Geometry of Spacetime from Quantum Measurements

Phys. Rev. D 105, 066011 (2022)

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in collaboration with

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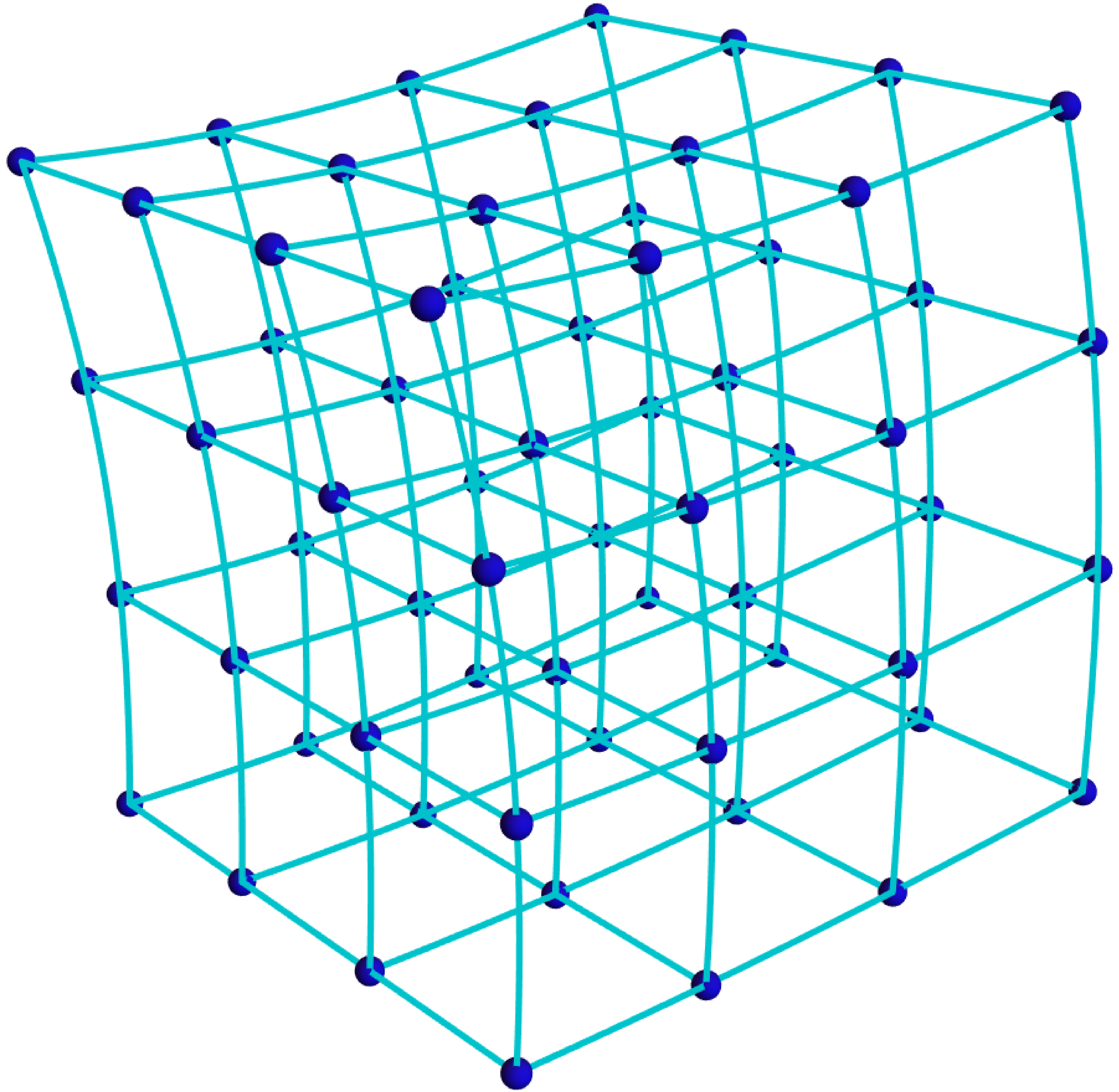
IQC

