The rise and fall of the black hole chemistry: what happens when we vary Lambda?

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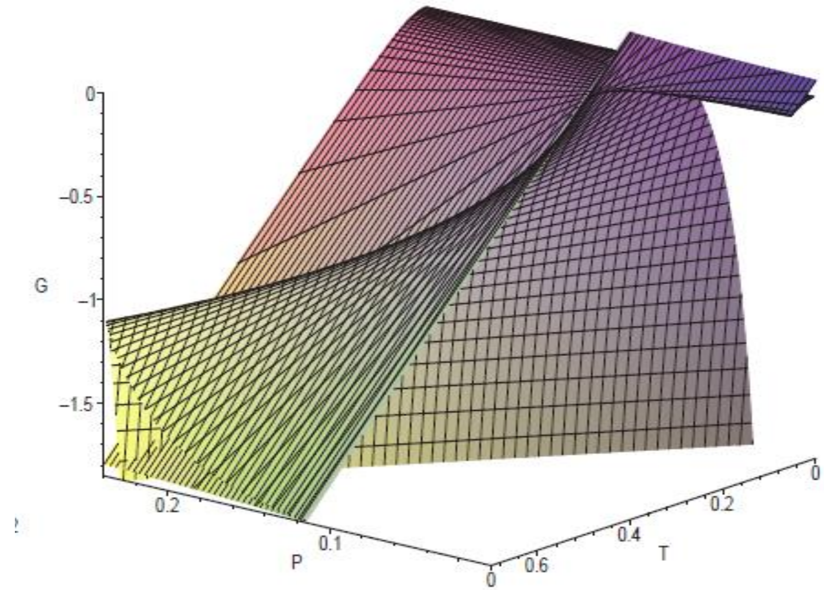


2022 Winter CAS-JSPS Workshop in Cosmology, Gravitation, and Particle Physics
CEICO, Prague, Czech Republic
December 12-17, 2022

Plan of the talk

I. Black holes as thermodynamic objects

II. The rise of black hole chemistry

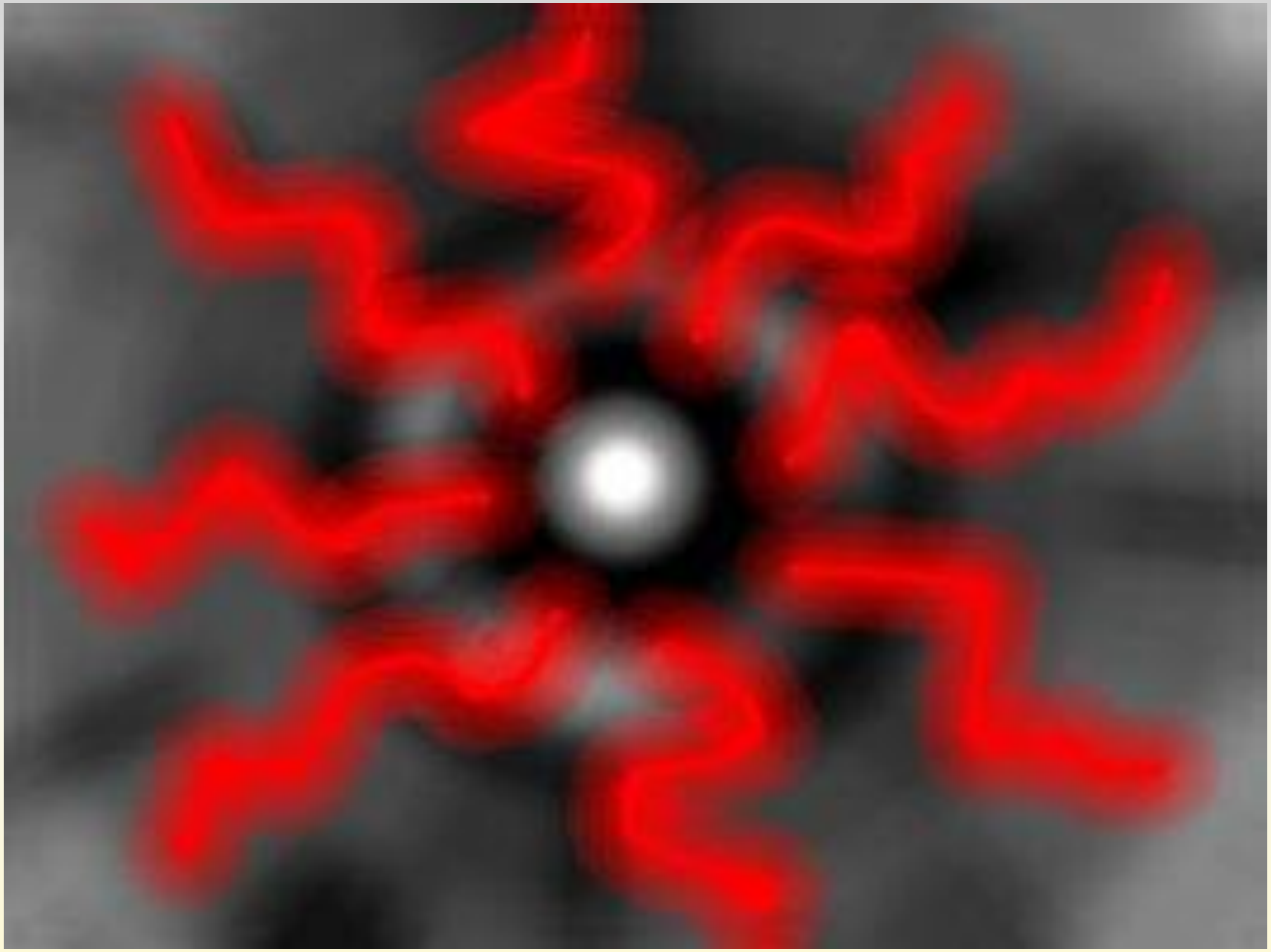
III. Extended bulk-boundary correspondence
(the fall of black hole chemistry)

IV. Summary

Based on:

- DK, R.B. Mann, *P-V criticality of charged AdS black holes*, JHEP 07 (2012) 033; ArXiv:1205:0559.
- W. Cong, DK, R.B. Mann, *Thermodynamics of AdS black holes: central charge criticality*, PRL 127 (2021) 9, 091301; Arxiv:2105.02223.
- W.Cong, DK, R.B. Mann, M.R. Visser, *Holographic CFT phase transitions and criticality of charged AdS black holes*, JHEP 08 (2022) 174.; Arxiv: 2112:14848.

