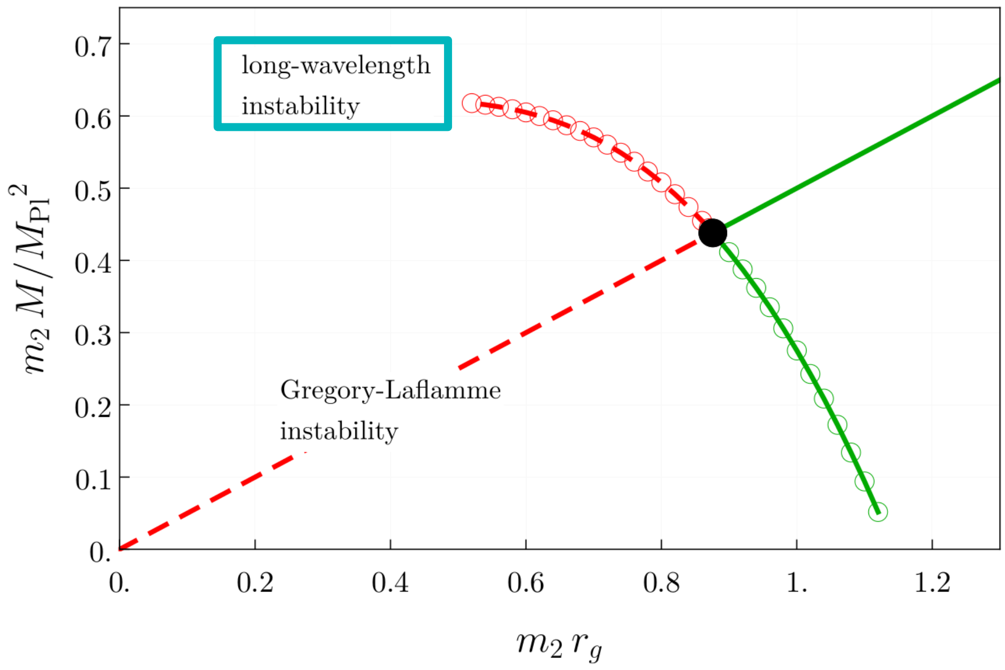

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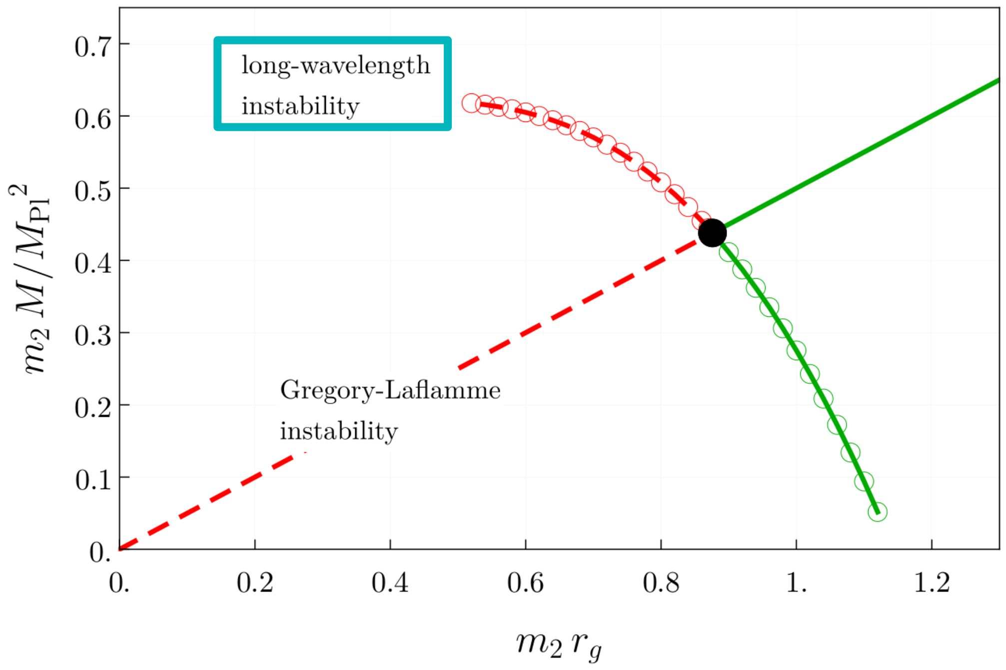

