Interaction between cosmic strings

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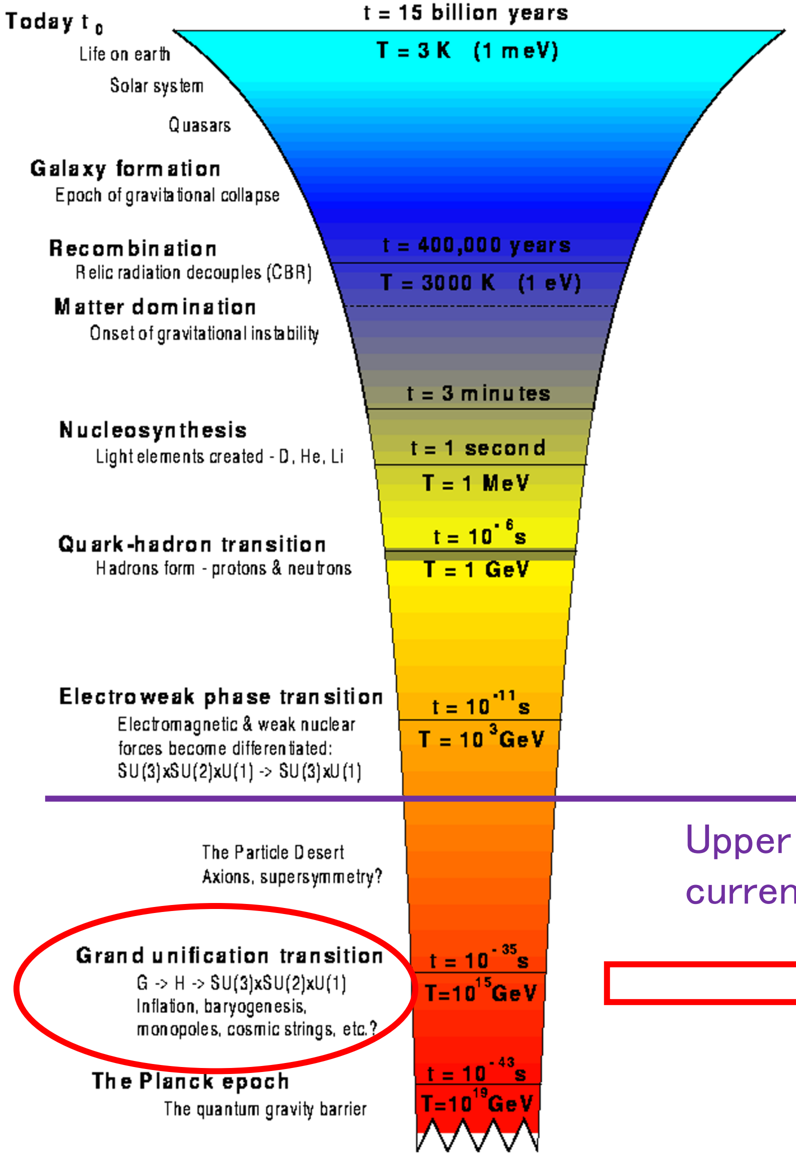
Contents

- Introduction
- Apply source method* to derive interaction energy of two strings for:
  - Bosonic Superconducting Cosmic Strings
  - Global cosmic string

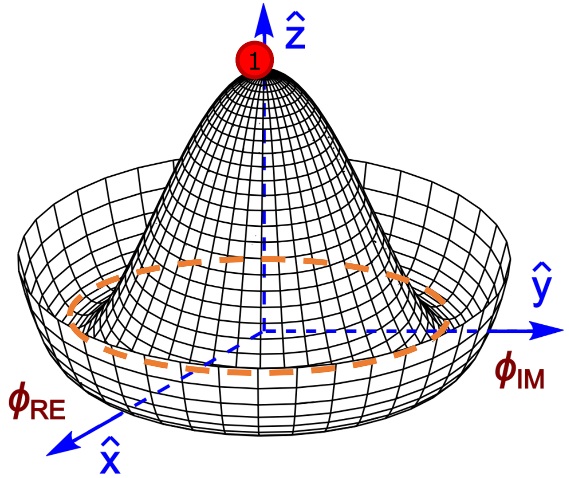
  *originated by J. M. Speight in Phys. Rev. D 55, 3830 (1997) for abelian-Higgs model
- Numerical results for two-string system
- Summary