1) Black Holes as Thermodynamic Objects



Black holes as thermodynamic objects

If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations-then so much the worse for Maxwell's equations. If it is found to be contradicted by observation-well these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

Sir Arthur Stanley Eddington

Gifford Lectures (1927), *The Nature of the Physical World* (1928), 74.

Black holes and their characteristics

Schwarzschild black hole:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2$$

- asymptotic mass (total energy)

$$M = -\frac{1}{8\pi}\int_{S_\infty} *dk\,, \quad k^a = (\partial_t)^a$$

- black hole horizon: (radius rh=2M)

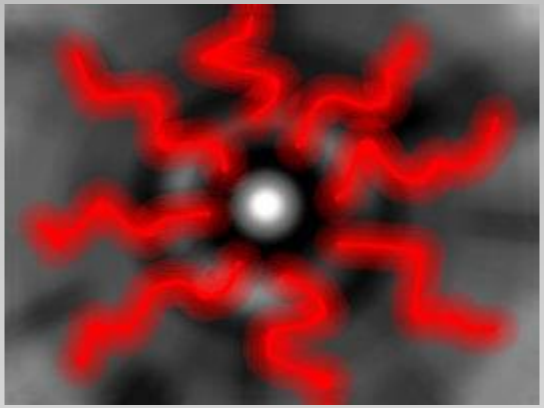
surface gravity $$(k^b \nabla_b k^a)_{|H} = \kappa k^a_{|H}$$ ⇒ $$\kappa = \frac{1}{4M}$$

surface area $$A = 4\pi r_h^2$$ never decreases

$$dM = \kappa dA$$ ⇔ Bekenstein? $$dE = TdS$$

Hawking (1974):

$$T = \frac{\kappa}{2\pi}\,, \qquad S = \frac{A}{4}$$



derivation used QFT in curved spacetime

Other approaches:

- Euclidean path integral approach (Gibbons & Hawking-1977)

$$Z = \int D[g_{ab}] e^{-S_E[g]} \approx e^{-S_E[g_c]}$$

$$F = -\frac{1}{\beta}\log Z \quad \Rightarrow \quad S = -\frac{\partial F}{\partial T} = \frac{A}{4}$$

Euclidean manifold non-singular if the imaginary time τ identified with a certain period Δτ. In QFT this corresponds to a finite temperature

$$T = \frac{1}{\beta}\,, \qquad \beta = \Delta\tau$$

- Tunneling approach, LQG, String theory, ….

Black hole thermodynamics

- First law of black hole thermodynamics:

$$\delta M = T\delta S + \sum_i \Omega_i \delta J_i + \Phi \delta Q$$

- Smarr-Gibbs-Duhem relation:

$$\frac{d-3}{d-2} M = TS + \sum_i \Omega_i J_i + \frac{d-3}{d-2} \Phi Q$$

- Specific heat of AF Schwarzschild BH is negative (cannot have thermal equilibrium)

Where is the PdV term?

2) The rise of black hole chemistry

Black hole chemistry

Simple idea:

• Consider an asymptotically **AdS black hole spacetime**

• Identify the cosmological constant with a thermodynamic pressure

$$P = -\frac{\Lambda}{8\pi G}\,, \quad \Lambda = -\frac{(D-1)(D-2)}{2l^2}$$

• Allow this to be a "dynamical" quantity

(Teitelboim and Brown – 1980's)

Immediate consequences

- **Extended black hole thermodynamics:**

D.Kastor, S.Ray, and J.Traschen, *Enthalpy and the Mechanics of AdS Black Holes*, Class. Quant. Grav. 26 (2009) 195011.

$$\delta M = T \delta S + \Theta \, \delta P + \ldots$$

• Introduces the standard -**PdV term** into black hole thermodynamics

• Black hole mass M no longer identified with energy but rather interpreted as **enthalpy**

$$U = M + \epsilon V = M - PV$$

Immediate consequences

- **Black hole volume:**

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,\ldots}$$

Schwarzschild(-AdS):

$$\Rightarrow \quad V = \frac{4}{3}\pi r_+^3$$

- More involved for more complicated black holes

- The fact this this provides a good definition of volume is supported by the **Reverse Isoperimetric Inequality** conjecture:

M. Cvetic, G.W Gibbons, DK, C.N. Pope, *Black hole enthalpy and an entropy inequality for the thermodynamic volume,* Phys. Rev. D84 (2011) 024037, [arXiv:1012.2888].

Immediate consequences

- **Consistent Smarr relation:**

$$\delta M = T\delta S + V\delta P + \phi\delta Q + \Omega\delta J\,,$$
$$M = \frac{D-2}{D-3}(TS + \Omega J) + \phi Q - \frac{2}{D-3}PV$$

- **Phase transitions:**

  - AdS black holes can be in **thermal equilibrium**
  - Exhibit interesting **phase transitions**
  - Provide dual description of CFT at finite temperature via **AdS/CFT correspondence**

Canonical Example: VdW behavior of charged AdS black holes

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_2^2\,, \quad A = -\frac{Q}{r}dt$$

$$f = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} + \frac{r^2}{l^2}\,,$$

- Basic thermodynamic quantities:

$$M = \frac{r_+(l^2 + r_+^2)}{2l^2G} + \frac{Q^2}{2r_+}\,, \quad T = \frac{3r_+^4 + l^2r_+^2 - GQ^2l^2}{4\pi l^2 r_+^3}$$

$$S = \frac{\pi r_+^2}{G}\,, \quad V = \frac{4\pi r_+^3}{3}\,, \quad \phi = \frac{Q}{r_+}\,,$$

$$\boxed{F = M - TS = \frac{3GQ^2l^2 + l^2r_+^2 - r_+^4}{4Gr_+l^2}}$$

P-V criticality

- DK, R.B. Mann, *P-V criticality of charged AdS black holes*, JHEP 1207 (2012) 033.