Bourses d'études supérieures du Canada Vanier Canada Graduate Scholarships



Outline

- Spacetime separations in General Relativity

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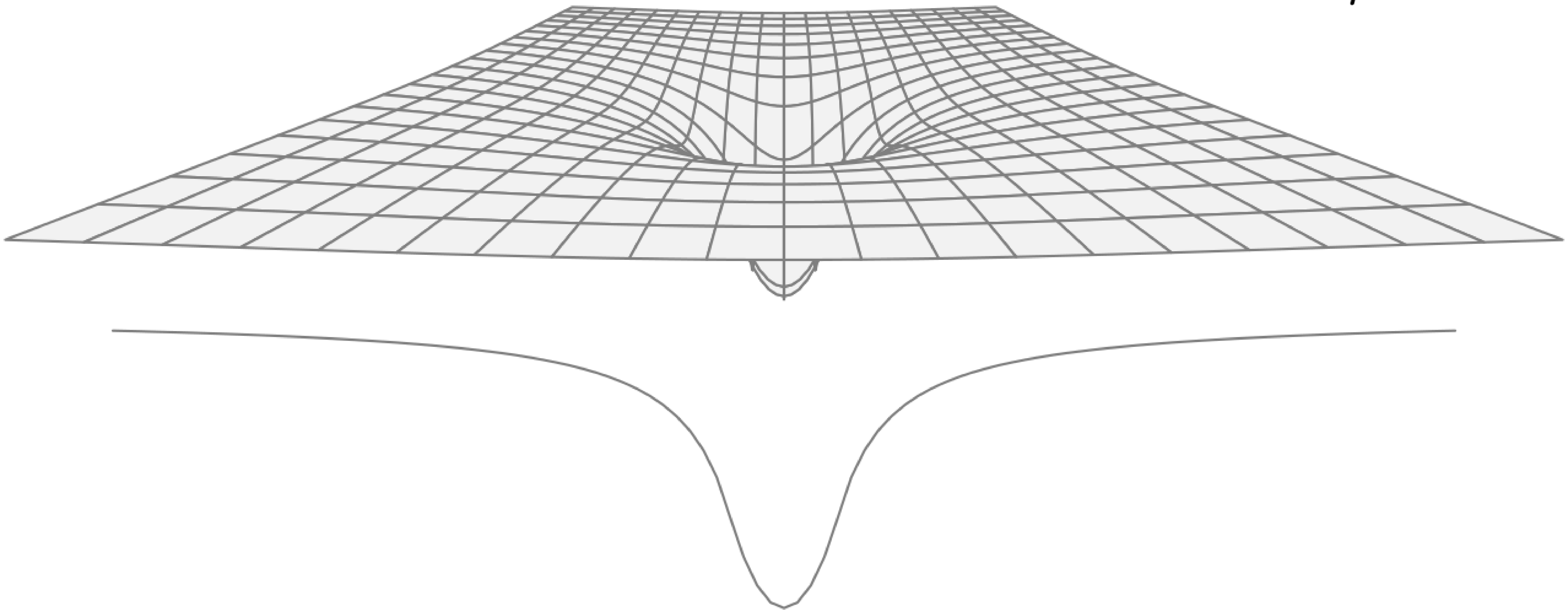
- The Geometry of Spacetime from Quantum Measurements

Spacetime separations in GR



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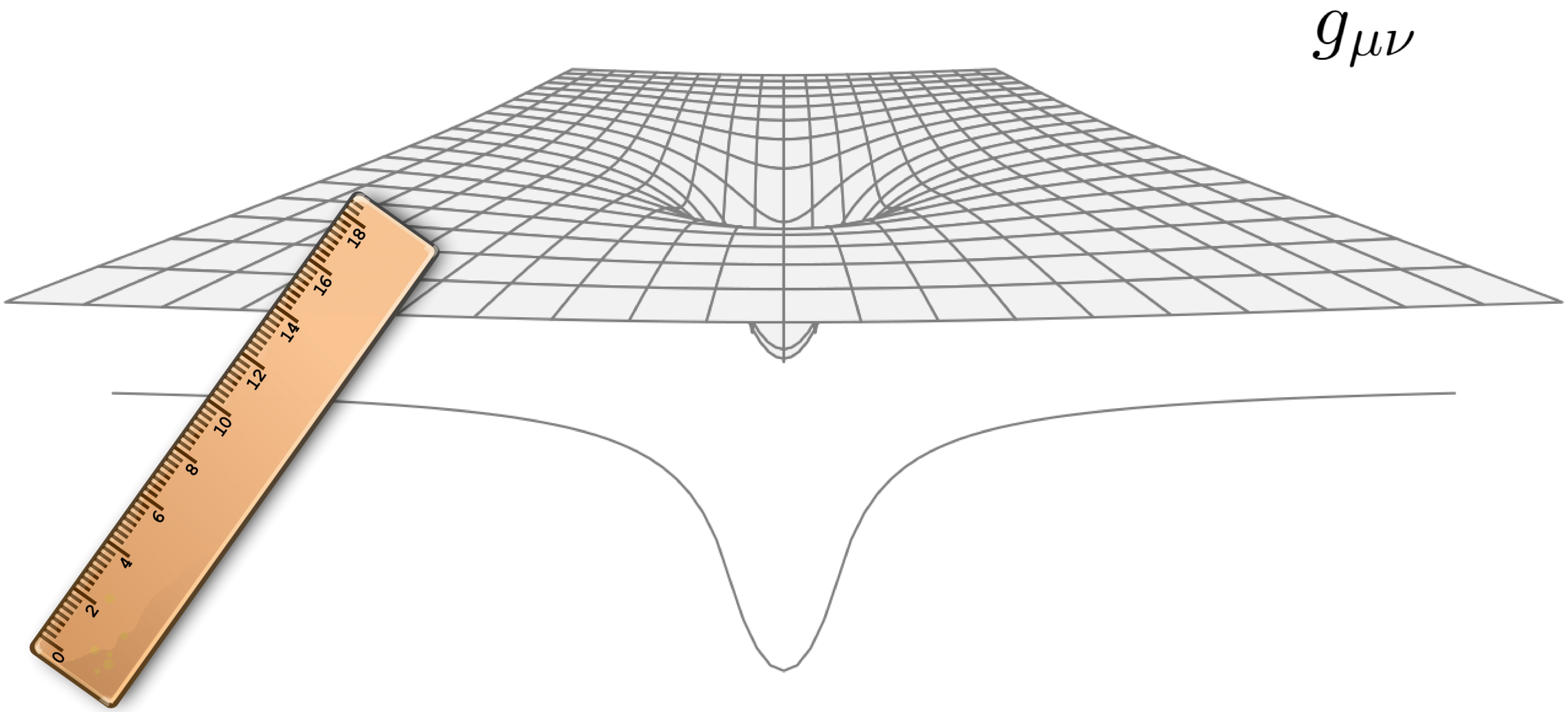