Introduction

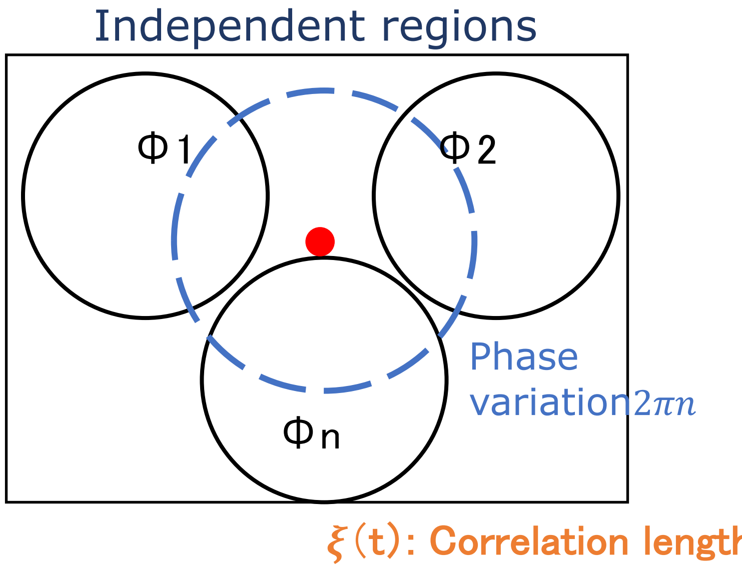


Upper limit of current experiment

Natural high energy experiment lab



Spontaneous symmetry breaking during phase transition

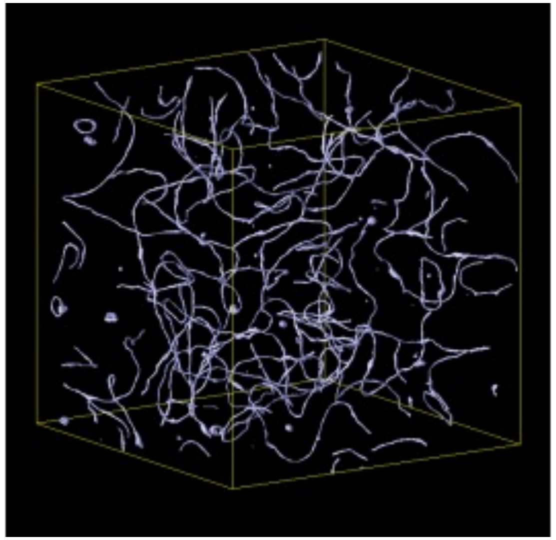


Cosmic strings (topological defects)

Introduction

Purpose of this work:
to investigate interaction between two cosmic strings

Cosmic strings network

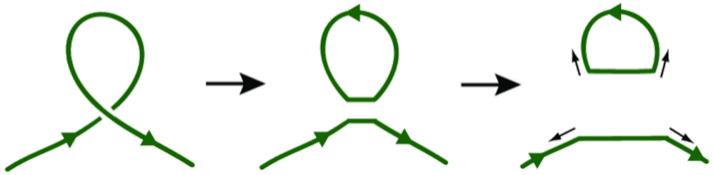


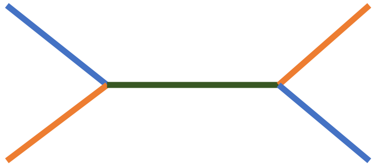
[Takashi.H et al. 2013]

interactions, dynamics...

reconnect, collapse···

kinks, cusps, Y-junctions





Y-junction

stochastic gravitational wave

+

gravitational wave burst

Local string models

Abelian-Higgs model

$$\mathcal{L}_{AH} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |D_\mu\phi|^2 - V(\phi),$$

$$V(\phi) = \frac{1}{4}\lambda(|\phi|^2 - \eta^2)^2$$

$U(1)$ gauge symmetry

Bosonic superconducting model

$$\mathcal{L}_{BC} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |D_\mu\phi|^2 - \frac{1}{4}\widetilde{F^{\mu\nu}}\widetilde{F_{\mu\nu}} + |\widetilde{D_\mu}\tilde{\phi}|^2 - V(\phi,\tilde{\phi}),$$

$$V(\phi,\tilde{\phi}) = \frac{1}{4}\lambda_\phi(|\phi|^2 - {\eta_\phi}^2)^2 + \frac{1}{4}\lambda_{\tilde{\phi}}\left(|\tilde{\phi}|^2 - {\eta_{\tilde{\phi}}}^2\right)^2 + \beta|\phi|^2|\tilde{\phi}|^2$$

[Witten, Nuclear Physics B, 1985]

static, straight, circular symmetric

Local string solution:

$$\phi = |\phi(r)|e^{in\theta},$$

$$A_\mu = A_\mu(r)\delta^\mu_\theta$$

in cylindrical coordinate system (r, θ, z)

[Nielsen, Olesen P. Nuclear Physics B, 1973]

