Dynamical Approach to the Cosmological Constant Revisited

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Two big problems in cosmology

$$3H^2 = 8\pi G\rho - \frac{3K}{a^2} + \Lambda$$

- Λ does not decay → cc problem: "Why is Λ as small as $8\pi G\rho$ now?"
- K/a^2 decays but only slowly → flatness problem: "Why is $3K/a^2$ smaller than $8\pi G\rho$ now?"

Two ways to tackle flatness problem

$$3H^2 = 8\pi G\rho - \frac{3K}{a^2}$$

- If ρ does not decay for an extended period then flatness problem solved → Inflation
- If $K/a^2 << 8\pi G\rho$ initially then flatness problem solved → Quantum cosmology

cc problem is more difficult than flatness one

$$3H^2 = 8\pi G\rho - \frac{3K}{a^2} + \Lambda$$

- Λ does not decay at all
  → more stringent fine-tuning
- $8\pi G\rho$ may contain terms mimicking Λ and changing at phase transitions
  → difficult to make the late cc small by early universe dynamics

Probably the most difficult problem in cosmology…

Two cc problems

1. **The old cc problem:**
   "Why is it so small?" The cc is not protected against quantum corrections. (This must be solved early on -- NOT just a problem for long distance physics.)

2. **The coincidence problem:**
   "Why now?" The cc is the same order of magnitude as other components of energy density TODAY.

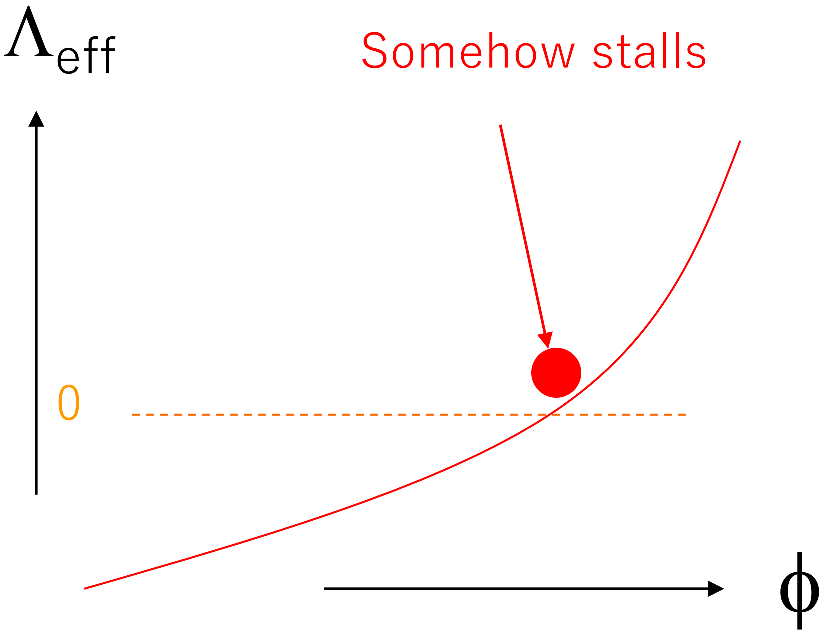
How to solve cc problem?

- Probably we need to solve the old cc problem BEFORE addressing the coincidence problem. [Dynamics with a large cc does not bring us to a low energy regime.]
- Previous Approaches:
  Find reasons for cc=0 to be at or near a local minimum of potential --- **unstable against radiative corrections!**

  or

  Anthropic principle?

Proposal



Assume that $\Lambda_{min} < 0$

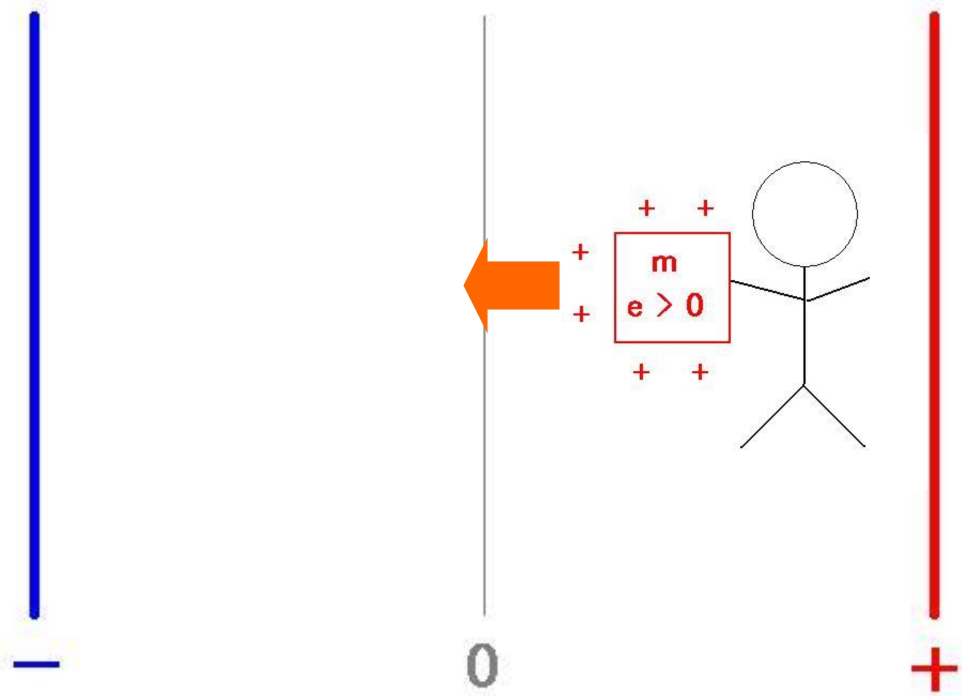
No fine-tuning of potential

Clearly, this would require unconventional dynamics.
But perhaps that is what the small cc is telling us.

Potential

- No fine-tuning
- Taylor expansion: $\Lambda_{eff} \simeq c\kappa^{-1}(\phi-\phi_0)+\Lambda_0$
- Can always shift ϕ_0 to set $\Lambda_0 = 0$
  (c will be changed but it does not matter.)
- Need to stop ϕ at ϕ_0, but ϕ wants to roll.
- Key to this model is NOT special potential, but special form for a kinetic term.

Analogue (charged box & electric field)



How can we stop him/her just before zero?