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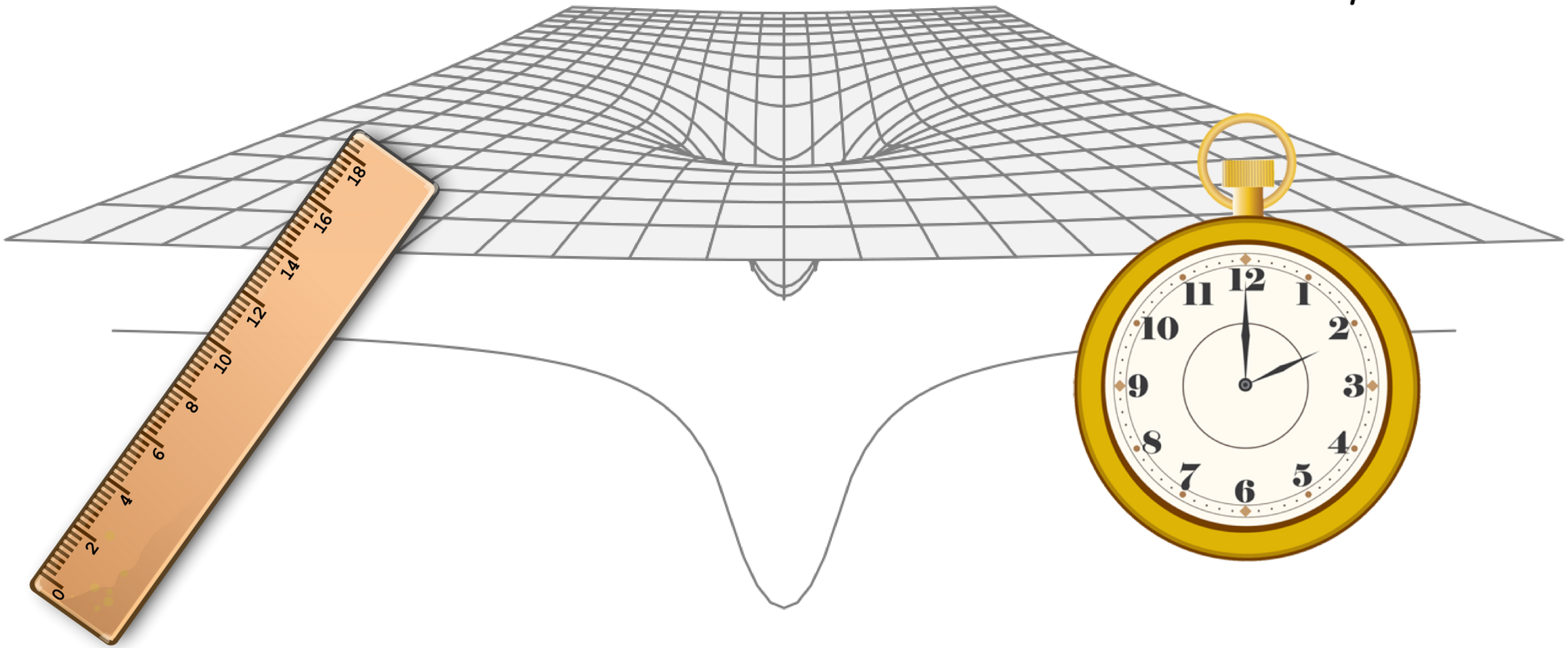