$U(1)\times\tilde{U}(1)$ gauge symmetry

- $U(1)$ symmetry to be spontaneously broken
  ⇩
  giving rise to ANO string
- $\tilde{U}(1)$ symmetry to be unbroken outside the string
  ⇩
  making the string carry persistent current

Bosonic superconducting strings

Parameter space:

- $\tilde{U}(1)$ symmetry unbroken outside string

$$m_{\tilde{\phi}}^2(\infty) = \beta\eta_{\phi}^2 - \frac{1}{2}\lambda_{\tilde{\phi}}\eta_{\tilde{\phi}}^2 > 0$$

- $|\phi| = \eta_\phi$, $|\tilde{\phi}| = 0$ should be global minimum

$$\lambda_\phi\eta_\phi^4 > \lambda_{\tilde{\phi}}\eta_{\tilde{\phi}}^4$$

- To make $|\tilde{\phi}| \neq 0$ energy favorable rather than trivial solution $|\tilde{\phi}| = 0$

(existence of negative energy state)

$$\beta < \lambda_{\tilde{\phi}}\eta_{\tilde{\phi}}^2/\eta_\phi^2$$

static, straight, circular symmetric

Note the lowest energy solutions as $\tilde{\phi} = \tilde{\phi}_r(r), \widetilde{A_\mu} = 0$.

Then a general ansatz is

[Alford M, Benson K, Coleman S, et al. Nuclear Physics B, 1991]

$$\tilde{\phi} = \tilde{\phi}_r(r), \widetilde{A_\mu} = \frac{1}{g}s(r)\partial_\mu\alpha(z).$$

$$r\partial_r s\partial_z^2\alpha = 0$$

$$\frac{1}{r}\partial_r(r\partial_r)s(r) - 2g^2 s(r)\tilde{\phi}_r^2 = 0$$

$\Rightarrow$

$$\alpha(z) = \omega z$$

London equation with penetration depth $\delta_A(r) = 1/g\tilde{\phi}_r(r)$

Superconducting current along the string

$$J_z = \int d^2x[-2g\omega s(r)\tilde{\phi}_r^2]$$

Asymptotic solutions

Boundary conditions:

- Regularity at the origin

$$|\phi| = 0, A_\mu = 0, \partial_r|\widetilde{\phi}| \to 0, \partial_r \tilde{s}(r) \to 0, \quad \text{at } r = 0$$

- Finite energy

$$|\phi| \to \eta_\phi,\ D_\mu\phi \to 0, |\tilde{\phi}| \to 0, \quad \text{at } r \to \infty$$

Asymptotic solutions at large distance:

$$\sigma(r) = k_\phi K_0(m_\phi r)$$

$$\mathrm{U}_\theta(r) = k_e r K_1(m_e r)$$

$$\tilde{\phi}_r(r) = k_{\tilde{\phi}} K_0(m_{\tilde{\phi}} r)$$

$$s(r) = k_s \ln r$$

modified Bessel function $K_i(mx) \propto e^{-mx}$ at $x \to \infty$

$k_\phi \propto |n|$

$k_e \propto -n$

$k_{\tilde{\phi}} \propto |\omega|$

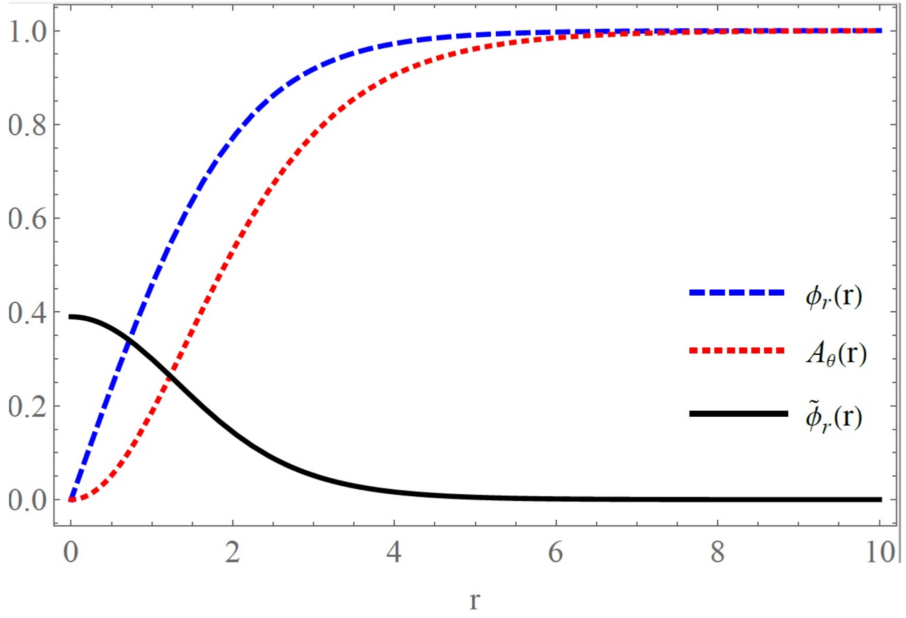
$k_s \propto \omega$

$m_\phi \equiv \sqrt{\lambda}\eta$

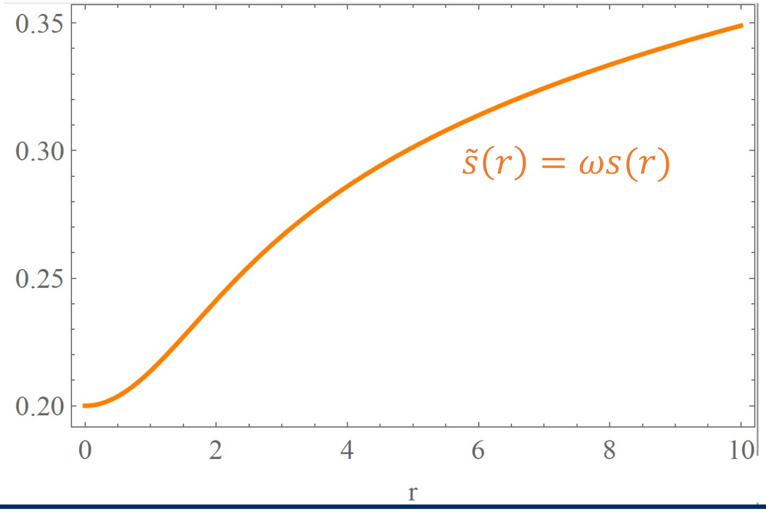
$m_e \equiv \sqrt{2}e\eta$

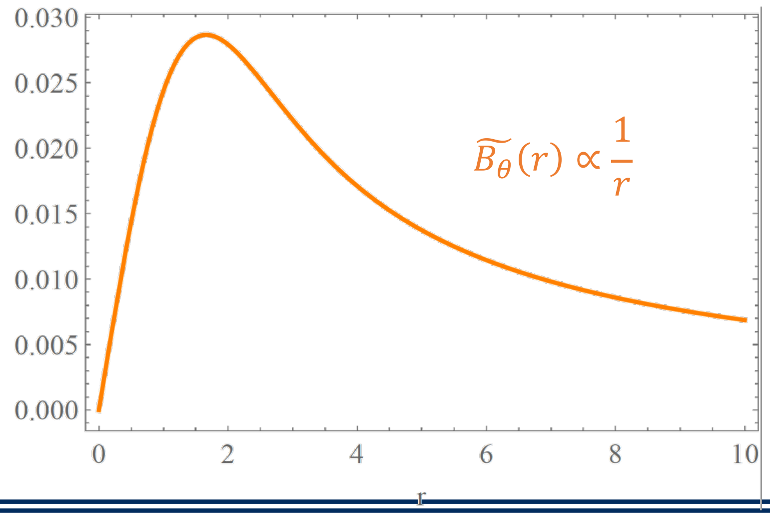
with parameterization

$$\phi(r) = \left(\eta_\phi + \frac{\sigma(r)}{\sqrt{2}}\right)e^{in\theta},\ A_\theta(r) = \mathrm{U}_\theta(r) + \frac{n}{e}$$



Field configurations





Interaction with source method

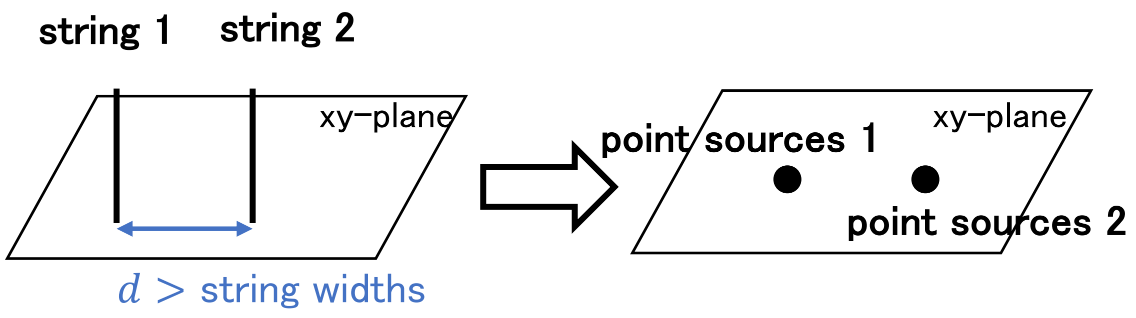
Define the external sources representing the asymptotic fields by linearized field theory,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}m_\phi^2 \sigma^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_e^2 U_\mu U^\mu + J_\sigma \sigma - j_\mu U^\mu$$