●Suppose that m can be controlled by someone watching his/her position.
●Make m → ∞ (with e fixed), when he/she approaches zero.

$$L = \frac{1}{2} m(t) \dot{x}^2 - V(x)$$

$$\begin{cases} \dot{p} = -V'(x) \\ \dot{x} = \dfrac{p}{m} \end{cases}$$

→ p remains finite within finite time.

→ x stops when m diverges.

Moreover,

$$L_{kin} = \frac{1}{2} m \dot{x}^2 = \frac{p^2}{2m}$$

vanishes when m diverges.

More singular m → More regular L_{kin}

Of course, this never happens in the real world. However, for a scalar field, this may be a theoretical possibility, as far as it does not conflict with any observations or any experiments.

Singular-looking kinetic term at R=0

$$L_{kin} = -\frac{1}{2f(R)}\partial^{\mu}\phi\partial_{\mu}\phi \qquad f(R) \approx (\kappa^4 R^2)^m \propto (\kappa^4 H^4)^m$$

$$(-+++)$$

- Stable under radiative corrections!
  Other kinetic terms can exist: the most singular-looking term dominates.
- Does NOT generate terms like 1/f(R).

Asymptotic behavior in FLRW

Equation of motion

$\kappa = 1,\ c = O(1)$

$$\dot{\pi} + 3H\pi + c = 0,\ \pi \equiv \frac{\dot{\phi}}{f} \quad \text{for} \quad V(\varphi) \approx c(\phi - \phi_0)$$

$\Rightarrow$ $H\pi \to const. < 0\ (t \to \infty)$ if $\dot{H}/H^2 \approx const. \leq 0$

At low energy

$$L_{kin} = f\pi^2 \approx H^{4m-2} << R \approx H^2 \quad \text{for} \quad m > 1$$

$$<< \alpha R^2 \approx \alpha H^4 \quad \text{for} \quad m > 3/2$$

By using the (effective) Friedmann equation $3H^2 \approx V$

$$\dot{V} \approx \dot{\phi} = f\pi \approx -H^{4m-1} \approx -V^{2m-1/2}$$

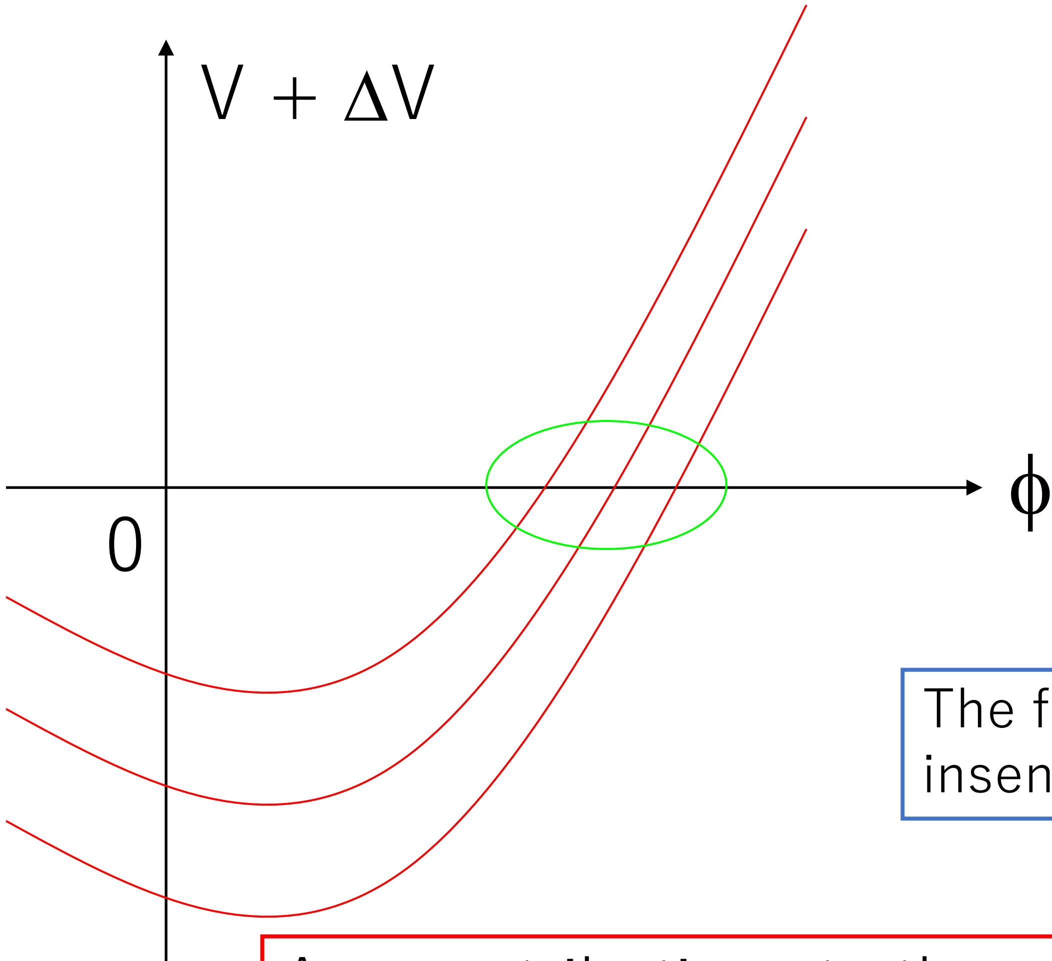
$$V \propto t^{-1/(2m-3/2)} \to 0\ (t \to \infty)$$

Feedback mechanism

$V(\phi) \approx c\kappa^{-3}(\phi - \phi_0)$ near $V \approx 0$

(ϕ rolls very slowly.)

Note that the potential includes all contributions to the cosmological constant (eg. Quantum corrections, etc.).



Shift of the ground state energy

Change of c and ϕ_0

The feedback mechanism is insensitive to c and ϕ_0 .

Any contributions to the cosmological constant can be dynamically cancelled.

Recovery of Einstein gravity and stability

hep-th/0306208

section V.A.4 of arxiv:1006.0281

- Surprisingly, despite the very singular-looking action, **the low energy gravity is stable and reduces to Einstein gravity**, provided that αR^2 (with $\alpha>0$) is added to the Lagrangian.
- **Two scalar d.o.f. satisfy the no-ghost conditions and have unit sound speeds**.

- Feedback mechanism $\Lambda_{eff} \to +0$.
- Recovery of the standard Friedmann eq.
- Recovery of the linearized Einstein eq.