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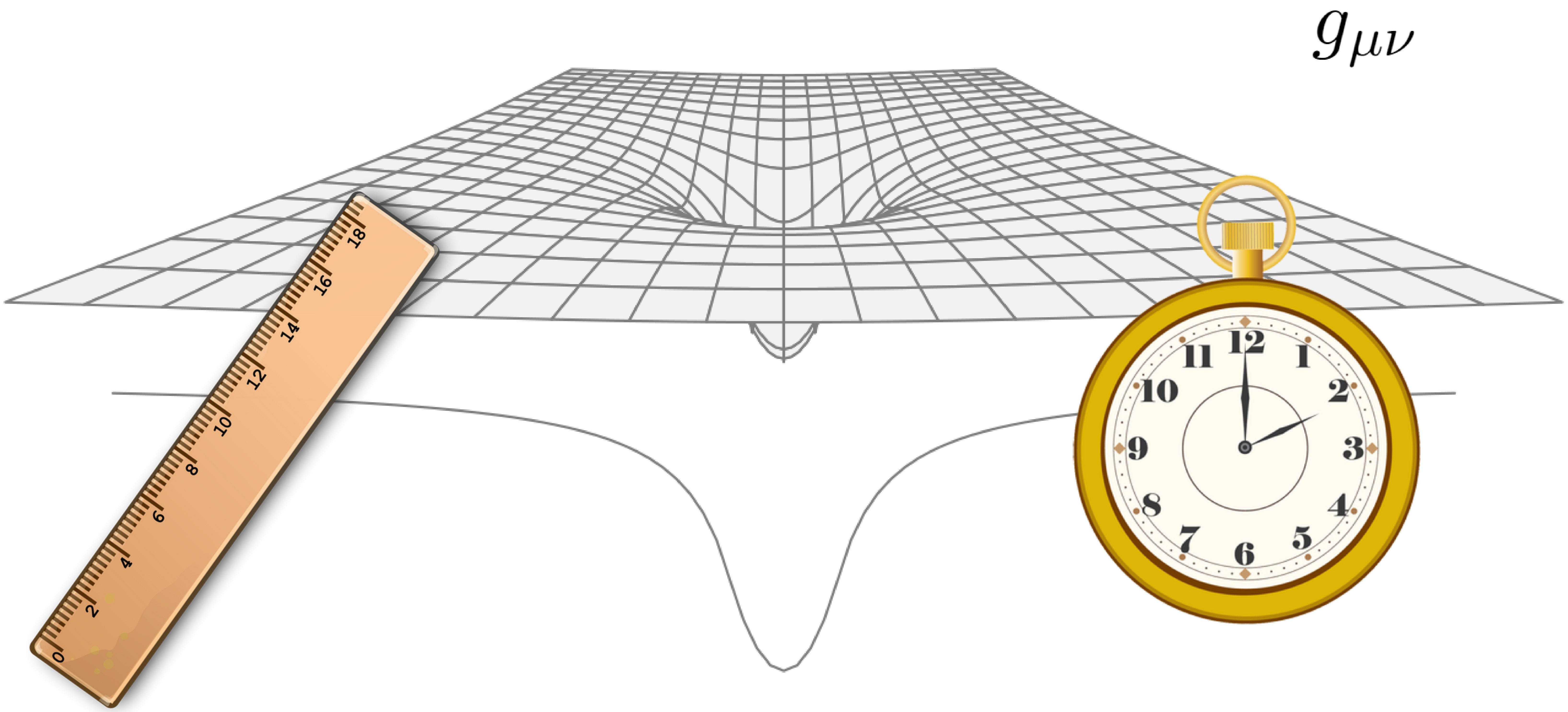
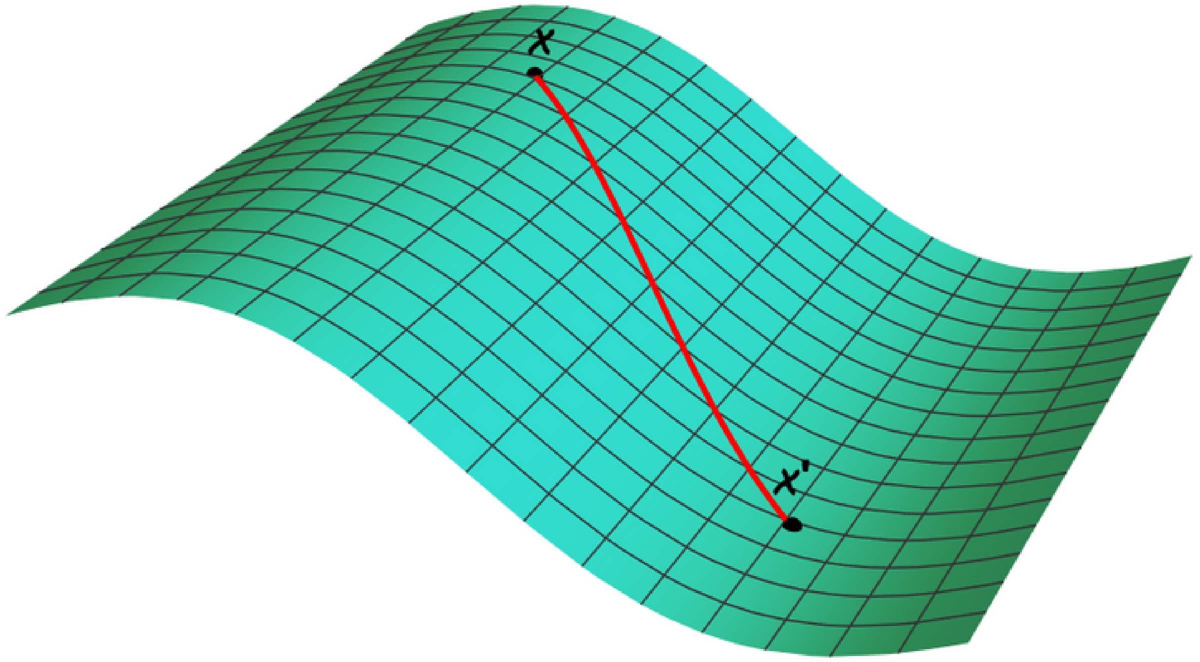


At its core, general relativity is a theory for rulers and clocks.

Synge's World Function

All information about spacetime separations in general relativity is contained in Synge's world function, $\sigma(x, x')$.



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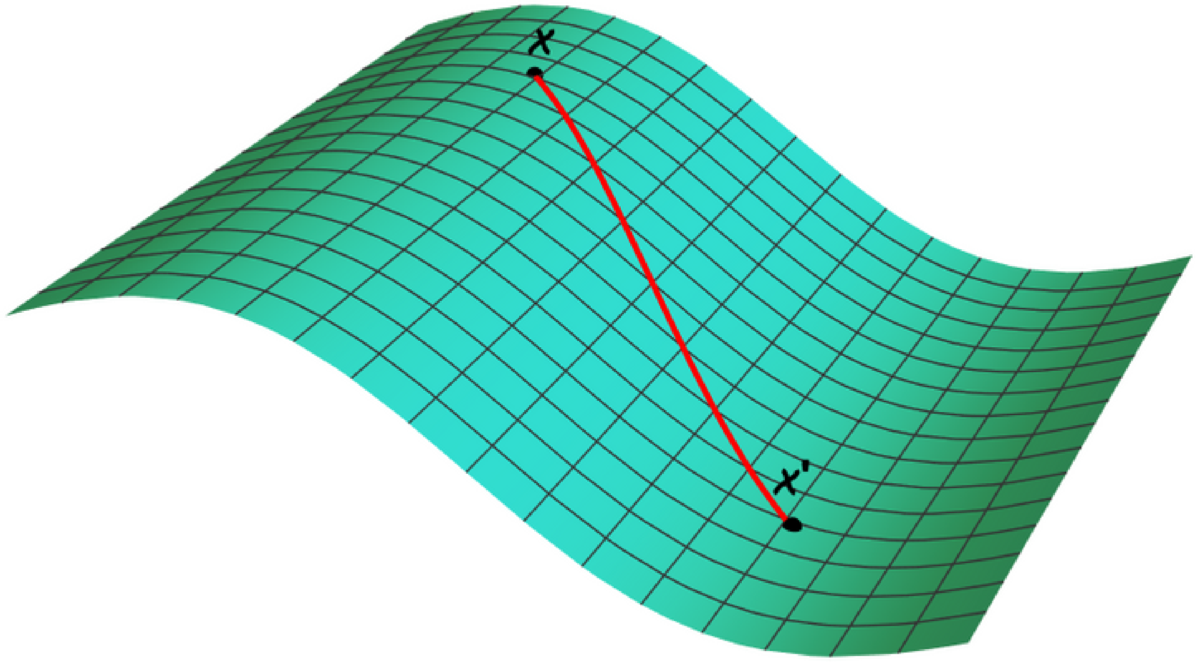

