Cosmological aspects of non-local infinite derivative gravity theories

Alexey Koshelev

Universidade da Beira Interior, Covilhã, Portugal

December 14, 2022

Mainly based on recent papers
with Sravan Kumar, Alexei Starobinsky, Anna Tokareva,

and works in progress

Introduction

Grand Problem – quantizing gravity

- In an attempt to understand how to modify the Einstein's gravity which is not UV-complete we turn to more fundamental approaches

Strings

- Strings and especially string field theory should in principle give a chance to understand quantum gravity

Strings feature non-local interactions in the form of higher- and even infinite-derivative form factors

- Aref'eva, Barvinskiy, Biswas, Koivisto, Krasnikov, Kuz'min, Mazumdar, Modesto, Sen, Siegel, Shapiro, Tomboulis, Witten, Zwiebach, ...

Physical excitations

Suppose we modify the propagators as follows

$$\Box - m^2 \to \mathcal{G}(\Box)$$

Recall, in $D = 4$ in $(- + ++)$

$$L = \frac{1}{2}\phi(\Box - m^2)\phi - \text{good field}$$

$-\Box$ gives a ghost, $+m^2$ gives a tachyon (for real m).

Consider

$$L = \frac{1}{2}\phi(\Box - m^2)(\Box - \mu^2)\phi$$

This Lagrangian describes 2 physical excitations and the second one is a ghost. The higher degree polynomial in $\Box$ will just produce more ghosts.

Analytic Infinite Derivative (AID) way around

To preserve the *perturbative* physics we demand

$$\mathcal{G}(\Box) = (\Box - m^2)e^{2\sigma(\Box)}$$

where $\sigma(\Box)$ must be an *entire* function resulting in the fact that the exponent of it has no roots.

Thus

$$L = \frac{1}{2}\phi(\Box - m^2)e^{2\sigma(\Box)}\phi$$

So, yes, we can incorporate infinite number of derivatives by employing properties of entire functions.

Non-local scalar field [arxiv:2103.01945]

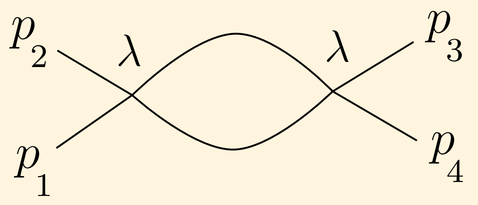
Consider AID scalar field action:

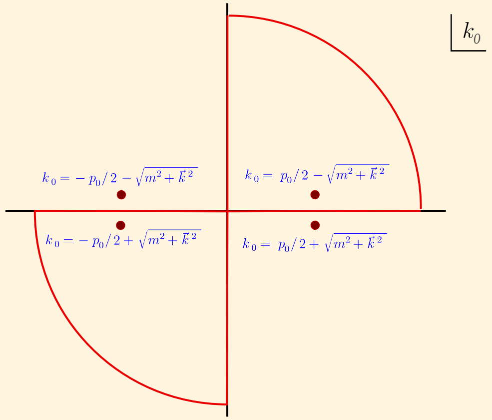
$$L = \frac{1}{2}\phi(\Box - m^2)f^{-1}(\Box)\phi - \frac{\lambda}{4!}\phi^4$$

Again, we demand the form-factor to be an exponent of an entire function. We also normalize it as $f(0) = f(m^2) = 1$ to preserve the local answers in the IR limit.

We can adjust the fall rate for large momenta by choosing the form-factor. Power-counting convergence requires the fall faster than $\sim 1/p^2$.

Fish graph and one-loop unitarity





As a matter of definition we write amplitudes in Euclidean signature and analytically continue the result to Minkowski values of external momenta. [Pius,Sen,arXiv:1604.01783]

$$\mathcal{M} = -i \frac{\lambda^2}{32\pi^4} I(p)$$

We compute the integral with euclidean internal momentum k and also account for poles shown above.

Result for the fish graph with $f(k^2) = f(-k^2)$

$$\mathcal{M}(p) = -\frac{\lambda^2}{64\pi^3 p}\int_0^\infty J_1(px)J_1(kx)J_1(qx)f(k^2)f(q^2)dkdqdx + i\frac{\lambda^2\pi}{32} + \frac{\lambda^2}{32p^2}\int_{-p^2}^{p^2} f(z)dz$$

If $f(z)$ is an integrable function than the last term gives an apparently universal $\sim 1/p^2$ contribution for any even form-factor.