Empty universe problem

- **The cc indeed goes to 0** .
- But, the energy density of matter and radiation approaches 0 even faster.
- Thus, **some form of reheating is required**. Otherwise, we would end up with a vanishing-cc but empty universe.

Summary so far

- **Feedback mechanism $\Lambda_{eff} \rightarrow +0$ is stable under radiative corrections!**
- Recovery of **the standard Friedmann eq.**
- Recovery of **the linearized Einstein eq.**
- However, realistic cosmology **requires reheating.**

Reheating without spoiling small cc

This is not easy …

- If we consume Λ_{eff} then Λ_{eff} after reheating would be negative.
- Λ_{eff} should go back to almost the same value after reheating.

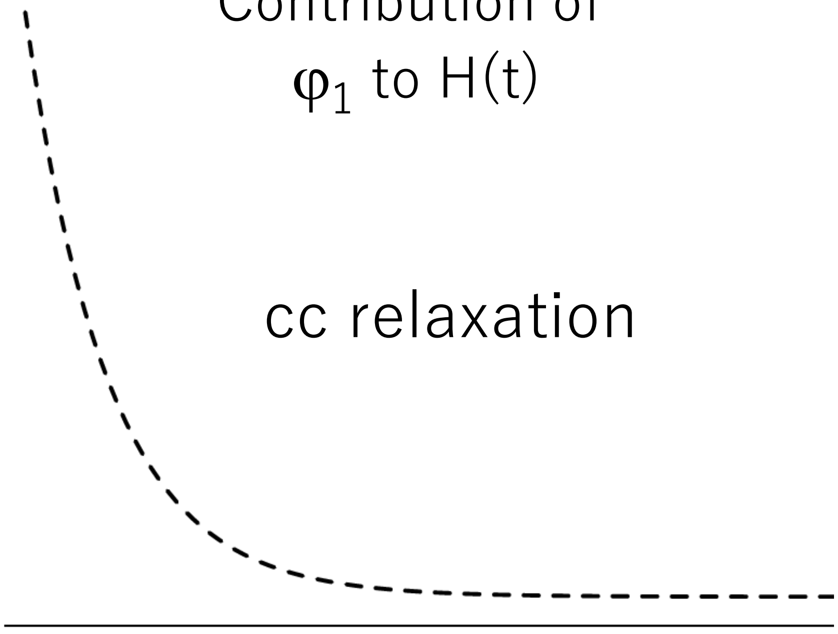
These suggest the following scenario.

c.f. $\dot{\rho} + 3H(\rho + P) = 0$

- Introduce a field **φ_2 violating the null energy condition (NEC)**, $\rho_2 + P_2 \geqq 0$, to increase the energy density.
- Couple φ_2 to **yet another field φ_3 that reheats the universe** by using the energy density created by φ_2 .
- Suppose that **the field space for φ_2 is periodic** so that the reheating process repeats.

Reheating without spoiling small cc

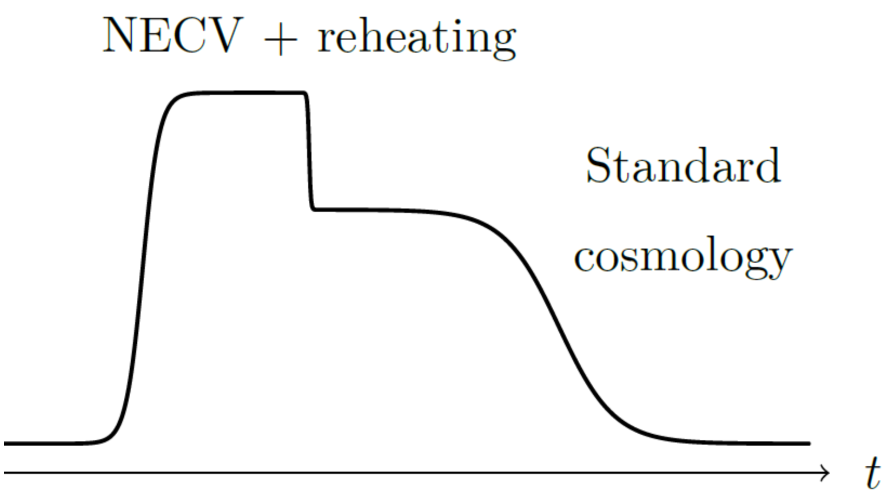
Contribution of φ_1 to $H(t)$



φ_1 relaxes cc slowly

(ϕ is renamed to φ_1)

Contribution of φ_2 & φ_3 to $H(t)$



φ_3 reheats the universe by using the energy density created by φ_2

φ_2 field space is periodic → cyclic

1st step: cc relaxation

$$X_i \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi_i \partial_\nu \varphi_i$$

$$\mathscr{L} = \overbrace{\underbrace{\frac{M_{\rm Pl}^2}{2} R}_{\mathscr{L}_{\rm E.H.}} + \alpha R^2}^{\rm gravity} + \underbrace{\frac{X_1}{f(R)} - V_1(\varphi_1)}_{\mathscr{L}_{\rm c.c.\ relax}}$$

(ϕ is renamed to φ_1)

$$f(R) \approx \left(\frac{R^2}{M_{\rm Pl}^4} \right)^m \quad \text{with} \quad m > 3/2$$

$$V_1(\varphi_1) \simeq c M_{\rm Pl}^3 (\varphi_1 - v)$$

(ϕ_0 is renamed to v)

- **Feedback mechanism Λ_{eff} → + 0 is stable under radiative corrections!**
- Recovery of **the standard Friedmann eq.**
- Recovery of **the linearized Einstein eq.**

Avoid overshooting during NECV & reheating

- Eom still implies $\dot{V}_1 \approx \dot{\varphi}_1 = f\pi_1 \approx -H^{4m-1}$. $\kappa = 1,\ c = O(1)$
- However, the contribution of φ_2 & φ_3 to H(t) breaks the relation $3H^2 \approx V_1$.
- Thus **φ_1 might overshoot to negative cc**.
- During one period of NECV & reheating,

$$\Delta V_1 \approx -\mathrm{H}^{4\mathrm{m}-1}\Delta t \approx -H^{4m-1}\frac{|\Delta\varphi_2|}{M^2} ,\qquad \mathrm{M}^2 \equiv |\dot{\varphi}_2|$$

  and H can be as large as max H_{23}, where H_{23} is the contribution of φ_2 & φ_3 to H(t).