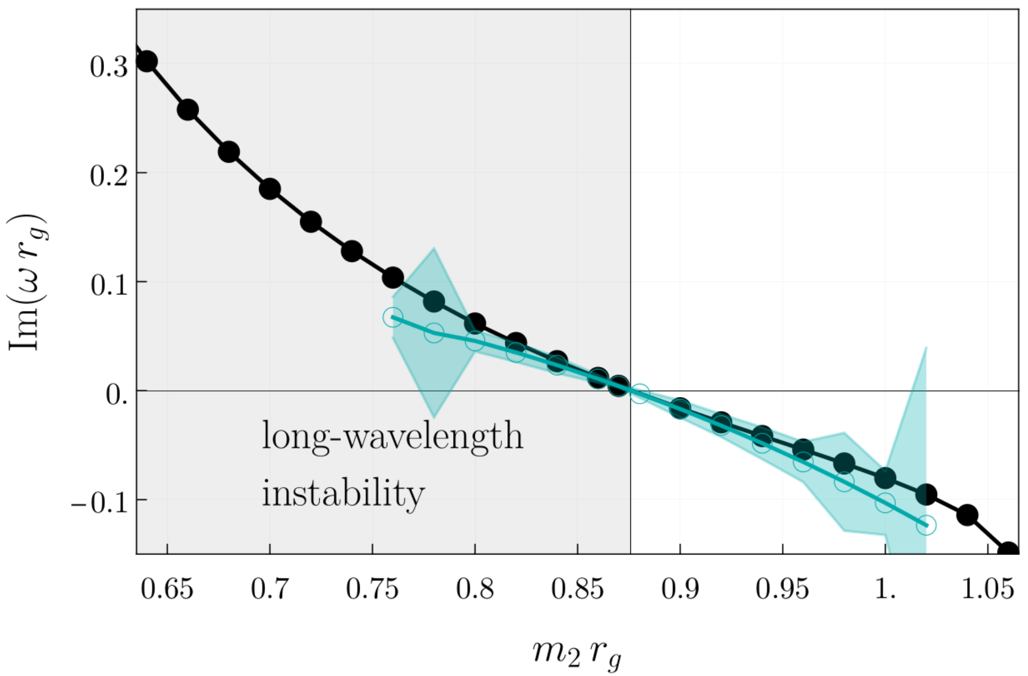

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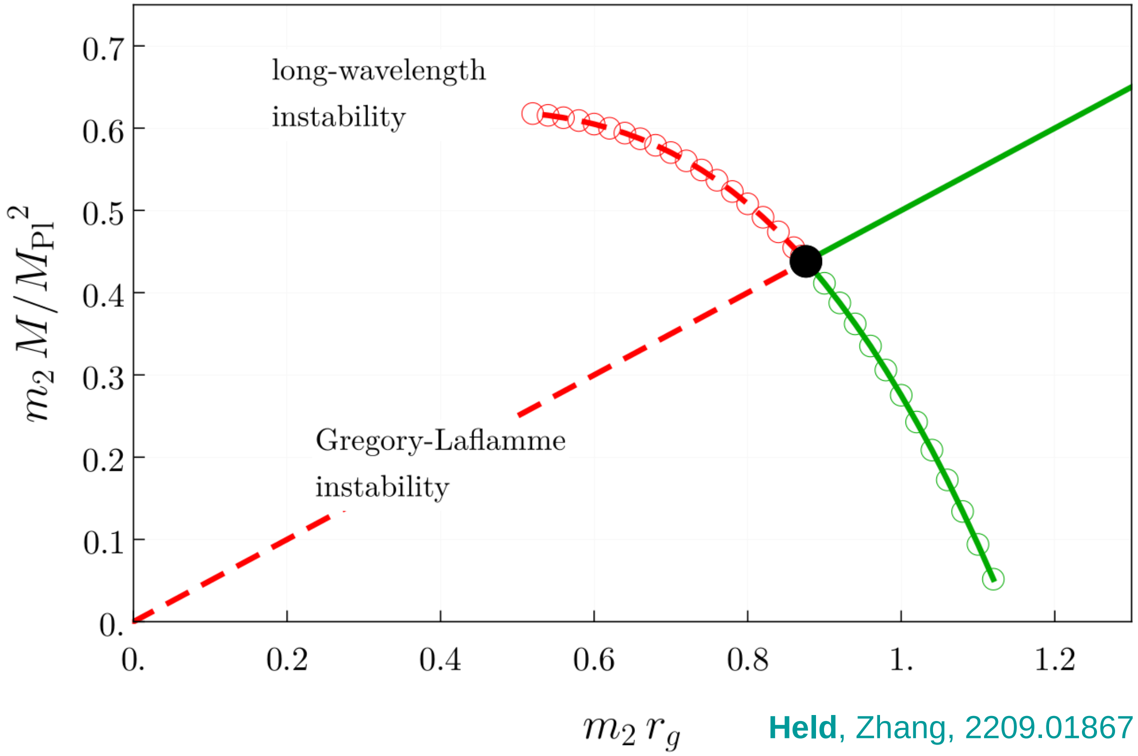