Van der Waals fluid

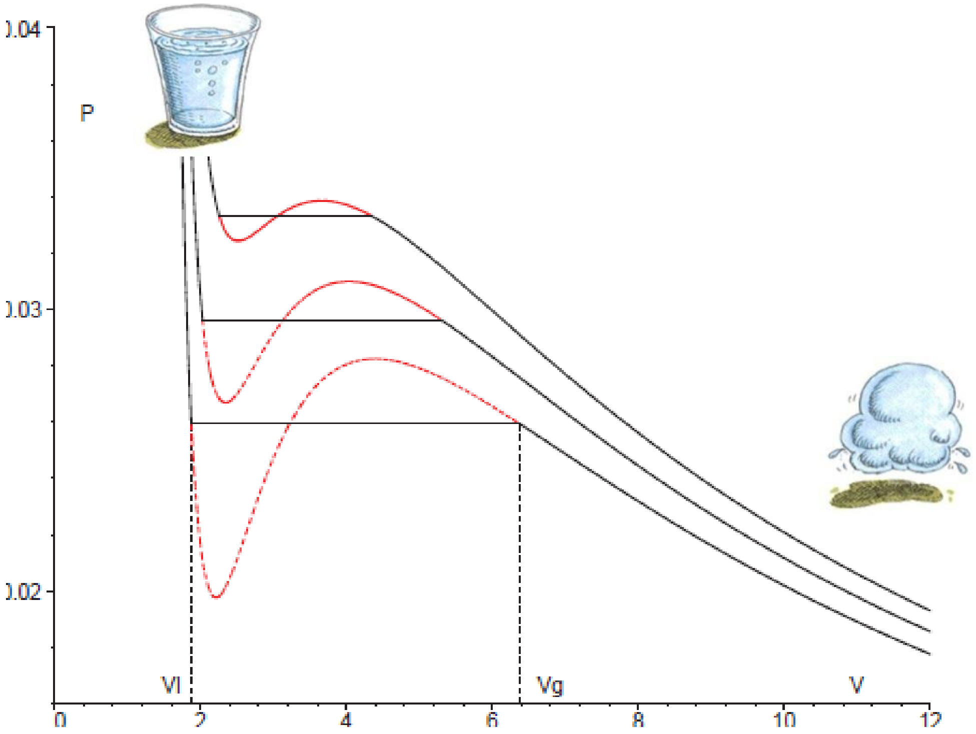


FIG. 2. Maxwell's equal area law. The 'oscillating' (dashed) part of the isotherm $T < T_c$ is replaced by an isobar, such that the areas above and below the isobar are equal one another.

$$\left(P + \frac{a}{v^2}\right)(v - b) = T$$

Parameter a measures the **attraction** between particles (a>0) and b corresponds to "**volume of fluid particles**".

Critical point:

$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}$$

P-V criticality

- DK, R.B. Mann, *P-V criticality of charged AdS black holes*, JHEP 1207 (2012) 033.

Charged black hole

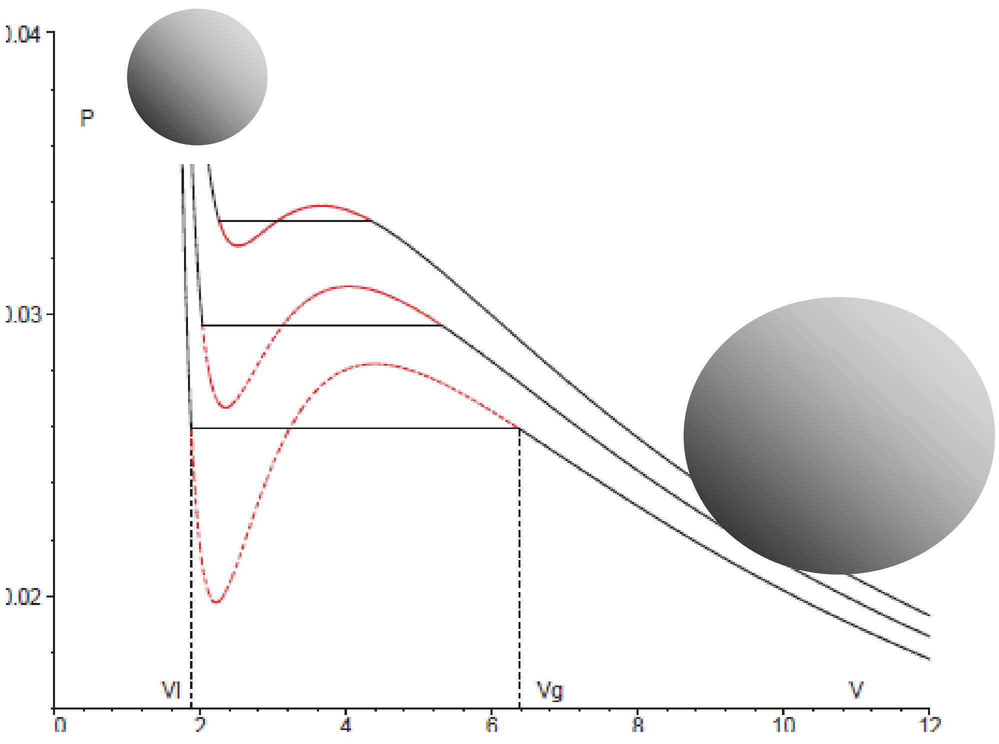


FIG. 2. Maxwell's equal area law. The 'oscillating' (dashed) part of the isotherm $T < T_c$ is replaced by an isobar, such that the areas above and below the isobar are equal one another.