- Avoidance of overshooting, i.e. $|\Delta V_1| < \Lambda_{obs}$, is ensured if

$$\frac{|\Delta\varphi_2|}{M} < \frac{\mathrm{M}}{\mathrm{c}^2\mathrm{M}_{\mathrm{Pl}}}\left(\frac{M_{Pl}}{\max H_{23}}\right)^{4m-1}\frac{\Lambda_{obs}}{M_{Pl}^2}$$

  where c and $\kappa = 1/M_{Pl}$ are recovered.

- **This is easily satisfied for a sufficiently large m.**

2nd & 3rd steps: NECV & reheating

c.f. Alberte, Creminell, Khmelnitsky, Pirtskhalava, Trincherini, arXiv: 1608.05715

$$\mathscr{L}_{\mathrm{NECV+reh}} = K(\varphi_2, X_2, \varphi_3) - G_3(\varphi_2, X_2, \varphi_3)\Box\varphi_2 + P(\varphi_3, X_3)$$

- Ansatz for K : up to 2nd order in X_2

$$K = f_1(\varphi_2, \varphi_3)X_2 + f_2(\varphi_2, \varphi_3)X_2^2 - V(\varphi_2, \varphi_3)$$

- Ansatz for G_3 : proportional to X_2

$$G_3 = f(\varphi_2, \varphi_3)X_2$$

- Ansatz for P: up to 1st order in X_3

$$P = X_3 - U(\varphi_3)$$

- The NECV sector:

$$\mathcal{L}_{\mathrm{NECV}} \equiv \mathcal{L}_{\mathrm{NECV+reheat}}\big|_{\varphi_3=0,\ P=0} = \tilde{K}(\varphi_2, X_2) - \tilde{G}_3(\varphi_2, X_2)\Box\varphi_2$$

and the reheating sector

$$P(\varphi_3, X_3)$$

couple to each other through the φ_3-dependence of

$$f_{1,2}(\varphi_2, \varphi_3), V(\varphi_2, \varphi_3), f(\varphi_2, \varphi_3).$$

Stability of NECV sector

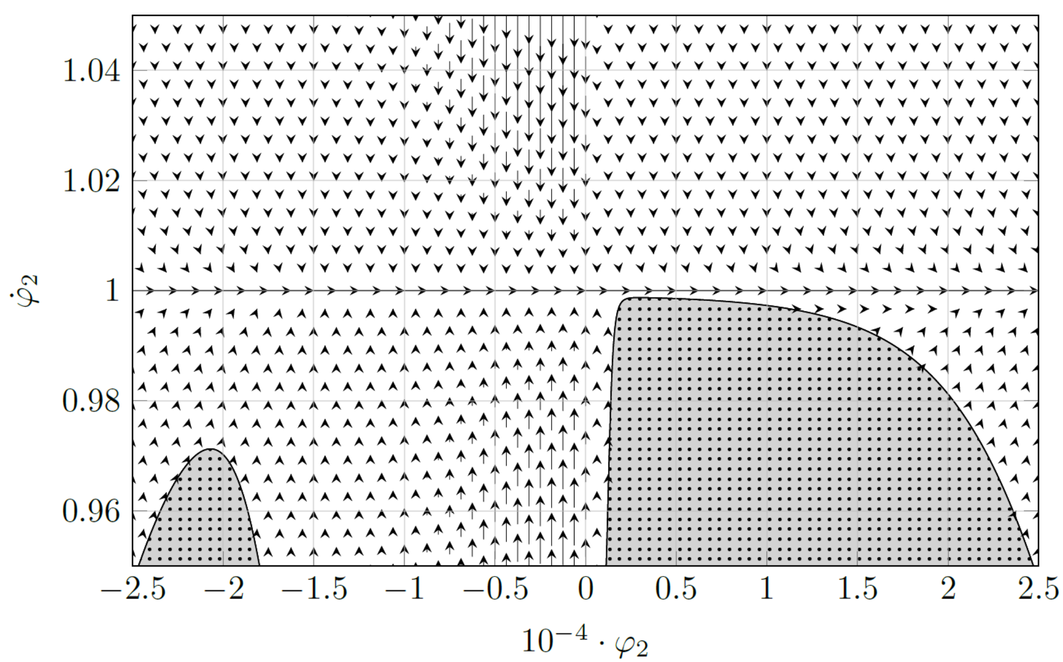
- Homogeneous & isotropic linear perturbation

$$\varphi_2(t) = \varphi^{(0)}(t) + \epsilon\,\varphi^{(1)}(t)\,, \qquad H(t) = H^{(0)}(t) + \epsilon\,H^{(1)}(t)$$

$$\partial_t \begin{pmatrix} \varphi^{(1)} \\ \pi_\varphi^{(1)} \end{pmatrix} = \mathcal{M} \begin{pmatrix} \varphi^{(1)} \\ \pi_\varphi^{(1)} \end{pmatrix}\,, \qquad \mathcal{M} = \begin{pmatrix} 0 & 1 \\ \widetilde{\mathcal{A}} & \widetilde{\mathcal{B}} \end{pmatrix}$$

- Simple subclass of models → $\widetilde{\mathcal{A}} = 0$
- In this case the given background is a local attractor if $\widetilde{\mathcal{B}} \leq 0$.

- Global stability can be studied by phase portraits.
- The phase portrait for a numerical example →



No-ghost condition & squared sound speeds for NECV & reheating sectors

- Homogeneous & isotropic background + inhomogeneous linear perturbation
- Decompose the perturbation into scalar, vector and tensor parts
- Tensor part same as GR; vector part non-dynamical
- **Two d.o.f. in scalar part**
  → two no-ghost conditions & two squared sound speeds
- Simple subclass of models → $\partial_{\varphi_3} G_3 = 0 \quad \& \quad \partial_{X_3 X_3} P = 0$
- In this case one of the no-ghost condition is simply $\partial_{X_3} P > 0$ and one of the squared sound speeds is unity.
- The remaining **no-ghost condition and squared sound speed** are

$$\mathcal{G}_S > 0 \quad \text{and} \quad c_s^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} - \frac{M_{\rm Pl}^4 X_3 \, \partial_{X_3} P}{\mathcal{G}_S \, \Theta^2} \,,$$

where $\mathcal{G}_S$, $\mathcal{F}_S$ and Θ are similar to the known expressions for the single-field case but contain **corrections due to φ_3** .