Held, Zhang, 2209.01867

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  Svarc et. Al, 2209.15089
- **Large BHs are stable**
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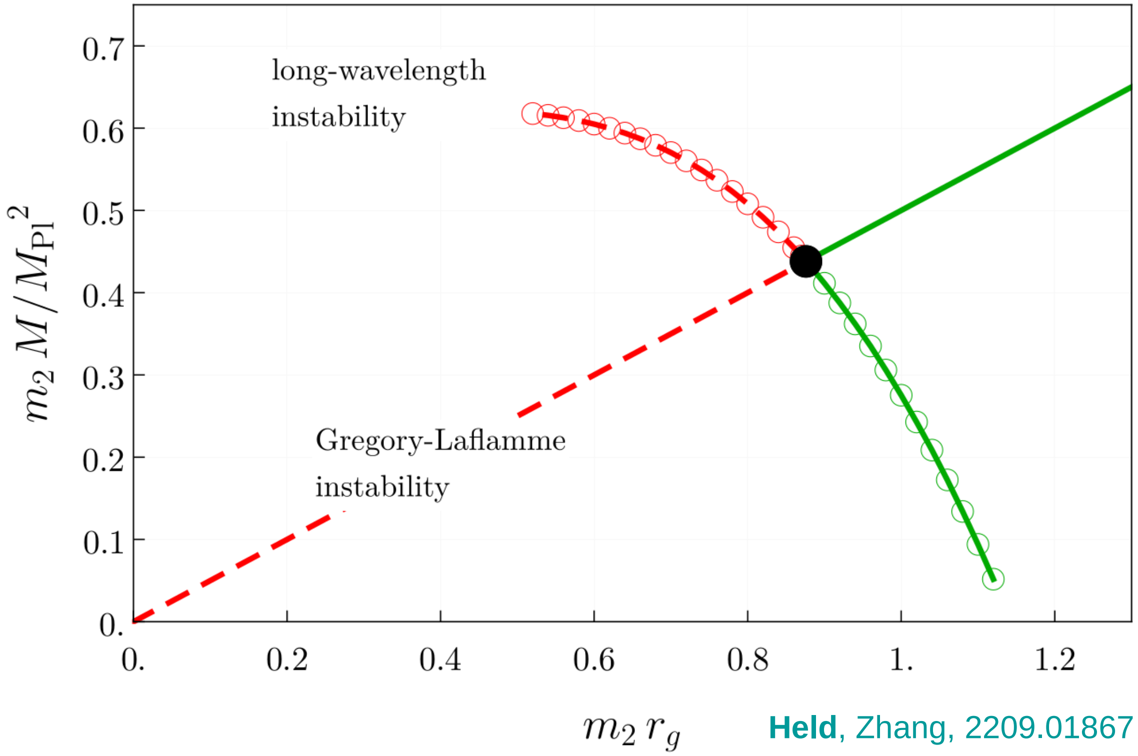
  Held, Zhang, 2209.01867
- **Slowly rotating BHs spin down** (superradiant instability)
  Brito et. Al, PRL 124, 211101 (2020)

Linear stability of BHs in Quadratic Gravity **Held**, Zhang, 2209.01867



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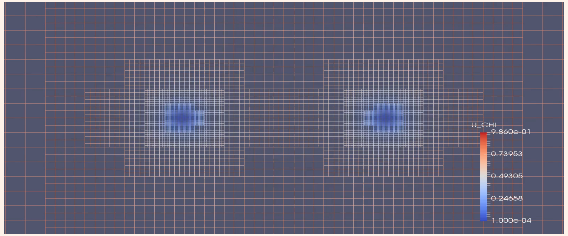
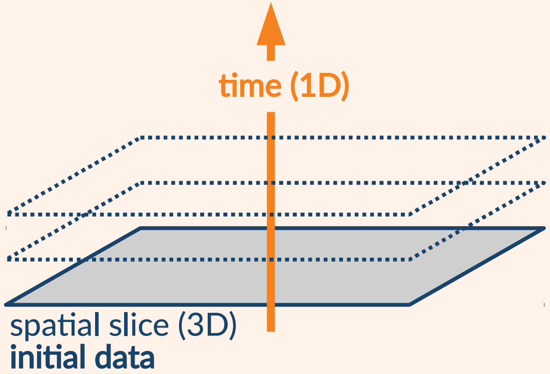
… as a potential source for primordial black holes?

Part II:
Nonlinear dynamics & stability



[benchmark simulation]
collaboration with Hyun Lim, LANL,
using **Dendro-GR** (Fernando et.Al. 2018),
https://github.com/paralab/Dendro-GR

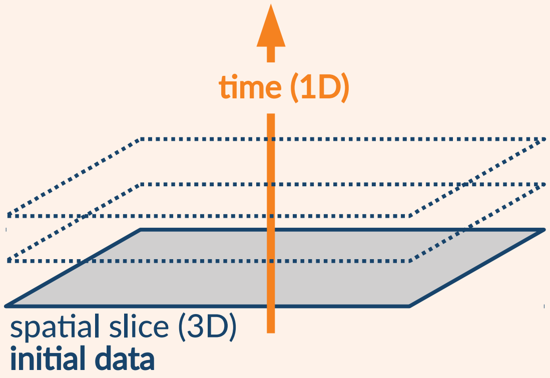
A well-posed initial value problem (IVP) ...