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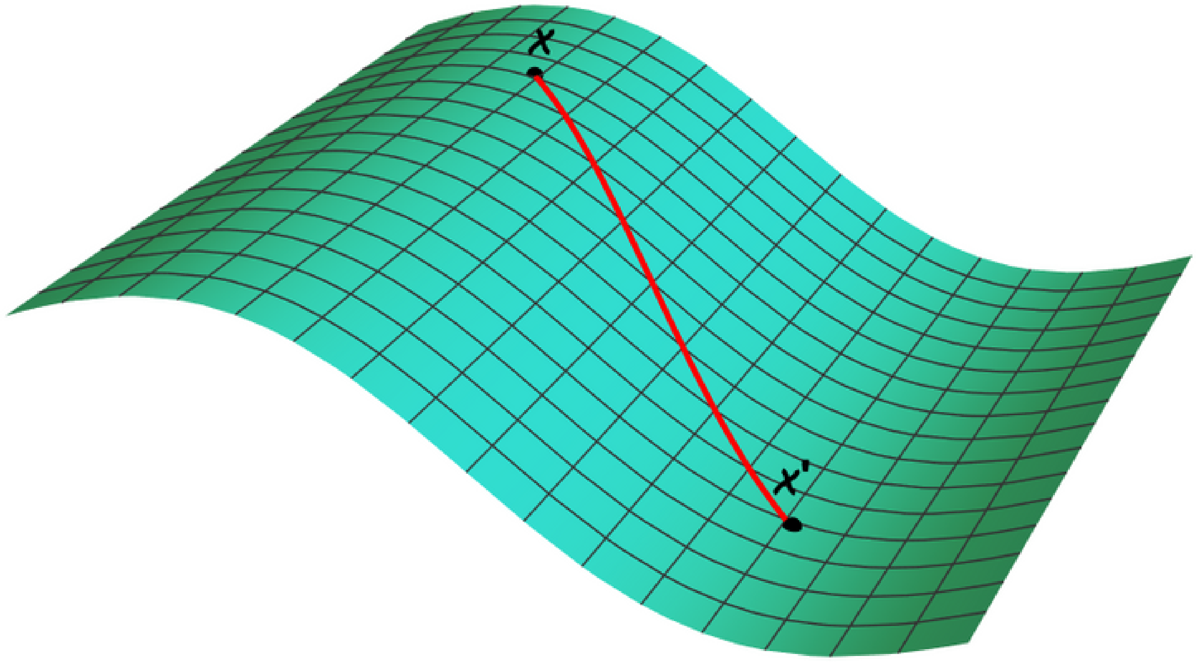

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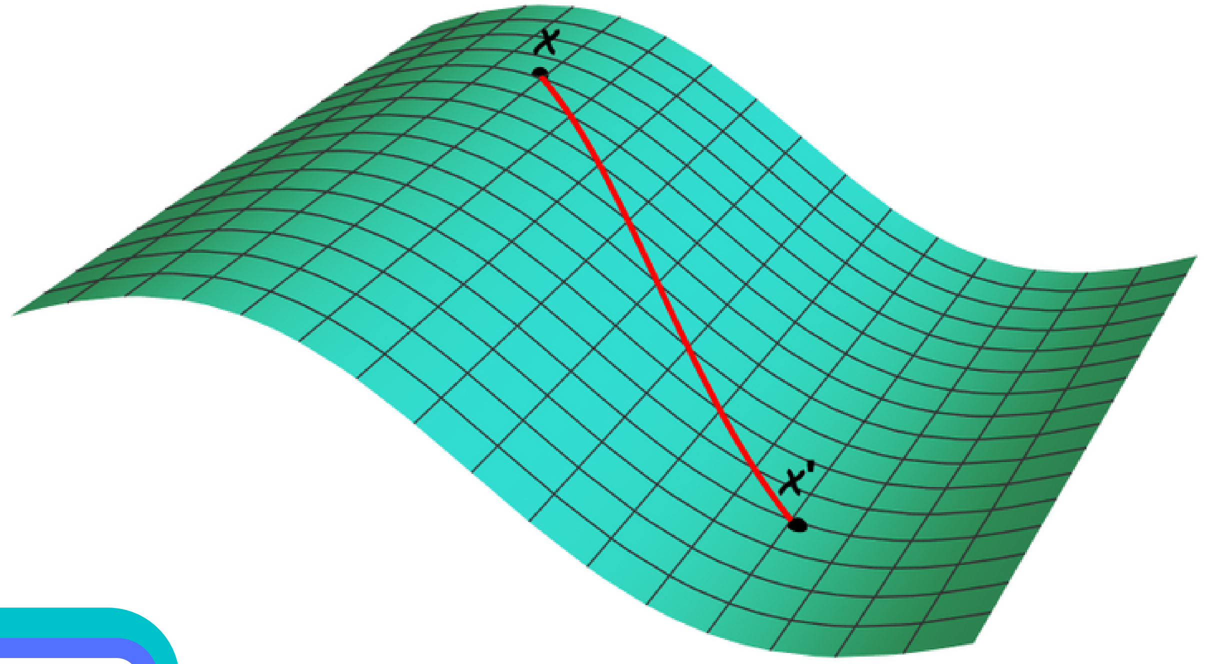