$$+\frac{1}{2}(\partial_\mu \tilde{\phi}_r)^2 - \frac{1}{2}m_{\tilde{\phi}}^2 \tilde{\phi}_r^2 - \frac{1}{4}\widetilde{F^{\mu\nu}}\widetilde{F_{\mu\nu}} - J_{\tilde{\phi}}\tilde{\phi}_r - \tilde{j}_\mu \tilde{A}^\mu$$

external sources

parallel, well-separated strings



$$J_\sigma = -2\pi k_\phi \delta^{(2)}(\boldsymbol{x})$$

$$j_\theta = -2\pi \frac{k_e}{m_e}\partial_r \delta^{(2)}(\boldsymbol{x})$$

$$J_{\tilde{\phi}} = -2\pi k_{\tilde{\phi}} \delta^{(2)}(\boldsymbol{x})$$

$$\tilde{j}_\mu = -2\pi \frac{k_s}{g}\delta^{(2)}(\boldsymbol{x})$$

point-like sources in 2-dimensions

$k_\phi \propto |n|$

$k_e \propto -n$

$k_{\tilde{\phi}} \propto |\omega|$

$k_s \propto \omega$

We assume superposition of two sources

$$J_{total}(\boldsymbol{x}) = J_1(\boldsymbol{x} - \boldsymbol{x}_1) + J_2(\boldsymbol{x} - \boldsymbol{x}_2)$$

determined by winding direction of vortex

$$E_{int} = 2\pi \int dz \left[-k_{\phi 1}k_{\phi 2}K_0(m_\phi d) + k_{A1}k_{A2}K_0(m_e d) - k_{\tilde{\phi}1}k_{\tilde{\phi}2}K_0(m_{\tilde{\phi}} d) + k_{\tilde{A}1}k_{\tilde{A}2}\ln d\right]$$

always attractive contribution

determined by direction of flowing current

Numerical calculation

- The solutions we are looking for are the static, lowest energy states of the system.
- Method: Gradient Flow
  - initial guess satisfying boundary conditions
  - evolve the fields with time

field $X_i(r,\theta) \rightarrow X_i(t,r,\theta)$

$$EOM\ of\ X_i = 0 \rightarrow EOM\ of\ X_i = \partial_t X_i$$

Diffusion equation

  - converge symbol: $\partial_t X_i = 0$

Numerical calculation for local strings

Abelian-Higgs model

$$\mathcal{L}_{AH} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left|D_\mu\phi\right|^2 - V(\phi),$$