“ An initial value problem is well-posed if a solution “

- **exists** for all future time
- is **unique**
- and depends **continuous**ly on the initial data

A well-posed initial value problem (IVP) …



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… for General Relativity

Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat '52

(3+1) numerical evolution
Frans Pretorius '05
Baumgarte, Shapiro, Shibata, Nakamura '87-'99
Sarbach et.Al '02-'04

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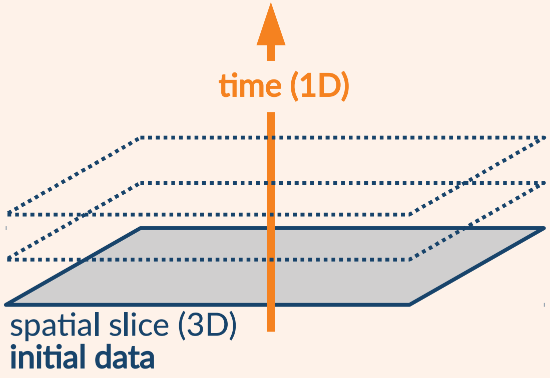

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… and for Quadratic Gravity

Formal proof of existence and uniqueness
Noakes ‘83

spherical symmetry: **Held**, Lim, PRD 104 (2021) 8
(3+1): **Held**, Lim, (to appear)

Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim (to appear)

2nd-order variables

$$R_{ab}(\Box g) = \widetilde{\mathcal{R}}_{ab} + \frac{1}{4} g_{ab}\mathcal{R} \equiv \widetilde{T}_{ab}$$

massless spin-2 (graviton)

$$\Box\mathcal{R} = m_0^2\mathcal{R} + 2T^c{}_c$$

massive spin-0 (scalar)

$$\Box\widetilde{\mathcal{R}}_{ab} = -\frac{1}{3}\left(\frac{m_2^2}{m_0^2} - 1\right)(\nabla_a\nabla_b\mathcal{R}) + 2\widetilde{\mathcal{R}}^{cd}C_{acbd} + \mathcal{O}_{\text{lower order}}$$

massive spin-2 (ghost)

Well-posed evolution in Quadratic Gravity

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massive spin-2 (ghost)

1st-order variables

$$\widetilde{V}_{ab} \equiv -n^c \nabla_c \widetilde{\mathcal{R}}_{ab}$$

$$\hat{\mathcal{R}} \equiv -n^c \nabla_c \mathcal{R}$$

2nd order quasilinear diagonal

+ constraints

(in harmonic gauge)

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2nd order quasilinear diagonal

+ constraints

(in harmonic gauge)

Leray's theorem guarantees well-posed IVP for $\mathcal{C}^\infty$ initial data

Leray '53
Choquet-Bruhat et.Al '77

Spherical symmetry

Held, Lim, PRD 104 (2021) 8

Numerical Evolution of Quadratic Gravity (sph-symm)

Held, Lim, PRD 104 (2021) 8

Alcubierre et.Al '01

Cartoon method to reduce to spherical symmetry

$$\mathbf{u} = \left(\mathsf{R},\, \mathsf{g}_{tt},\, \mathsf{g}_{tx},\, \mathsf{g}_{xx},\, \mathsf{g}_{yy}\right) \qquad \partial_t^2 \mathbf{u} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \partial_t \mathbf{u}\right)$$

$$\mathbf{v} = \left(\widetilde{\mathsf{R}}_{tt},\, \widetilde{\mathsf{R}}_{tx},\, \widetilde{\mathsf{R}}_{xx}\right) \qquad \partial_t^2 \mathbf{v} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \partial_t \mathbf{u},\, \partial_t \mathbf{v},\, \partial_t^2 \mathbf{u}\right)$$

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Held, Lim, PRD 104 (2021) 8

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Diagonalization to quasi-linear 2nd-order form

$$\partial_t^2 \dot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \partial_t \dot{\mathbf{u}},\, \partial_t \mathbf{v}\right) \qquad \partial_t \dot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}}\right)$$

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Held, Lim, PRD 104 (2021) 8

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Diagonalization to quasi-linear 2nd-order form

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Reduction to 1^{st} order in time

$$\partial_t \ddot{\mathbf{u}} = \mathcal{O}(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \ddot{\mathbf{u}},\, \dot{\mathbf{v}}) \qquad \partial_t \dot{\mathbf{u}} \equiv \ddot{\mathbf{u}} \qquad \ddot{\mathbf{u}} = \mathcal{O}(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}})$$

$$\partial_t \dot{\mathbf{v}} = \mathcal{O}(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \ddot{\mathbf{u}},\, \dot{\mathbf{v}}) \qquad \partial_t \mathbf{u} \equiv \dot{\mathbf{u}}$$