$$\left(P + \frac{a}{v^2}\right)(v - b) = T$$

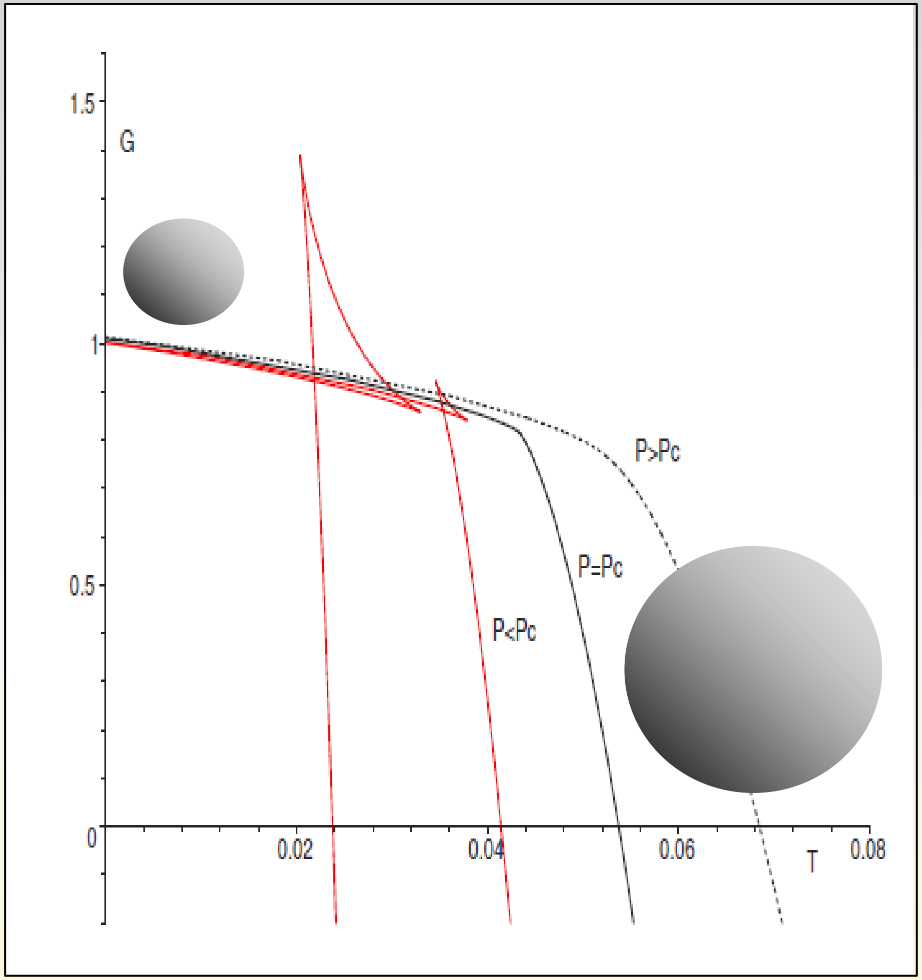
$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}$$

Critical point:

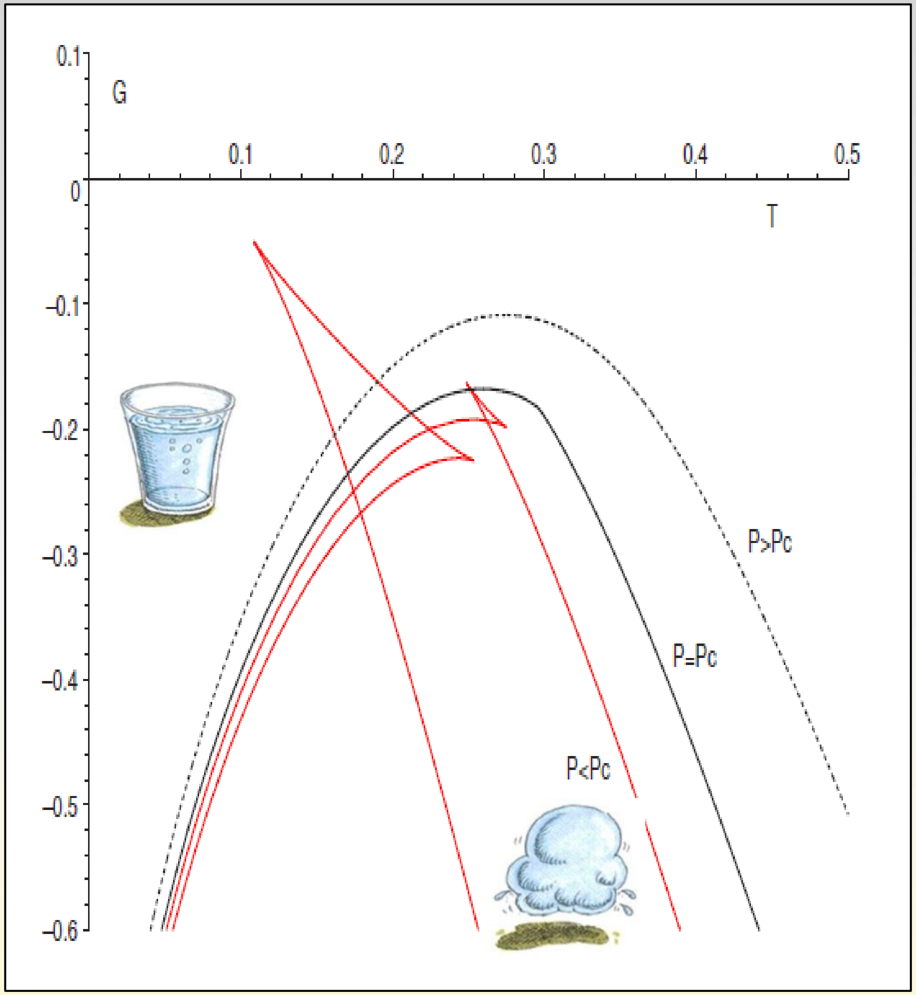
$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}$$