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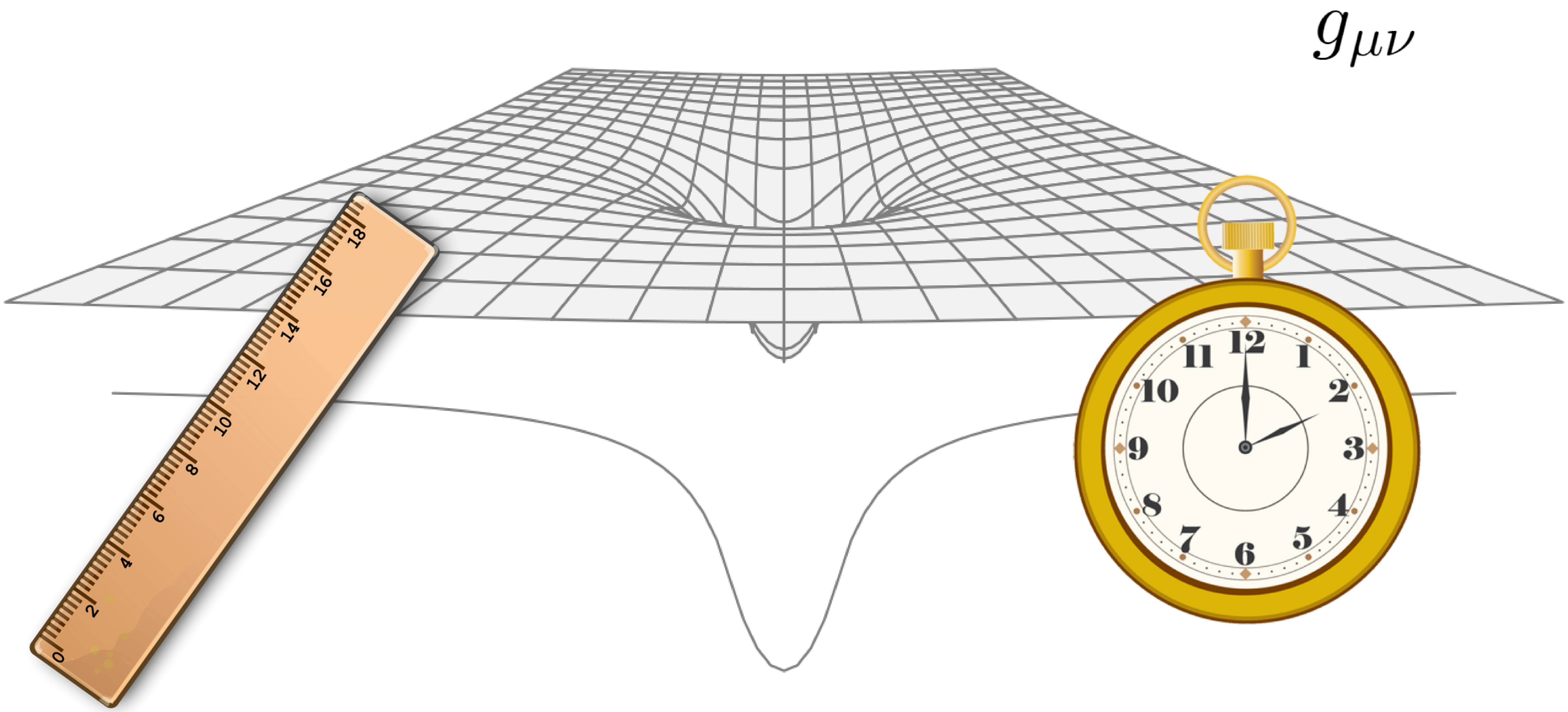

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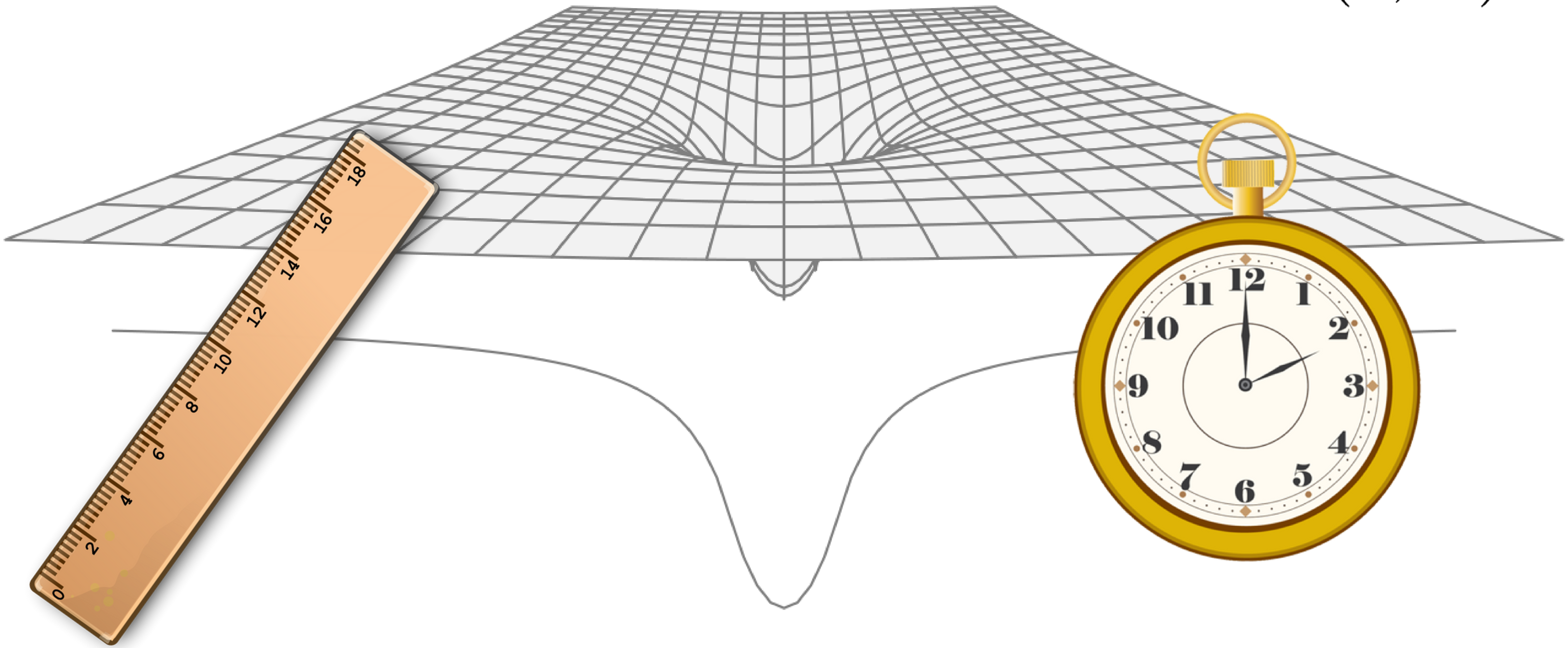