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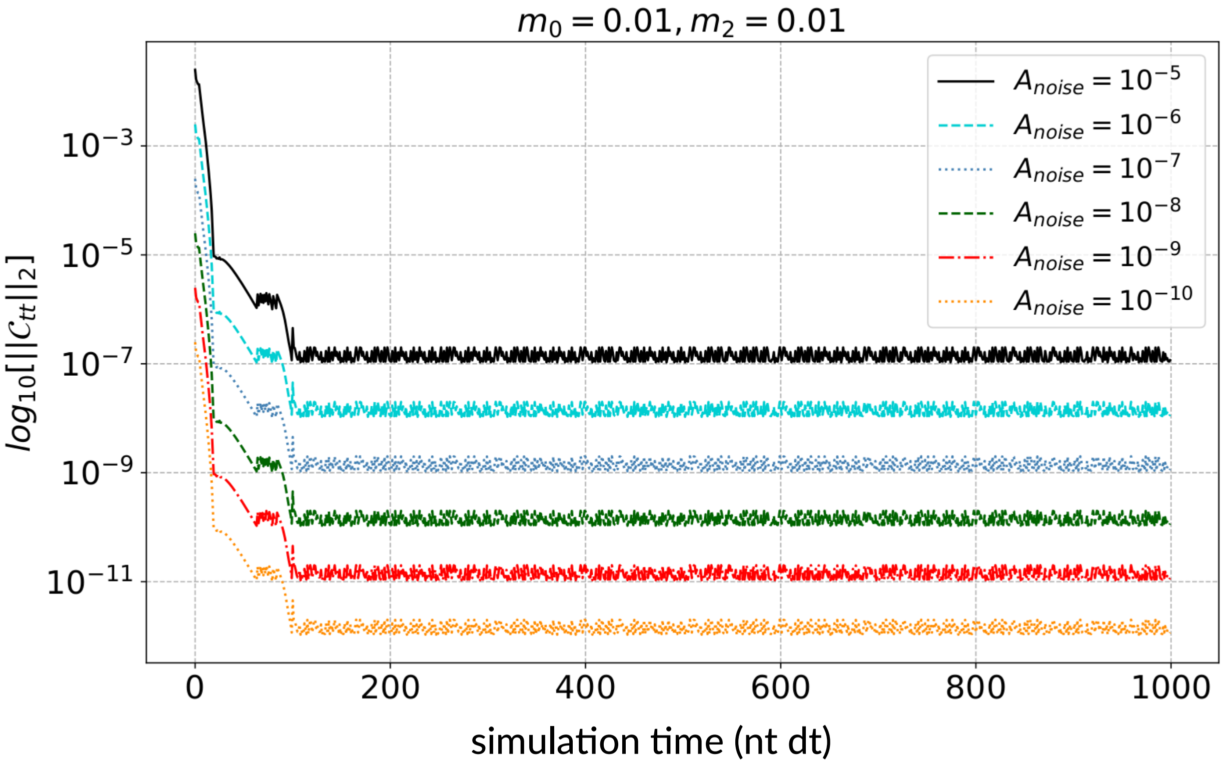
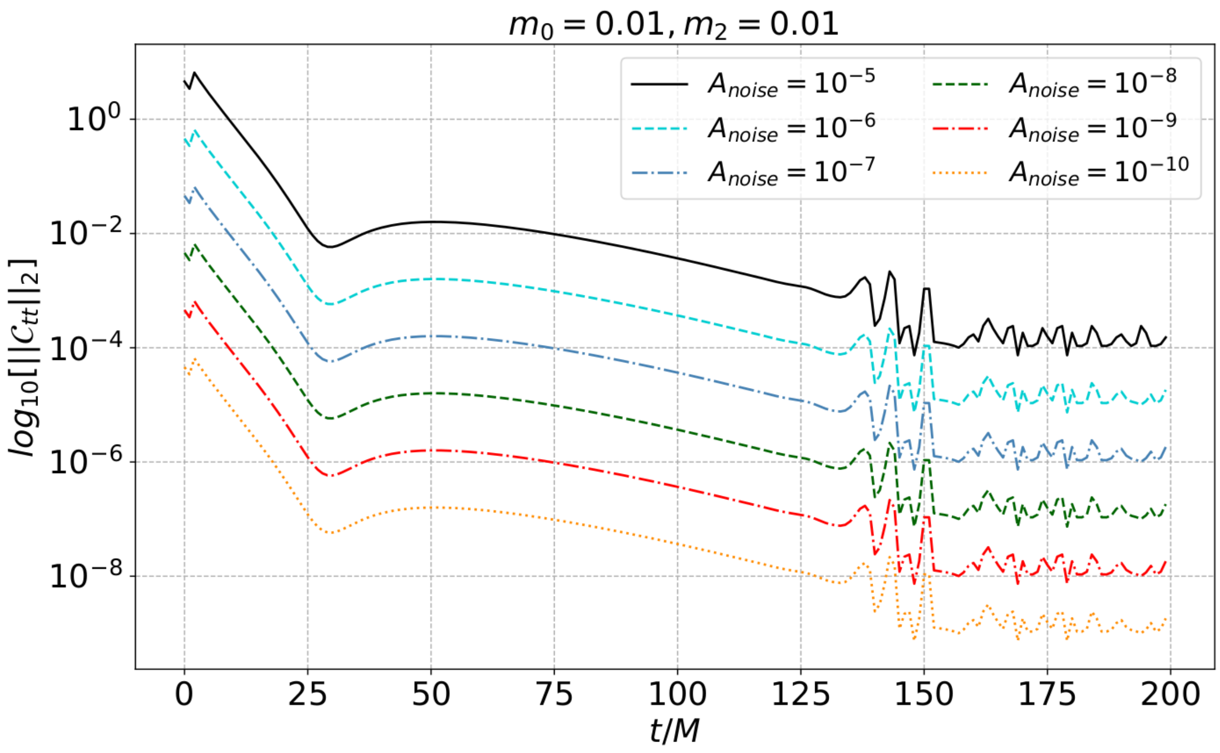
Numerical stability …