Free energy: demonstrates standard swallow tail behavior

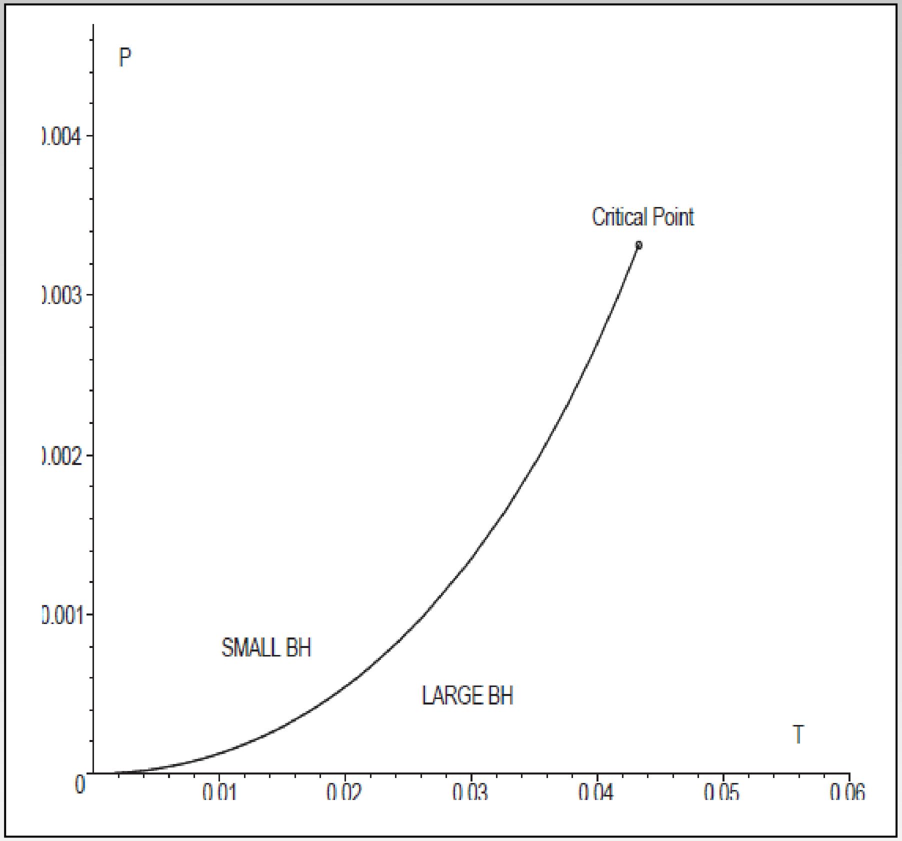
$$F = F(T,P,Q) = \frac{1}{4}\left(r_+ - \frac{8\pi}{3}Pr_+^3 + \frac{3Q^2}{r_+}\right)$$



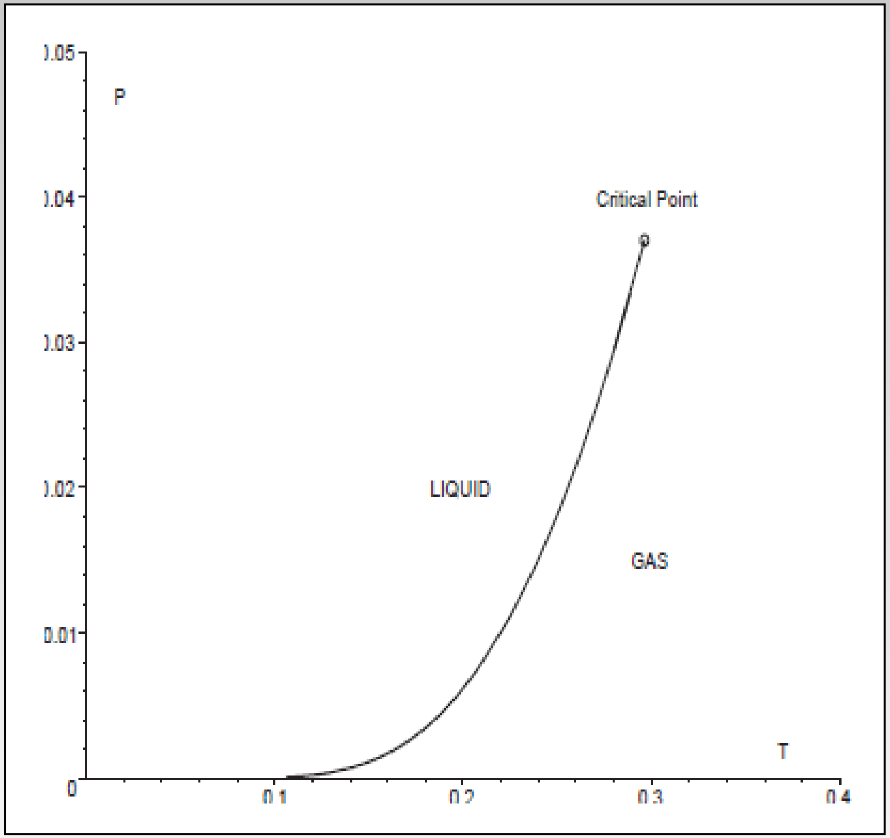
vs.



Phase diagrams: complete analogy



vs.