We can show numerically that the model remains weakly coupled in contrast to $f(p^2) = e^{-\alpha p^2}$

Examples used were $f = e^{-p^4}$ and $f = e^{-\Gamma(0,p^4)-\gamma-\log(p^4)}$

Can we modify GR using AID operators

Interesting development unfortunately leading to a non-unitary theory

- **Stelle's 1977 and 1978 papers show that R^2 gravity is renormalizable gravity with the price of a physical (Weyl) ghost**

Big success

- **Starobinsky inflation is based on R^2 and works perfectly**

The idea is to make use of AID form-factors to exorcise the Weyl ghost

Outline of the general idea

We start with constructing propagator in at least Minkowski background in a generic gravity theory

See manifestly that one cannot avoid an infinite tower of derivatives if a ghost-free theory is expected

How to deal with such theories? SFT would teach

What does it mean for the proposed gravity generalization? Towards quantum gravity

Develop Analytic infinite derivative gravity and see, whether it is observationally viable

Pure gravity arguments why infinite derivatives appear

We start with

$$S = \int d^D x \sqrt{-g} \left(\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right)$$

Here $\mathcal{P}$ and $\mathcal{Q}$ depend on curvatures and $\mathcal{O}$ are operators made of covariant derivatives.

Everywhere the respective dependence is *analytic* in IR.

Let's name it *general analytic gravity*

Question:
formulate an action which would reproduce the linear perturbations of the general analytic gravity around MSS

Action to study: first step

Excluding all the terms which vanish around MSS and massaging others we arrive to the following action [1602.08475, 1606.01250]:

$$S = \int d^Dx\sqrt{-g}\Bigg(\frac{M_P^2 R}{2} - \Lambda$$

$$+\frac{\lambda}{2}\left(R\mathcal{F}_R(\Box)R + L_{\mu\nu}\mathcal{F}_L(\Box)L^{\mu\nu} + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\right)\Bigg)$$

Here $\mathcal{F}_X(\Box) = \sum_{n\geq 0} f_{X_n}\Box^n$ and $L_{\mu\nu} = R_{\mu\nu} - \frac{1}{D}Rg_{\mu\nu}$

Thanks to the Bianchi identities one can further achieve $\mathcal{F}_L(\Box) = 0$ in $D = 4$ and $\mathcal{F}_L(\Box) = \text{const}$ in $D > 4$.

Action to study [1711.08864]

$$S = \int d^4x\sqrt{-g}\Bigg(\frac{M_P^2 R}{2} - \Lambda$$

$$+\frac{\lambda}{2}\Big(R\mathcal{F}_R(\Box)R + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\Big)\Bigg)$$

Recall, that $\mathcal{F}_X(\Box) = \sum_{n\geq 0} f_{X_n}\Box^n$ and we often use $\mathcal{F} \equiv \mathcal{F}_R$

We assume that $\Box$ enters form-factors in a combination $\Box/\mathcal{M}_s^2$ where the mass parameter is the non-locality scale. We put $\mathcal{M}_s = 1$ for a while.

This is the most general action to study linear perturbations around MSS (in 4 dimensions).

We name it Analytic infinte derivative (AID) gravity.

Covariant spin-2 propagator on MSS:

$$S_2 = \frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\, h^{\perp}_{\nu\mu} \left(\bar{\Box} - \frac{\bar{R}}{6}\right) \left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda \mathcal{F}_W \left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

The Stelle's case corresponds to $\mathcal{F}_W = 1$ such that

$$\mathcal{P}(\bar{\Box})_{Stelle} = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda \cdot 1 \cdot \left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

This is an obvious second pole which will be the ghost.

Physical propagators around Minkowski, AID form-factors:

$$\begin{aligned}\mathcal{O}_s =&\frac{(6\lambda\Box\mathcal{F}(\Box) - M_P^2)(2\lambda\Box\mathcal{F}_W(\Box)/M_P^2 + 1)}{6\lambda(\mathcal{F}(\Box) + \frac{1}{3}\mathcal{F}_W(\Box))}\\ =&(\Box - \mu^2)e^{2\sigma_0(\Box)}\end{aligned}$$

$$\mathcal{O}_t = \Box(2\lambda\Box\mathcal{F}_W(\Box)/M_P^2 + 1) = \Box e^{2\sigma(\Box)}$$

Then, avoiding all odds:

$$\mathcal{F}_W(\Box) = M_P^2\frac{e^{2\sigma(\Box)} - 1}{2\lambda\Box}$$

$$\mathcal{F}(\Box) = \frac{M_P^2}{6\lambda\Box}\left[\left(\frac{\Box}{\mu^2} - 1\right)e^{2\sigma(\Box)} + 1\right]$$

Outline of the general idea

We have constructed propagator in the Minkowski background (in fact even on MSS) in a generic gravity theory