… about **flat spacetime**

Numerical stability ...



... about **Schwarzschild** spacetime

(3+1) à la BSSN

Held, Lim (to appear)

Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim (to appear)

2nd-order variables

$$R_{ab}(\Box g) = \tilde{\mathcal{R}}_{ab} + \frac{1}{4} g_{ab} \mathcal{R} \equiv \tilde{T}_{ab}$$

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$$g_{ab} = \gamma_{ab} + n_a n_b$$

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Numerical Evolution of Quadratic Gravity (3+1)

Held, Lim (ongoing)

massless spin-2

$$(n^c\nabla_c\gamma_{ij}) = -2\,D_{(i}n_{j)} + \mathcal{O}_{ij}$$

$$(n^c\nabla_c K_{ij}) = -(n^c\nabla_c n_i)(n^c\nabla_c n_j) - 2\,D_{(i}n^c\nabla_c n_{j)} - 2\,K_{m(i}D_{j)}n^m + {}^{(3)}R_{ij} + \mathcal{O}_{ij}$$

massive spin-0

$$n^a\nabla_a\mathcal{R} = \mathcal{O}$$

$$n^a\nabla_a\hat{\mathcal{R}} = -D_iD^i\mathcal{R} + \mathcal{O}$$

constraints

$$0 = D_jK^j_i - D_iK + \mathcal{C}_i$$

$$0 = {}^{(3)}R - K_{ij}K^{ij} + K^2 - \frac{1}{2}\mathcal{R}$$

$$\mathcal{E}_a = -K^b_a\mathcal{C}_b - K\mathcal{C}_a - D^b\mathcal{A}_{ab} - \frac{1}{3}D_a\mathcal{A} + \frac{1}{4}D_a\mathcal{R}$$

$$\hat{\mathcal{R}} = -4\,D^b\mathcal{C}_b$$

constraint evolution

$$n^c\nabla_c\mathcal{C}_i = -\mathcal{E}_i + \mathcal{O}_i \qquad n^c\nabla_c\mathcal{E}_i = \ \ldots$$

massive spin-2

$$n^c\nabla_c\mathcal{A} = \mathcal{O}$$

$$n^c\nabla_c\mathcal{A}_{ij} = \frac{2}{3}\mathcal{A}\,D_{(i}n_{j)} + \mathcal{O}_{ij}$$