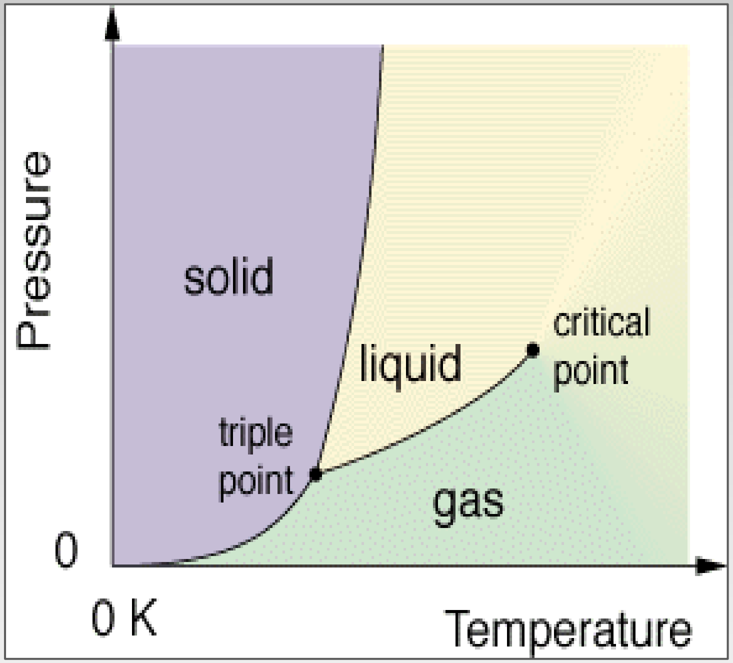
- Coexistence & critical point described by **Clausius-Clapeyron** and **Ehrenfest** equations

- **MFT critical exponents**

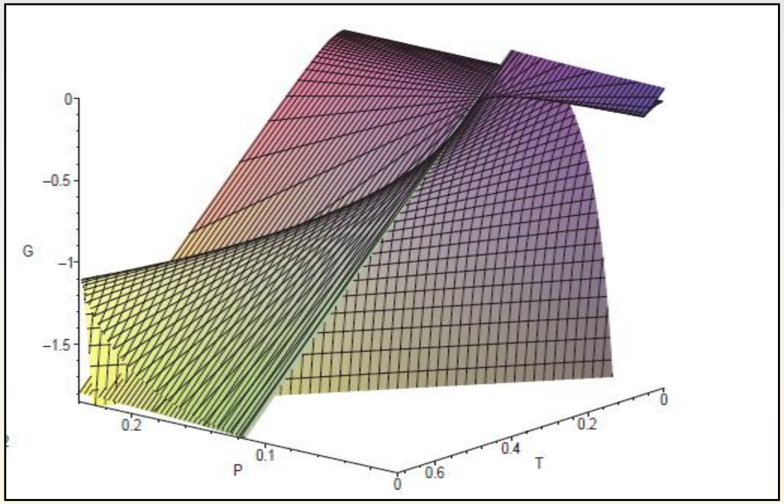
$$\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$$

More generally: black hole chemistry

- **Triple point** and solid/liquid/gas analogue**:**



- **Many other**:
  - Isolated critical point
  - Reentrant PT
  - superfluid PT



- DK, Mann, Teo, *Black hole chemistry: thermodynamics with Lambda*, CQG 34 (2017) 063001, Arxiv:1608.0614.

Moral

- We have a very **rich structure** of phase transitions

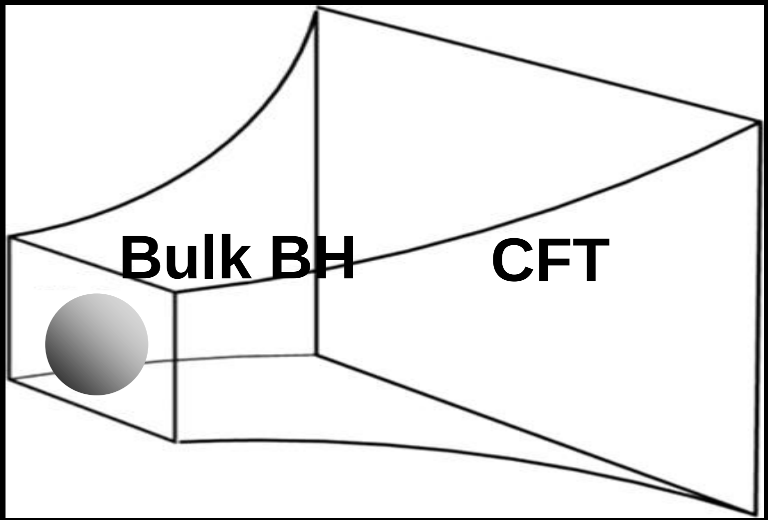
- **Thermodynamic pressure P** (cosmological constant) plays a role of "**control parameter**"

- But what is the **interpretation on the dual CFT side**?

$$+V\delta P$$

https://www.thoughtco.com

3) Extended bulk-boundary correspondence (the fall of black hole chemistry)



AdS/CFT interpretation

- Interpretation on boundary field theory

Varying Λ corresponds to **varying number of dof N**

$$l \sim N$$

e.g. n=4 SU(N) SYM

$$l \propto N^{\frac{1}{4}}$$

$$V\delta P \leftrightarrow \mu\delta N$$

$$C \propto N^2$$

- C. Johnson, Holographic Heat engines, CQG 31 (2014) 205002, Arxiv:1404:5982.
- B. Dolan, Bose condensation and branes, JHEP 10 (2014) 179, ArXiv:1406.7267.
- D.Kastor, S.Ray, J.Traschen, Chemical potential in the frst law of holographic entanglement entropy, Arxiv:1409.3521.

AdS/CFT interpretation

- However, this is not the full story, as the **CFT volume** in principle also changes

- A. Karch and B. Robinson, *Holographic black hole chemistry*, JHEP 12 (2015 073, Arxiv:1510.0247.
- M. Visser*, Holographic thermodynamics requires a chemical potential for color*, Axiv:2101.04145.

$$ds^2 = -\frac{r^2}{L^2}dt^2 + \frac{L^2}{r^2}dr^2 + r^2 d\Omega_{d-1}^2$$

$$\lambda = R/r$$

$$ds^2 = -\frac{R^2}{L^2}dt^2 + R^2 d\Omega_{d-1}^2$$

AdS/CFT interpretation

- **Standardly,** one choses

$$R = L \quad \Rightarrow \quad \mathcal{V} = \Omega_{d-1} L^{d-1}$$

and the volume also changes with variations of L.

- **The second** (quite natural from CFT side) **possibility** is to keep R general.

  In this case, variations of L are independent of variations of the CFT volume. However, what do the variations of R correspond to in the bulk?