We just have seen manifestly that one cannot avoid an infinite tower of derivatives if a ghost-free theory is expected

And, yes, SFT has taught us how to deal with such theories

What does it mean for the proposed gravity generalization? Towards quantum gravity

Develop Analytic infinite derivative gravity and see, whether it is observationally viable

UV completeness [1710.07759]

Minkowski propagator:

$$\Pi = -\left(\frac{P^{(2)}}{k^2 e^{2\sigma(-k^2)}} - \frac{P^{(0)}}{2k^2 e^{2\sigma(-k^2)}\left(1+\frac{k^2}{\mu^2}\right)}\right)$$

To guarantee that the QFT machinery works we arrange a polynomial decay of the propagator near infinity. The rate of the decay is our choice.

Recall that we need the functions σ to be entire.

An example of such a function can be, for instance

$$\sigma \sim \Gamma\left(0, p(z)^2\right) + \gamma_E + \log\left(p(z)^2\right)$$

where $p(z)$ is a polynomial.

Beyond 1-loop the powercounting arguments work just like in the higher derivative regularization.

Amplitudes and Cross-sections

Power-counting works because we have chosen the polynomial decay at infinity

Slavnov-Taylor identities work thanks to the presence of the diffeomorphism invariance

Exponential decay of form-factors may render the system to be in the strong-coupling regime. This way amplitudes become divergent for large external momenta.

The recent works with A.Tokareva have shed light on several questions including dealing with the essential singularity of the form-factor at inffinity and maintaining unitarity of the AID models. (see above)

[2006.06641, 2103.01945]

What else can AID quadratic action serve for?

- If we just start with the above proposed quadratic in curvature action it can accommodate many interesting solutions without requiring any other more general gravity model.
- For example, any conformally flat metric which satisfies $\Box R = r_1 R$ with constant r_1 is a solution.
- In particular, Starobinsky inflation is an exact solution here.
- Solution representing a ghost-free bouncing scenarios also were found.

Starobinsky inflation in non-local gravity

[1604.03127, 1711.08864] and the recent development [2209.02515]

For any scalar curvature satisfying:

$$\Box R = r_1 R + r_2$$

with $r_1,\ r_2$ constants we have a solution of AID gravity if:

$$\mathcal{F}^{(1)}(r_1) = 0,\ \frac{r_2}{r_1}(\mathcal{F}_1 - f_0) = -\frac{M_P^2}{2\lambda} + 3r_1\mathcal{F}_1,\ \mathcal{F}_1 \equiv \mathcal{F}(r_1),$$
$$4\Lambda r_1 = -r_2 M_P^2,\ \text{but for us } \Lambda = 0 \Rightarrow r_2 = 0$$

In a wide range of assumptions it exhuasts all the space of solution.

Notice that the Weyl part does not contribute to the background because the configuration of interest is conformally flat.

Tensor to scalar ratio r

$$r = \frac{2|\delta_h|^2}{|\delta_{\mathcal{R}}|^2} = 48\frac{\dot{H}^2}{H^4}e^{2\sigma(\bar{R}/6)}$$

All quantities here are at the horizon crossing $k = Ha$.

Analogously

$$N = \int_{t_i}^{t_f} H dt = \frac{1}{2\epsilon_1} \Rightarrow r = 48\epsilon_1^2 e^{2\sigma(\bar{R}/6)} = \frac{12}{N^2}e^{2\sigma(\bar{R}/6)}$$

We have gained an extra factor $e^{2\sigma(\bar{R}/6)}$ compared to the local R^2 inflation.

Non-Gaussianities, briefly

[2003.00629] and the recent development [2210.16459]

- f_{NL} *can* be made large.
- The consistency relation known to hold for a single scalar field model of inflation is violated here.
- We however have constraints on this number from observations and this crucially shows up in our AID gravity model as the constraint on the scale of non-locality which we denote $\mathcal{M}_s$
- Namely

$$\mathcal{M}_s > 10^{-4} M_P$$

- Moreover, it can be either sign of f_{NL} based on the lowest power of polynomial of our entire function, i.e. whether it is odd or even.

Conclusions and Outlook

- **A class of analytic infinite derivative (AID) theories has been considered targeting the goal of constructing a UV complete and unitary gravity. These models have clear connection with SFT.**
- **We have shown that there is a consistent prescription to maintain unitarity.**
- **This gravity model features many nice properties, like native embedding of the Starobinsky inflation, finite Newtonian potential at the origin, presence of a non-singular bounce, etc.**
- **Corrections to the inflationary observables are explicitly computed in terms of model parameters**
- **We provide arguments in favor of this approach as the quantum gravity candidate.**

Thank you for listening!