Reconstruction of NECV sector

c.f. Alberte, Creminell, Khmelnitsky, Pirtskhalava, Trincherini, arXiv: 1608.05715

- **Four free functions of φ_2** in NECV sector can be reconstructed so that the system admits the following solution.

$$H = H_{\rm necv}(\varphi_2)\ , \quad \varphi_2 = t\ , \quad \bar{N} = 1$$

- We fix two free functions by hand. We choose them as follows.

$$v(\varphi_2) = -v_0 \exp\left(-\frac{\varphi_2^2}{2T_{\rm dip}^2}\right) \qquad F_{\rm kb}(\varphi_2) = F_{\rm kb,0} + F_{\rm kb,1}\exp\left(-\frac{\varphi_2^2}{2T_{\rm kb}^2}\right)$$

so that

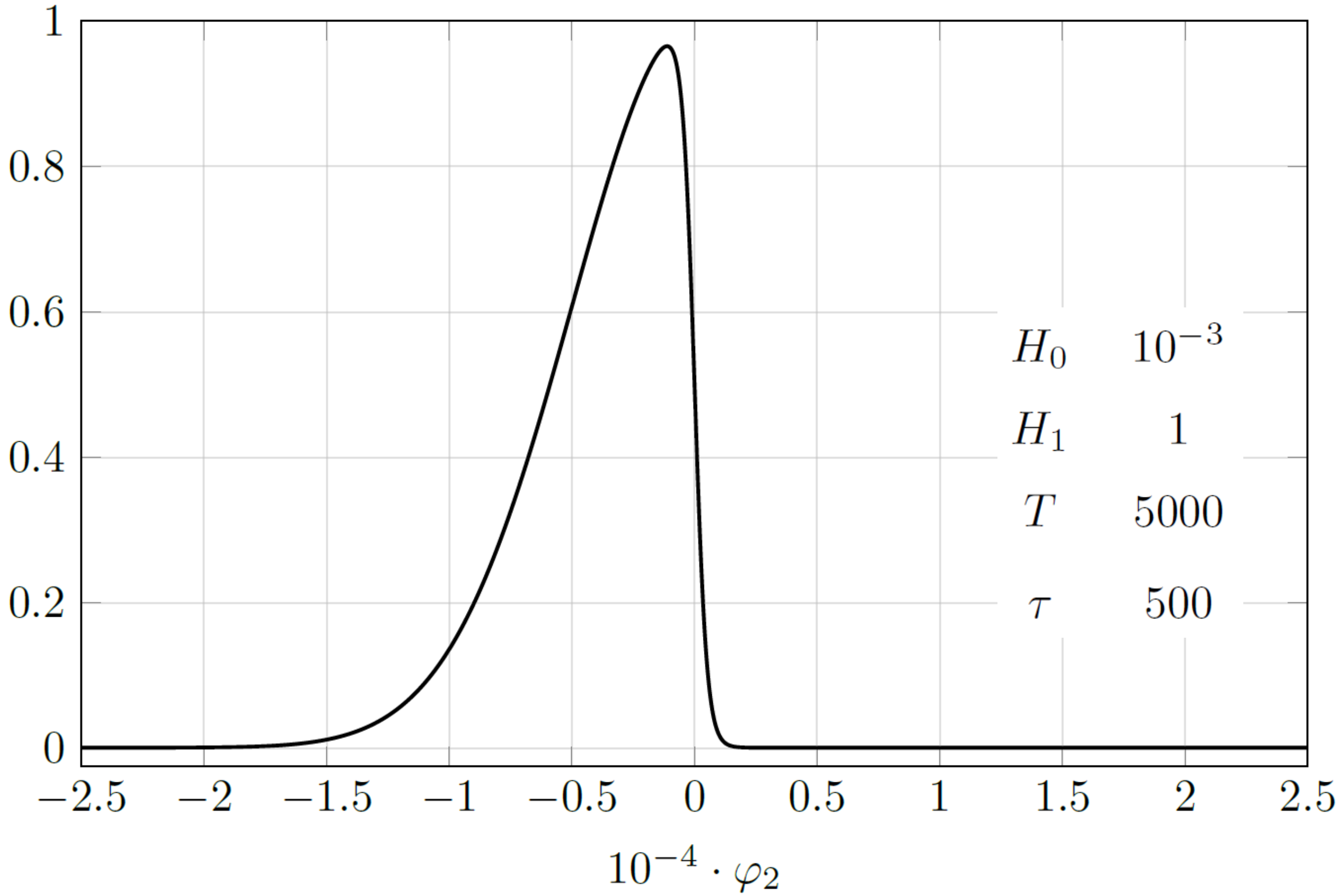
$$\tilde{K} = M_{\rm Pl}^2 H_{\rm dip}^2 \left[F_1(\varphi_2)\, X_2 + F_2(\varphi_2)\, X_2^2 - v(\varphi_2)\right]$$

$$\tilde{G}_3 = M_{\rm Pl}^2 H_{\rm necv}(\varphi_2) F_{\rm kb}(\varphi_2)\, X_2$$

- By requiring that the solution given above should satisfy the eom's, the remaining two free functions $F_1(\varphi_2)$ and $F_2(\varphi_2)$ are written in terms of **$H_{\rm necv}(\varphi_2)$ and $(v_0, T_{\rm dip}, F_{\rm kb,0}, F_{\rm kb,1}, T_{\rm kb})$** .

Choice of $H_{necv}(\varphi_2)$

$$H_{\mathrm{necv}}(\varphi_2) = H_0 + H_1 \exp\left(-\frac{\varphi_2^2}{2T^2}\right) \frac{1 - \tanh\left(\frac{\varphi_2}{\tau}\right)}{2}$$



Reheating sector and Coupling between NECV & reheating sectors

- Linear kinetic term and bare potential for reheating sector

$$P(\varphi_3, X_3) = M_{\rm Pl}^2 \beta_{\rm kin}^2 X_3 - U(\varphi_3)$$

$$U(\varphi_3) = 3M_{\rm Pl}^2 H_I^2 \left(1 - {\rm e}^{-\beta_{\rm I}\varphi_3}\right)^2$$