- In any case, we have the following CFT TDs:

Holographic first law (Visser 21)

$$\delta E = T\delta S - p d\mathcal{V} + \tilde{\phi}\delta\tilde{Q} + \Omega\delta J + \tilde{\mu}\delta C$$

- Here, E is the **internal energy** not **enthalpy**!
- Allows to study μ-C criticality

- **Accompanied by** $E = (D-2)p\mathcal{V}$

$$E = TS + \tilde{\phi}\tilde{Q} + \Omega J + \tilde{\mu}C$$

- The first equality - **equation of state** - comes from the dimensional analysis

- The second equality – **holographic Smarr** – comes from "central charge extensivity" (note that it does not have D-dependent factors!)

Corresponding bulk first law

- Holographic dictionary

$$C = \frac{\Omega_{d-1} L^{d-1}}{16\pi G_N}$$ (Einstein gravity) $$\tilde{Q} = QL\,.$$

$$S = \frac{A}{4G_N}\,, \quad E = M\frac{L}{R}\,, \quad T = \frac{\kappa}{2\pi}\frac{L}{R} \quad \tilde{\Phi} = \frac{\Phi}{L}\frac{L}{R}\,,$$

- Yields the following **bulk first law**:

$$dM = \frac{\kappa}{8\pi G_N} dA + \Phi dQ + \frac{\Theta}{8\pi G_N} d\Lambda - (M - \Phi Q)\frac{dG_N}{G_N}$$

  - Reduces to the previous when G fixed
  - Note also, that one of dG or dΛ is not "required" if R also varied.

Bulk first law with "mixed variables"

- Starting with the above **bulk first law**:

$$\delta M = \frac{\kappa}{8\pi G}\delta A + \Omega\delta J + \phi\delta Q - \frac{V}{8\pi G}\delta\Lambda - \alpha\frac{\delta G}{G}$$

- We can change variables and obtain a **Mixed first law**:

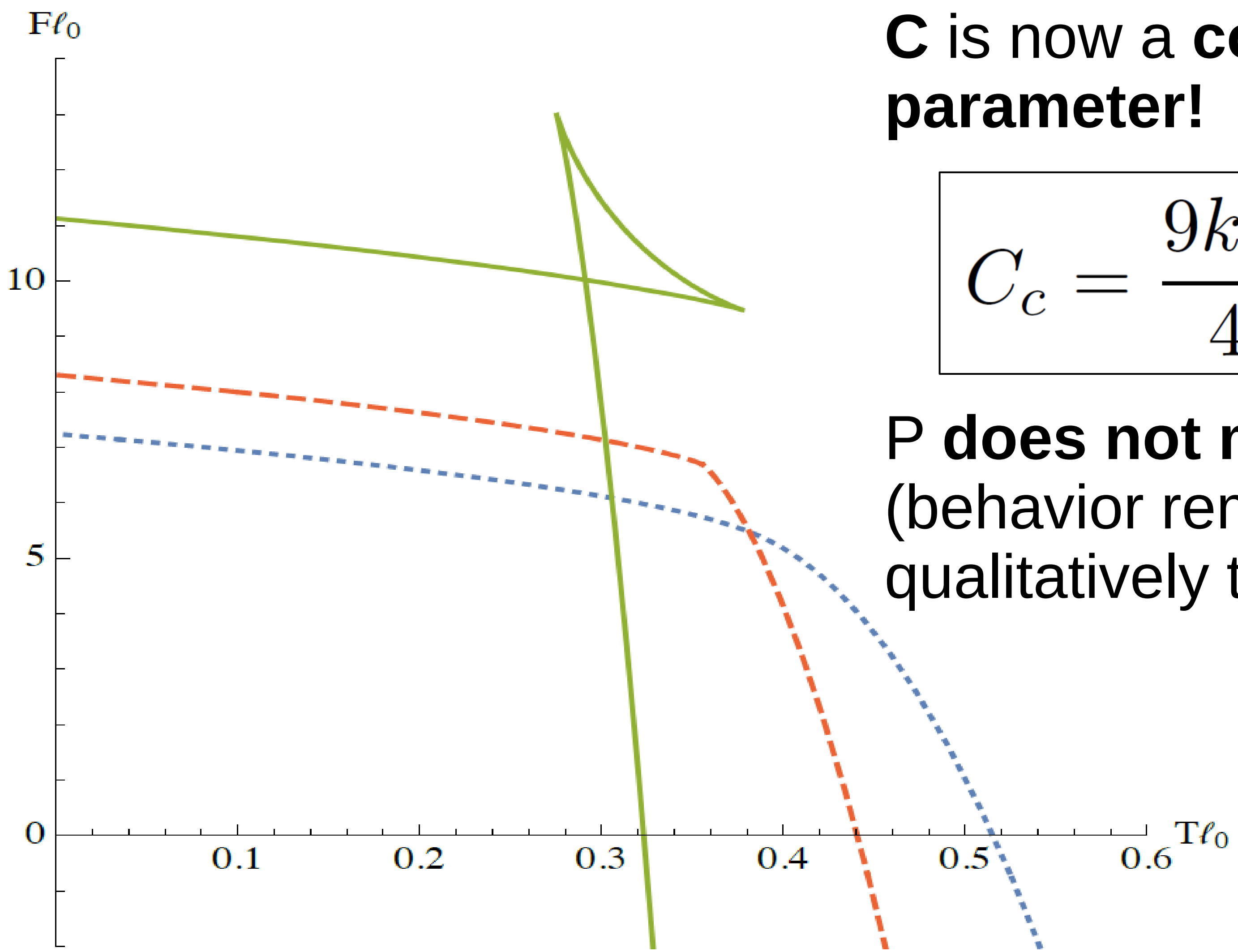
$$C = k\frac{l^{D-2}}{16\pi G} \quad \Rightarrow \quad \frac{\delta G}{G} = -\frac{2}{D}\frac{\delta C}{C} - \frac{D-2}{D}\frac{\delta P}{P}$$

$$\delta M = T\delta S + \Omega\delta J + \phi\delta Q + V_C\delta P + \mu\delta C$$

- Enables one to study **bulk** TDs at **fixed central charge C** (while both G and l vary)