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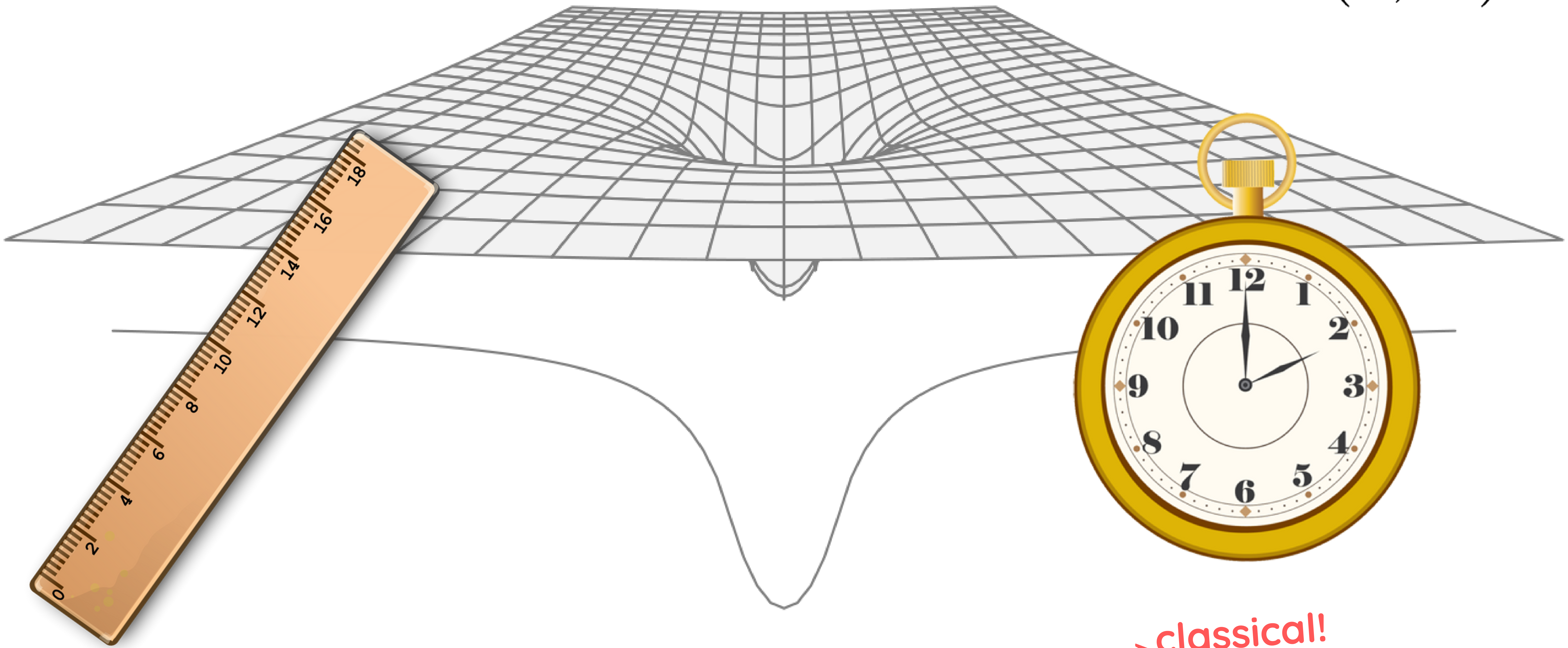