$$V(\phi) = \frac{1}{4}\lambda(|\phi|^2 - \eta^2)^2$$

$$\beta \equiv \frac{m_\phi}{m_e} = \frac{\lambda}{2e^2}$$

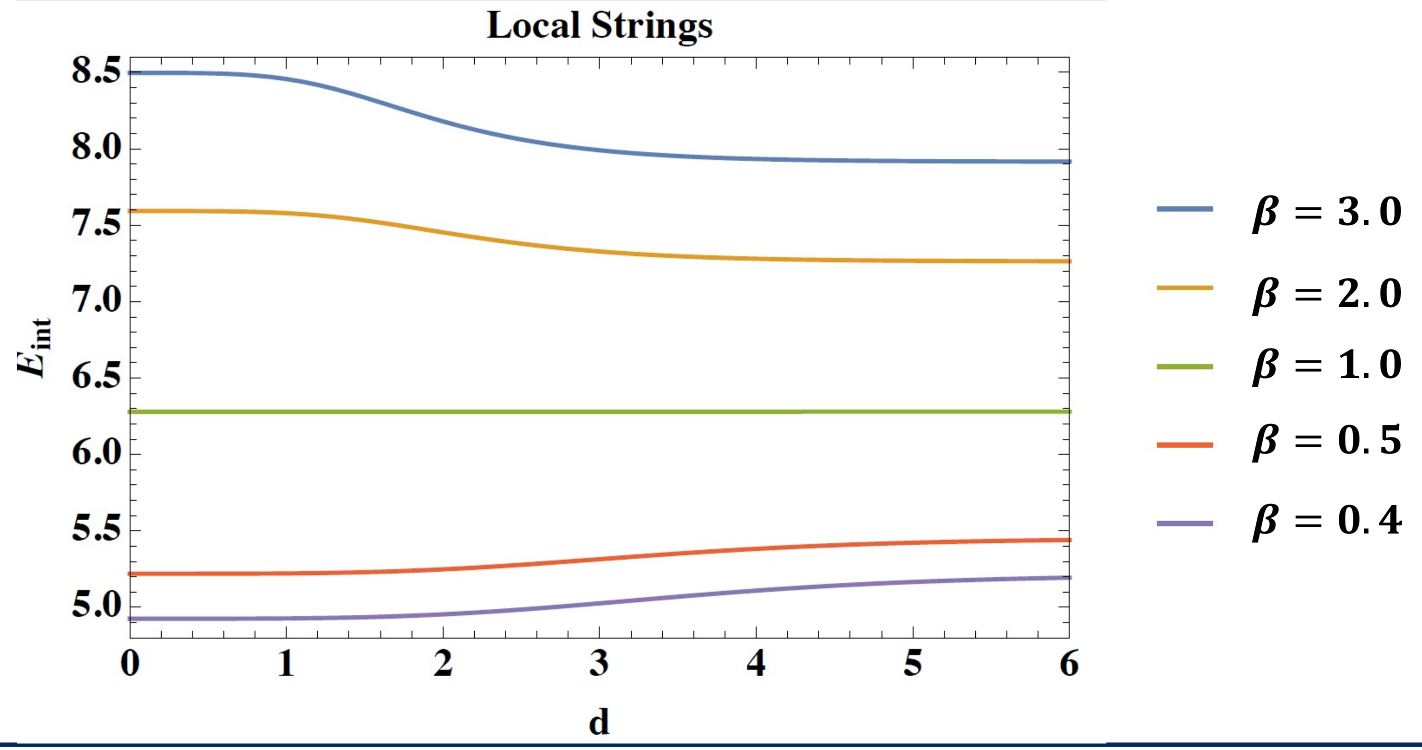
point source approximation

$$E_{int} = 2\pi \int dz\, [-k_{\phi 1}k_{\phi 2}K_0(m_\phi d) + k_{A1}k_{A2}K_0(m_e d)]$$