3 parameters: **β_{kin}, β_I and H_I**

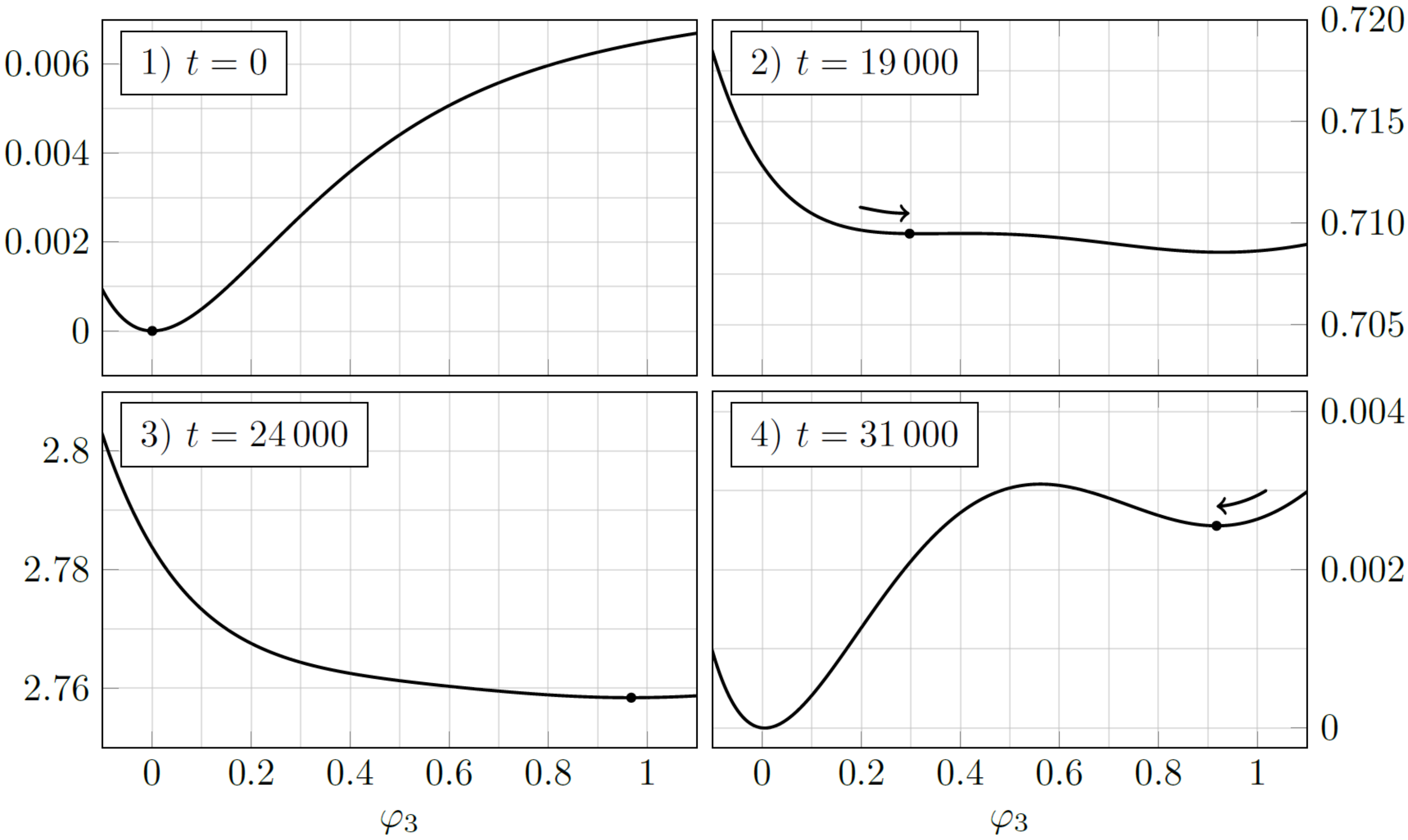
- Coupling the two sectors by the following replacement

$$F_1 \to \frac{1 + \alpha_{\rm kick}\, {\rm e}^{-\beta_{\rm kick}\varphi_3}}{1 + \alpha_{\rm kick}} F_1 \qquad F_2 \to \frac{1 + \alpha_{\rm kick}\, {\rm e}^{-\beta_{\rm kick}\varphi_3}}{1 + \alpha_{\rm kick}} F_2$$

$$v \to \exp\left[-\beta_{\rm dip}^2 \left((\varphi_3 - 1)^2 - 1\right)\right] v$$

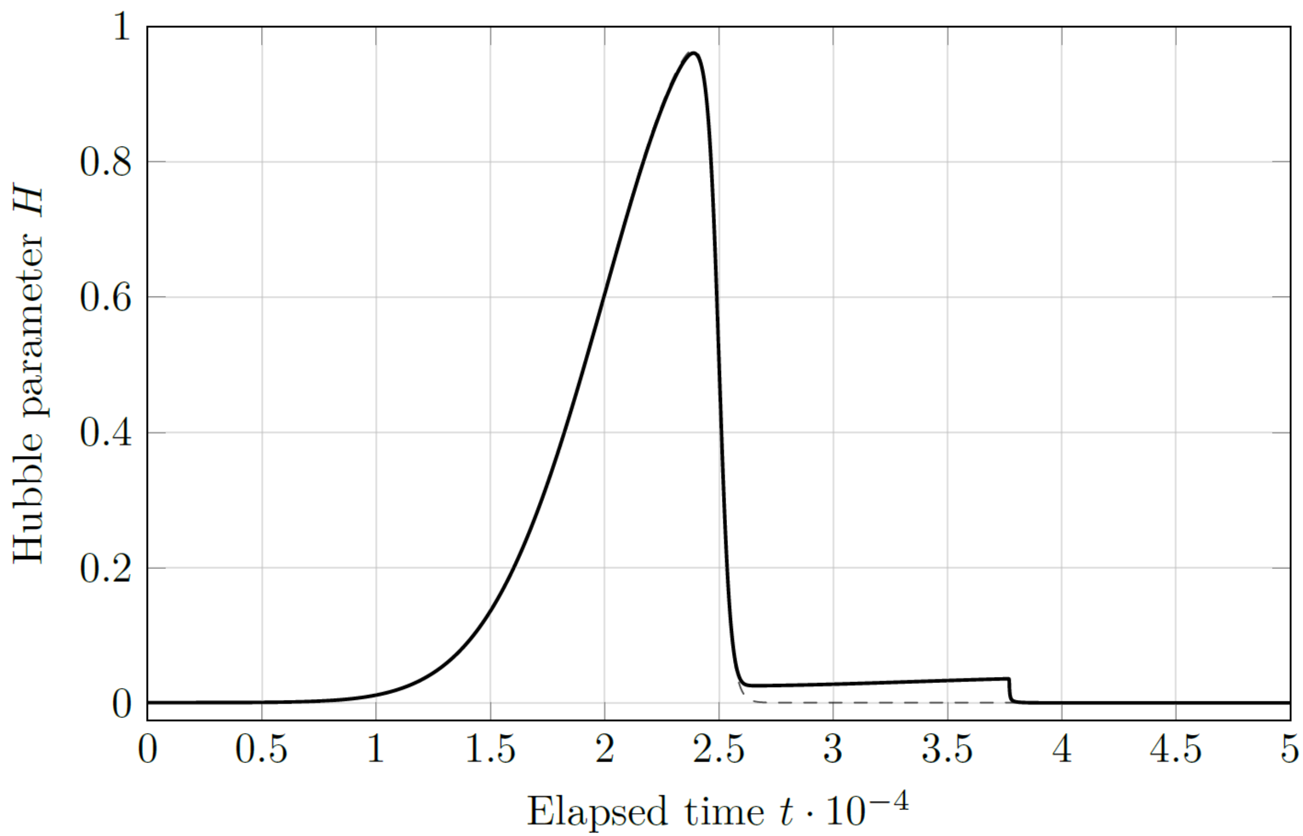
3 parameters: **α_{kick}, β_{kick}, β_{dip}**