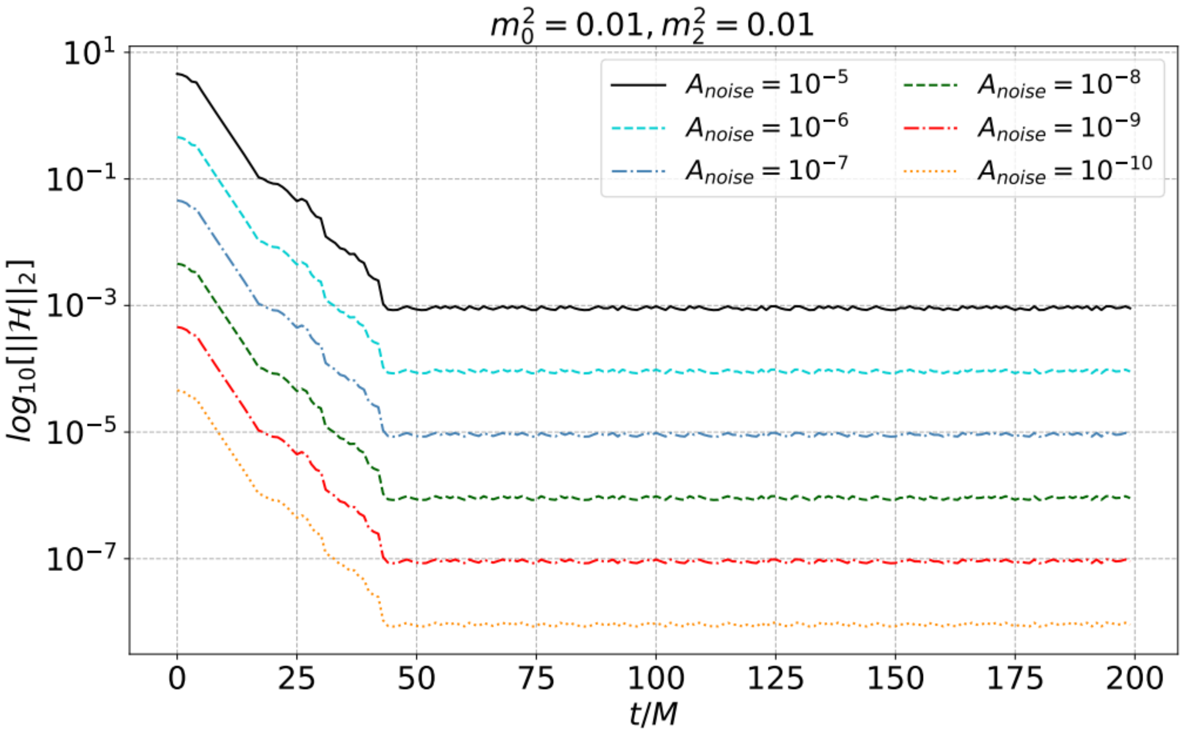
$$n^c\nabla_c\mathcal{B} = +2\left(\mathcal{A}^{ij} + \frac{1}{3}\gamma^{ij}\mathcal{A}\right){}^{(3)}R_{ij} - \frac{1}{3}\left(\frac{m_2^2}{m_0^2} + 1\right)D_iD^i\mathcal{R} - D_iD^i\mathcal{A} + 2\,a^k\mathcal{E}_k - a_iD^i\mathcal{A} + 4\,\mathcal{C}^j\left(D^iK_{ij} - D_jK\right) + \mathcal{O}$$

$$n^c\nabla_c\mathcal{B}_{ij} = +2\left(\mathcal{A}^{kl} + \frac{1}{3}\gamma^{kl}\mathcal{A}\right){}^{(3)}R_{ikjl} - \frac{1}{3}\left(\frac{m_2^2}{m_0^2} + 1\right)D_iD_j\mathcal{R} - \left(D_kD^k + a_kD^k\right)\left(\mathcal{A}_{ij} + \frac{1}{3}\gamma_{ij}\mathcal{A}\right) + \frac{2}{3}\mathcal{B}\,D_{(i}n_{j)} + 2\,a^c\gamma_{c(i}\mathcal{E}_{j)} - \frac{1}{3}\gamma_{ij}\left(n^c\nabla_c\mathcal{B}\right) + 4\,\mathcal{C}^k\left(D_{[i}K_{k]j} + D_{[j}K_{k]i}\right) + \mathcal{O}_{ij}$$

- massive spin-0/spin-2 do **not impact** the massless spin-2 **principal part**
- amenable to **1st-order strong-hyperbolicity analysis**

Sarbach et.Al '02-'04 (for GR)

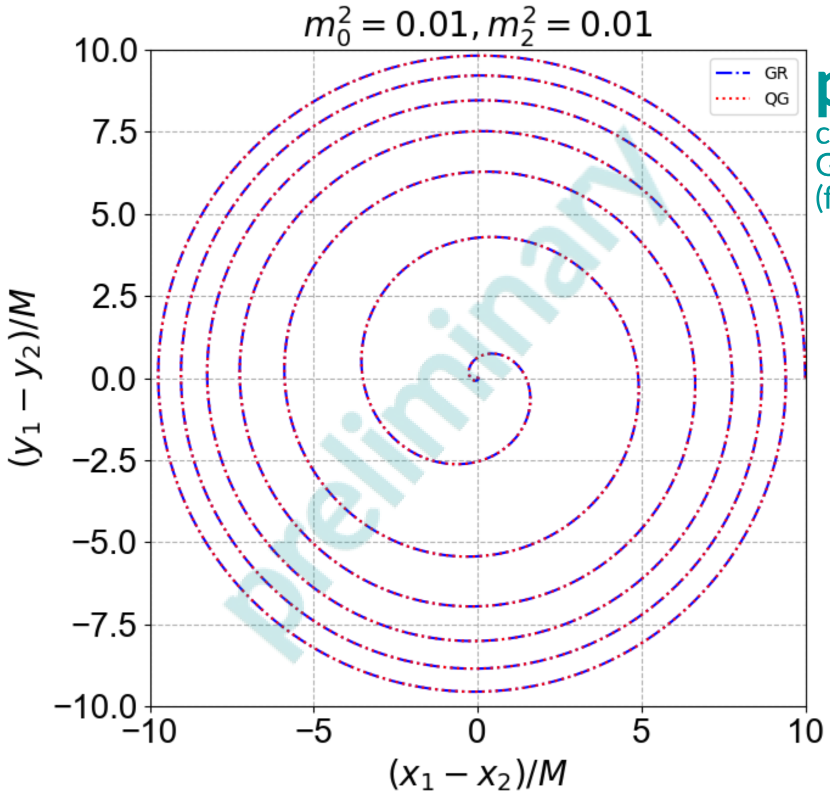
Numerical stability ...