attractive

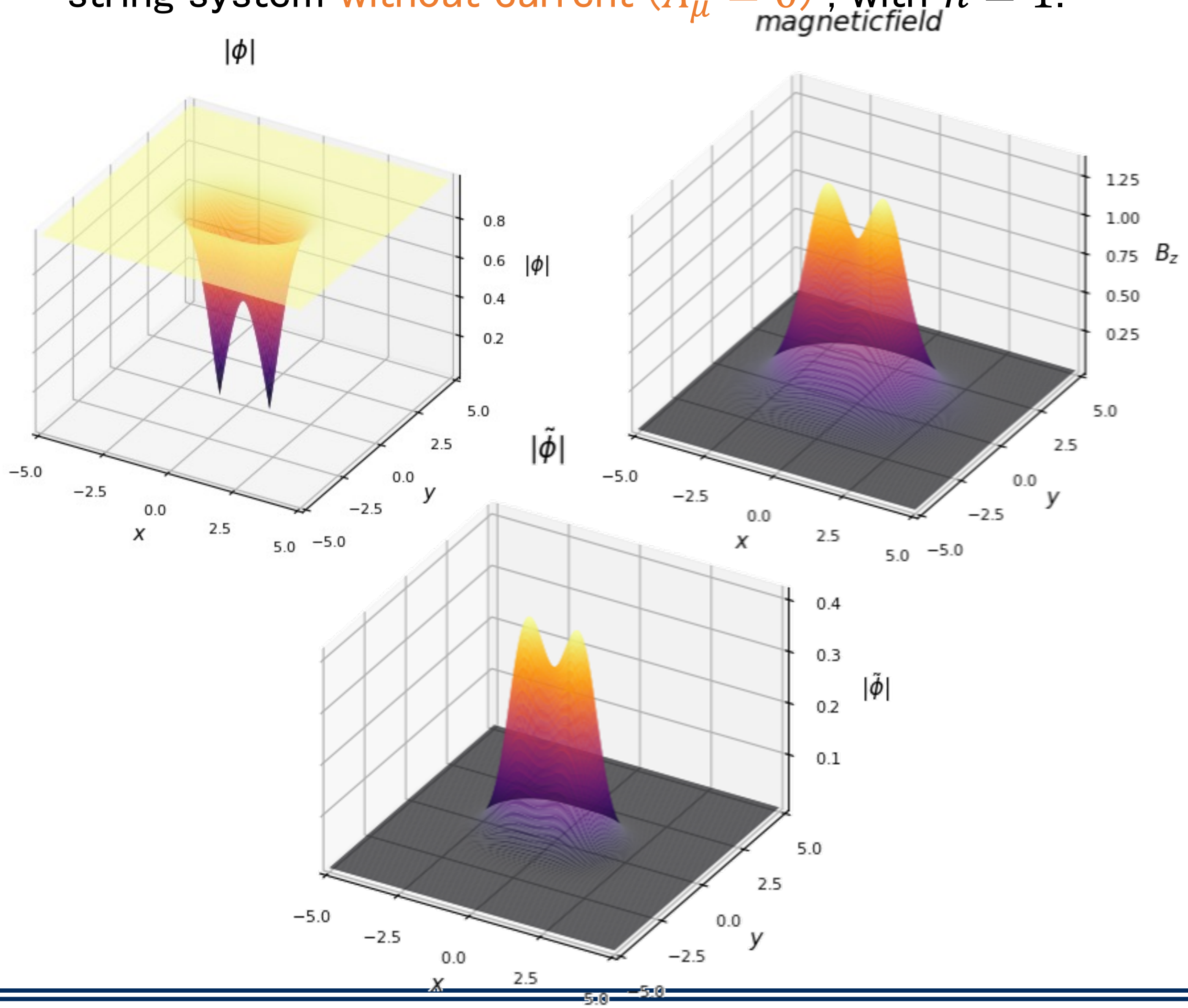
repulsive

cancel at β=1



Numerical result for bosonic superconducting

Field configurations of a two bosonic superconducting string system without current $(\tilde{A}_\mu = 0)$, with $n = 1$.

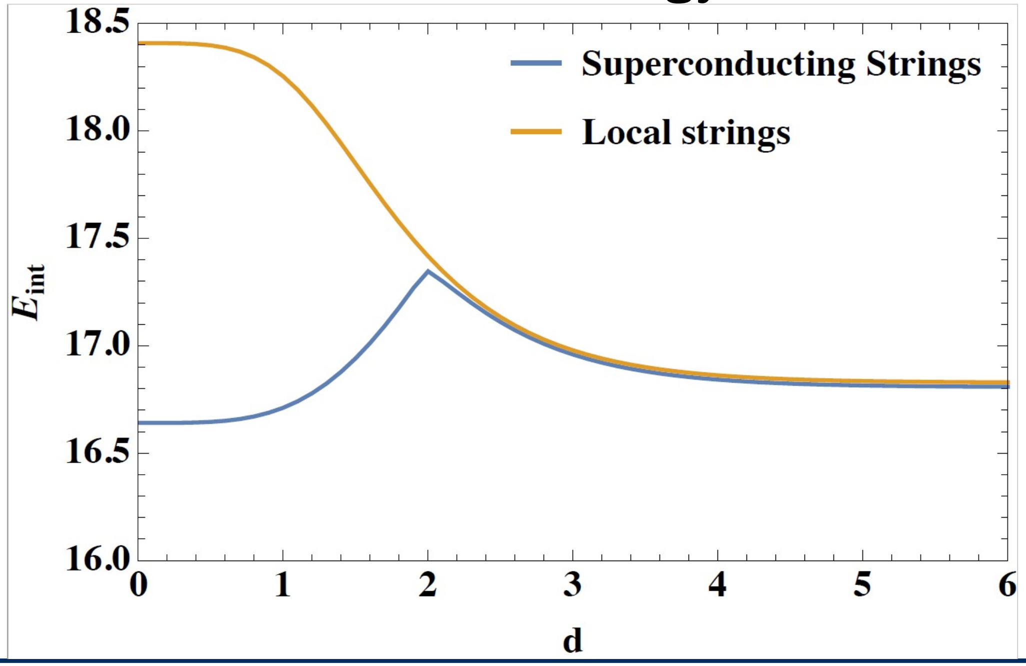


$$E_{int} = 2\pi \int dz \left[-k_{\phi1}k_{\phi2}K_0(m_\phi d) - k_{\tilde{\phi}1}k_{\tilde{\phi}2}K_0(m_{\tilde{\phi}} d)\right] + k_{A1}k_{A2}K_0(m_e d)$$

always attractive contribution

determined by winding direction of vortex

Interaction energy



Global string

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi),$$
$$V(\phi) = \frac{1}{4}\lambda(|\phi|^2 - \eta^2)^2$$

global $U(1)$ symmetry

$$\phi(r) = \left(\eta + \frac{\sigma(r)}{\sqrt{2}}\right) e^{i\pi(\theta)}, \qquad \pi(\theta) = n\theta$$

massless Nambu−Goldston boson

$$\sigma(r) = -\frac{\sqrt{2}n^2\eta}{m^2 r^2} \qquad \mathrm{m} \equiv \sqrt{\lambda}\eta$$

$$J_\sigma = \frac{\sqrt{2}n^2\eta}{r^2}$$
$$J_\pi = 2\pi n\eta\delta^{(2)}(\boldsymbol{x})$$