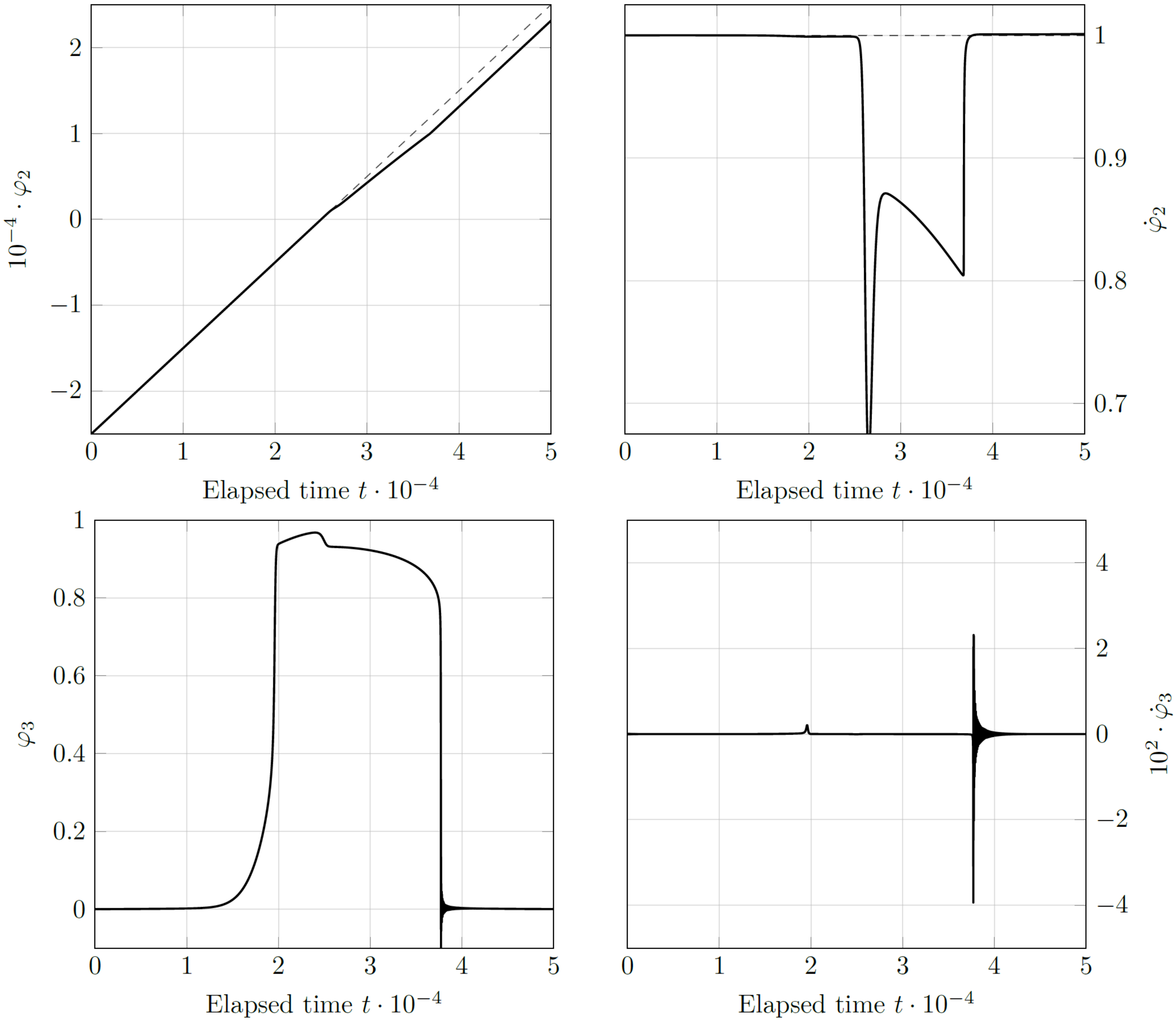
Shape of reheating potential



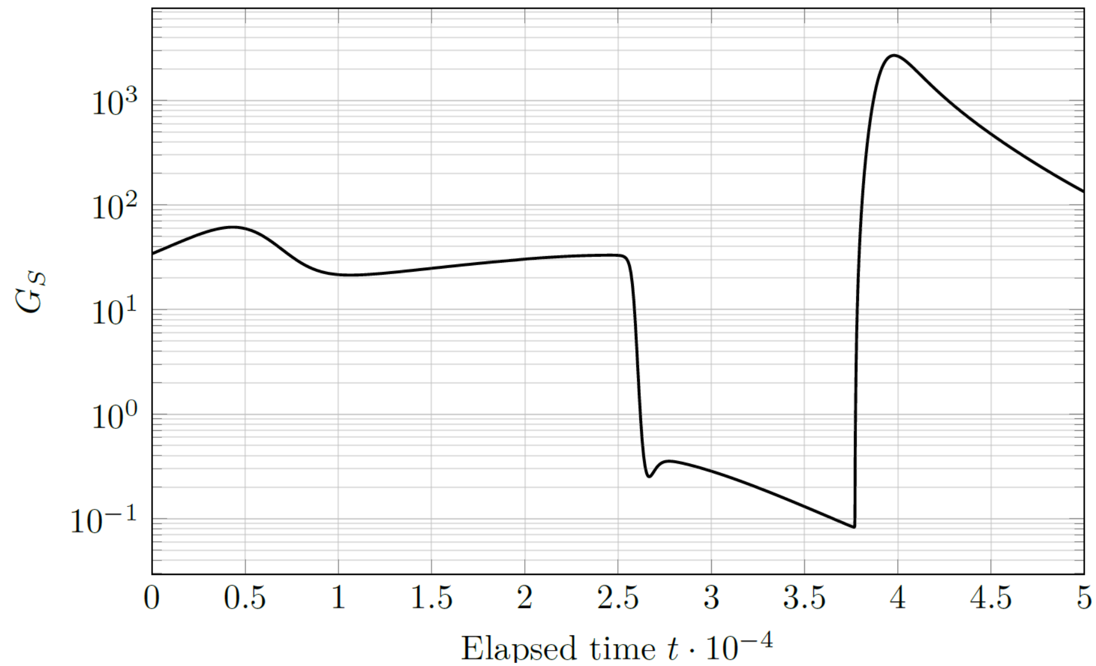
Numerical evolution of NECV & reheating sectors

Function H_{necv}		Function F_{kb}		Function v		Reheating φ_3 modulation		Reheating φ_3 sector	
H_0	10^{-3}	$F_{\mathrm{kb},0}$	10^{-3}	v_0	$5 \cdot 10^{-2}$	α_{kick}	10^{-2}	β_{kin}	1
H_1	1	$F_{\mathrm{kb},1}$	1	T_{dip}	$2T$	β_{kick}	5	β_{I}	3
T	5000	T_{kb}	$3T$	H_{dip}	$\frac{4H_1}{10}\sqrt{\frac{\alpha_{\mathrm{kick}}}{1+\alpha_{\mathrm{kick}}}}$	β_{dip}	2	H_I	$\frac{5H_1}{10}\sqrt{\frac{\alpha_{\mathrm{kick}}}{1+\alpha_{\mathrm{kick}}}}$
τ	500								

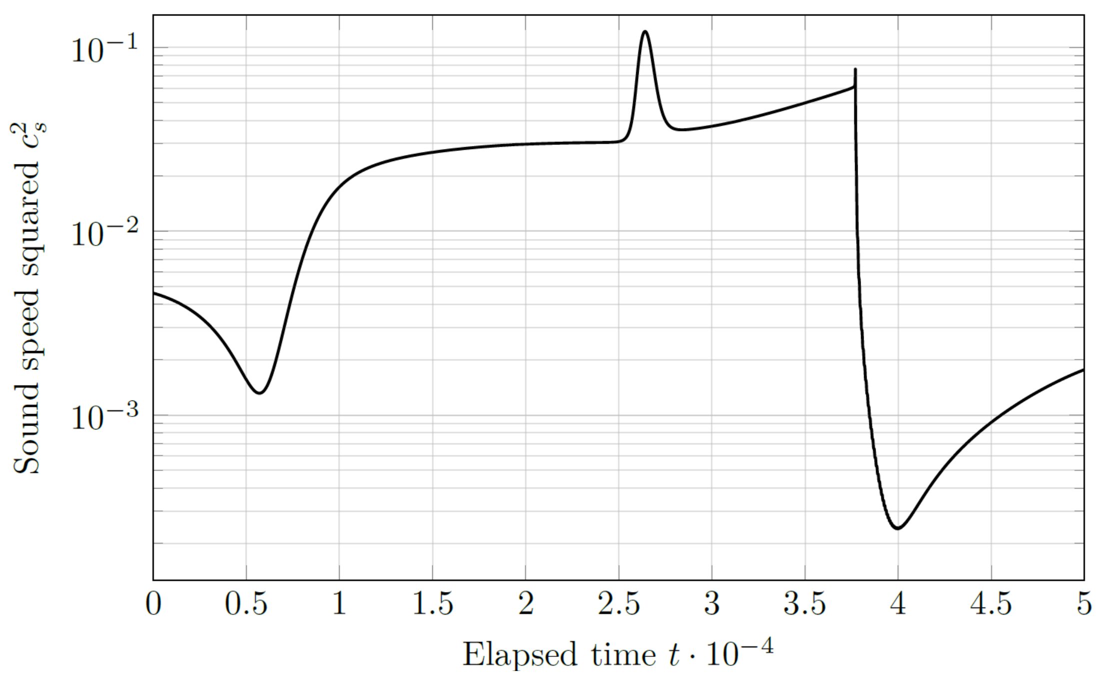




$10^{-4} \cdot \varphi_2$
Elapsed time $t \cdot 10^{-4}$
$\dot{\varphi}_2$
Elapsed time $t \cdot 10^{-4}$
φ_3
Elapsed time $t \cdot 10^{-4}$
$10^2 \cdot \dot{\varphi}_3$
Elapsed time $t \cdot 10^{-4}$



G_S
10^3
10^2
10^1
10^0
10^{-1}
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
Elapsed time $t \cdot 10^{-4}$



Sound speed squared c_s^2
10^{-1}
10^{-2}
10^{-3}
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
Elapsed time $t \cdot 10^{-4}$

Summary

- **Feedback mechanism $\Lambda_{eff} \to +0$ is stable under radiative corrections!**
- **The standard Friedmann eq.** recovered @ low E
- **The linearized Einstein eq.** recovered @ low E
- However, realistic cosmology **requires reheating.**
- **A proof-of-concept model of reheating that does not spoil the small Λ_{eff}** that was achieved by the feedback mechanism.
- The model consists of (i) a NEC violating field with a periodic field space, and (ii) a reheating field.
- The model is **stable under both homogeneous and inhomogeneous perturbations**.

Future work

- How robust is the reheating mechanism? If the NECV sector couples to a sufficiently complex system including SM and if NECV sufficiently quickly turns off, then it is expected that the universe is anyway reheated. Can we show this more quantitatively?
- Can we perform numerical evolution with a large hierarchy?
- Can the reheating field (or/and the NECV field) be the inflaton?
- Any link to fundamental theories?
- Variant modes?
- …

Thank you very much!

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