Bulk mu-C criticality

$$F = M - TS = F(T, P, Q, C)$$

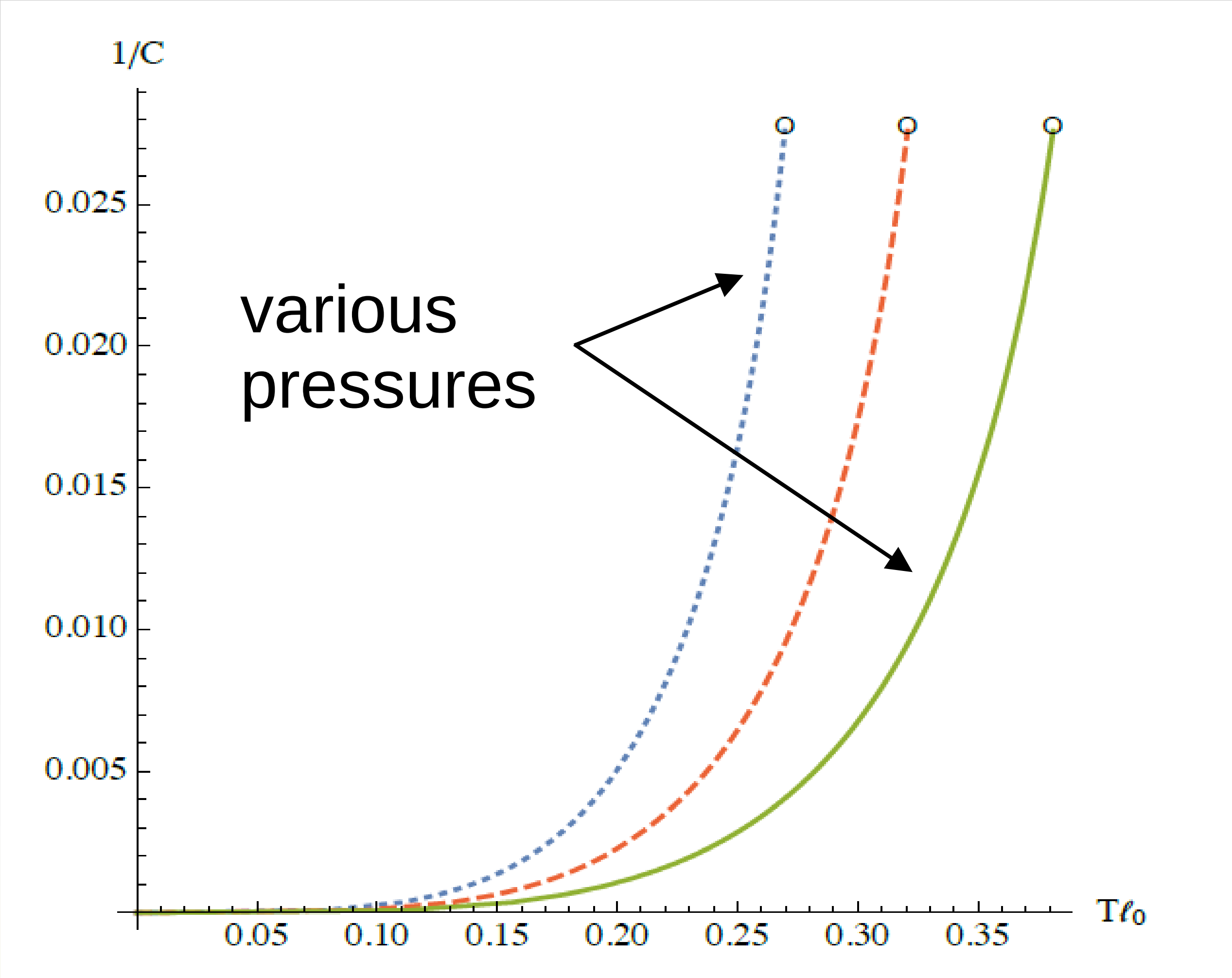


C is now a **control parameter!**

$$C_c = \frac{9kQ^2}{4\pi}$$

P **does not matter**! (behavior remains qualitatively the same)

Phase diagram



Only when CFT has **large number of dof**, C>Cc, the bulk black hole can experience a phase transition. Thus, **BH chemistry interpretation seems lost!**

Summary

1) **Black hole chemistry** (TDs with variable Lambda) provides an interesting framework for AdS black hole thermodynamics:

$$\delta M = T\delta S + V\delta P + \phi\delta Q + \Omega\delta J$$

- **Extended first law** and consistent Smarr
- Black hole mass is **enthalpy**
- Definition for **black hole volume**
- Uncovers various **phase transitions** & similarities with TDs of ordinary systems:

e.g. **Van der Waals criticality** of charged BHs

2) On CFT side, **varying Lambda** corresponds to both – varying the **central charge C** and varying the **CFT volume**.

$$\delta\Lambda \Longleftrightarrow \delta C \quad \& \quad \delta\mathcal{V}$$

Summary

3) The full **CFT first law**:

$$\delta E = T\delta S - p d\mathcal{V} + \tilde{\phi}\delta\tilde{Q} + \Omega\delta J + \tilde{\mu}\delta C$$

standardly translates to **varying both** G and Λ in the bulk!

- However, by considering CFT on a sphere of **radius R**, it is possible to vary only one of these (e.g. Λ)

- This then corresponds to the "**standard black hole chemistry** with its nice bulk interpretation, and its "**old" holographic interpretation** with variable central charge.

Summary

4) For the bulk thermodynamics (with variable G and Λ) we can write the following **mixed law**:

$$\delta M = T\delta S + \Omega\delta J + \phi\delta Q + V_C\delta P + \mu\delta C$$

This allows to study TDs of bulk black holes for fixed **CFT central charge C** (kind of TD ensemble).

Interestingly, thermodynamic pressure P is no longer a "control parameter" – **criticality only depends** on the **number of dof** of the dual CFT.

The nice interpretation of **black hole chemistry seems to fall** in this case**!**