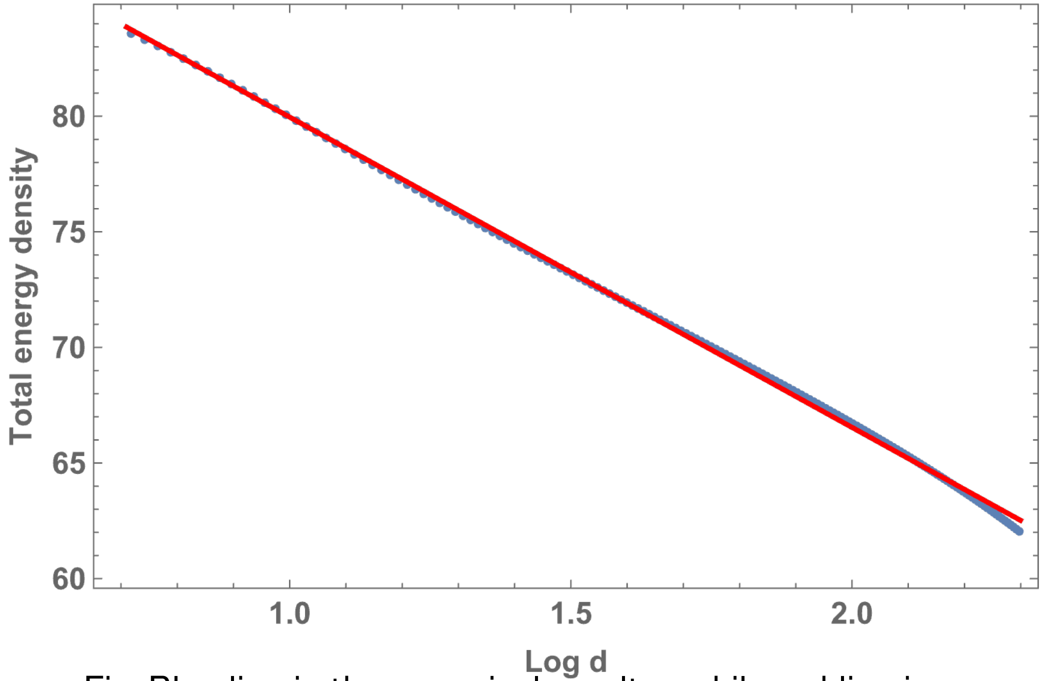


Fig. Blue line is the numerical results , while red line is source method result.

$$E_\pi = 2\pi\eta^2 \int dz\ [-n_1 n_2 \ln(\epsilon d)\ ]$$

cutoff at $\delta = \frac{1}{m}$

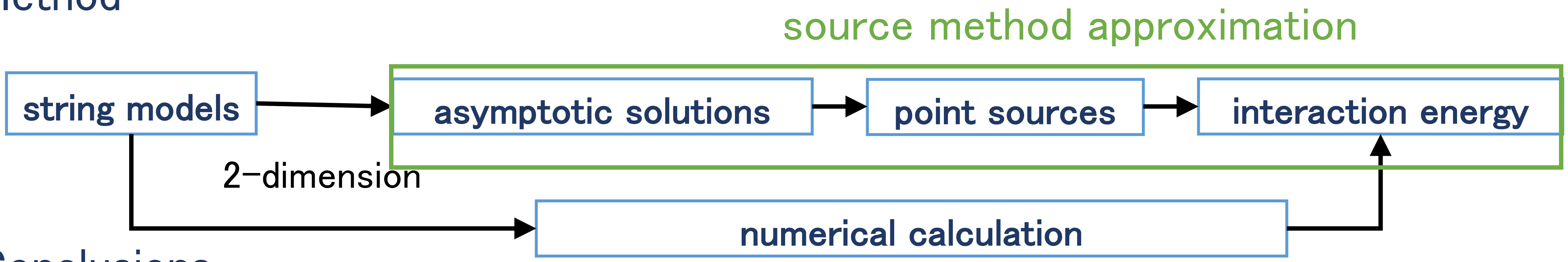
$$E_{int} = 2\pi\eta^2 \int dz\ \left[-n_1 n_2 \ln(md) - \frac{1}{\lambda} n_1^2 n_2^2 \frac{1}{d^2} \ln(m^2 d^2)\right]$$

suppressed at large distance

Summary

- We investigated interaction between two straight, static, cylindrical symmetric cosmic strings .

- Method



- Conclusions
  - Bosonic superconducting string: asymptotic configurations can be represented by **scalar monopoles** at string center and **a static current flowing along string**. Long-distance force is dominated by the **logarithmic contributions** from massless gauge field.
  - Global string: dominant by **logarithmic contributions** from NG-boson

- Future work
  - Simulation of cosmic string network; Formation and distribution of substructures; Prospective observations…

Tokyo Tech

Thank you very much for attention.