Held, Lim (to appear)

... in **(3+1)** dimensions

Physical stability ...



preliminary

comparison to GW150914 initial data (from EinsteinToolkit)

Held, Lim (to appear)

... in **(3+1)** dimensions

Where to go from here ...

in spherical symmetry

- non-linear stability of black hole branches
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- stable Ricci-flat sector?
- onset of GL instability in nonlinear dynamics
- **gravitational waveforms**

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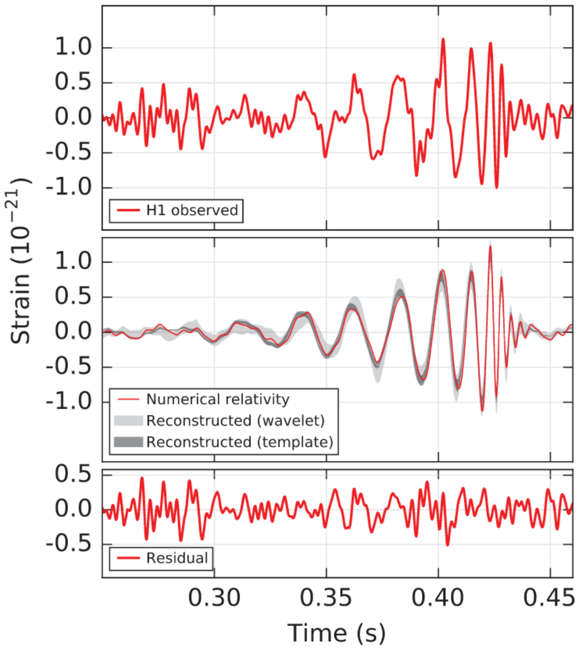
LIGO, PRL 116, 061102



???
waveform in Quadratic Gravity
???

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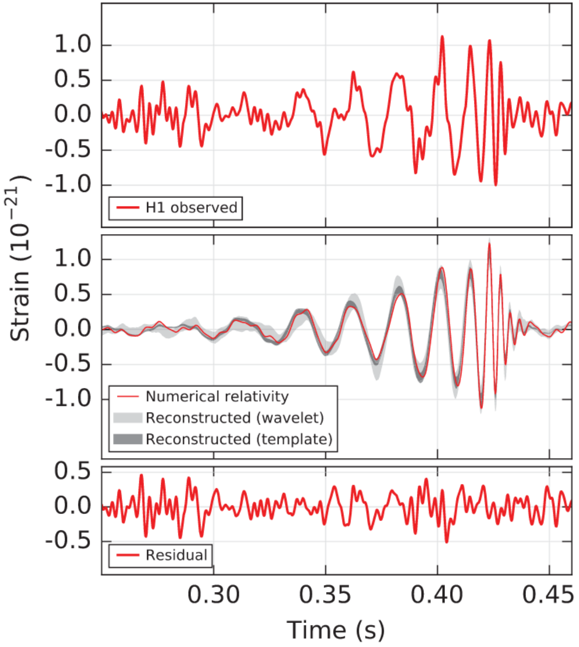

??? waveform in Quadratic Gravity ???

probe the EFT of gravity

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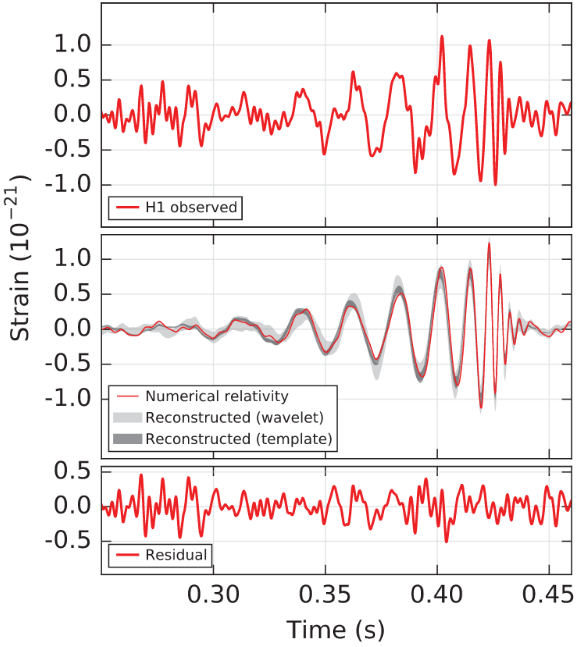
LIGO, PRL 116, 061102



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... thank you and stay tuned!