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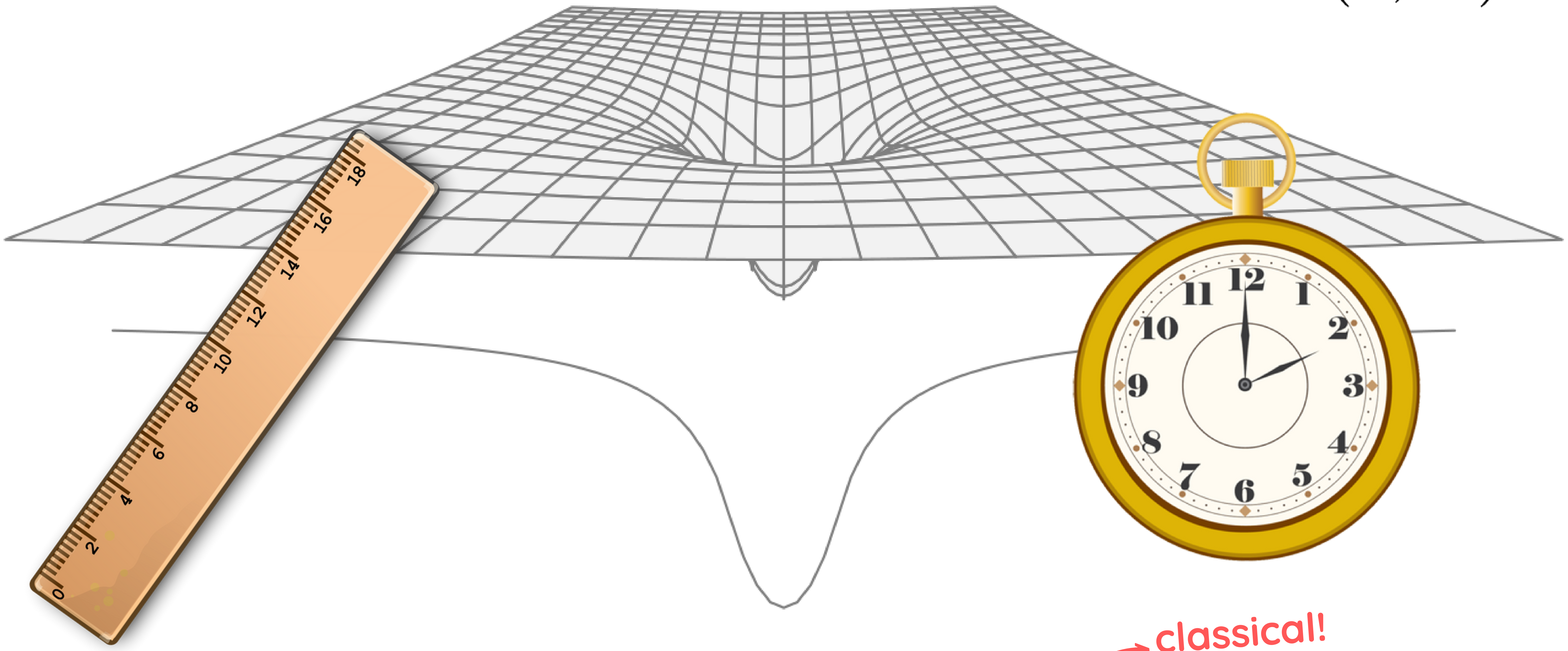

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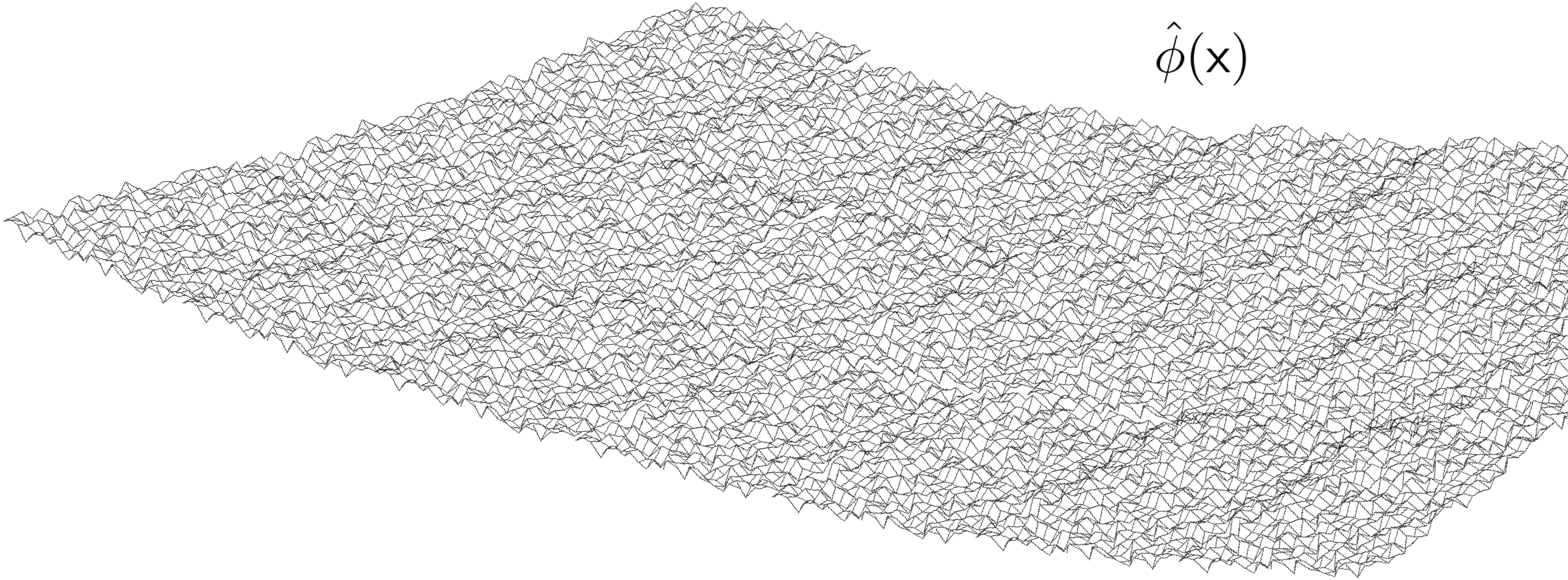

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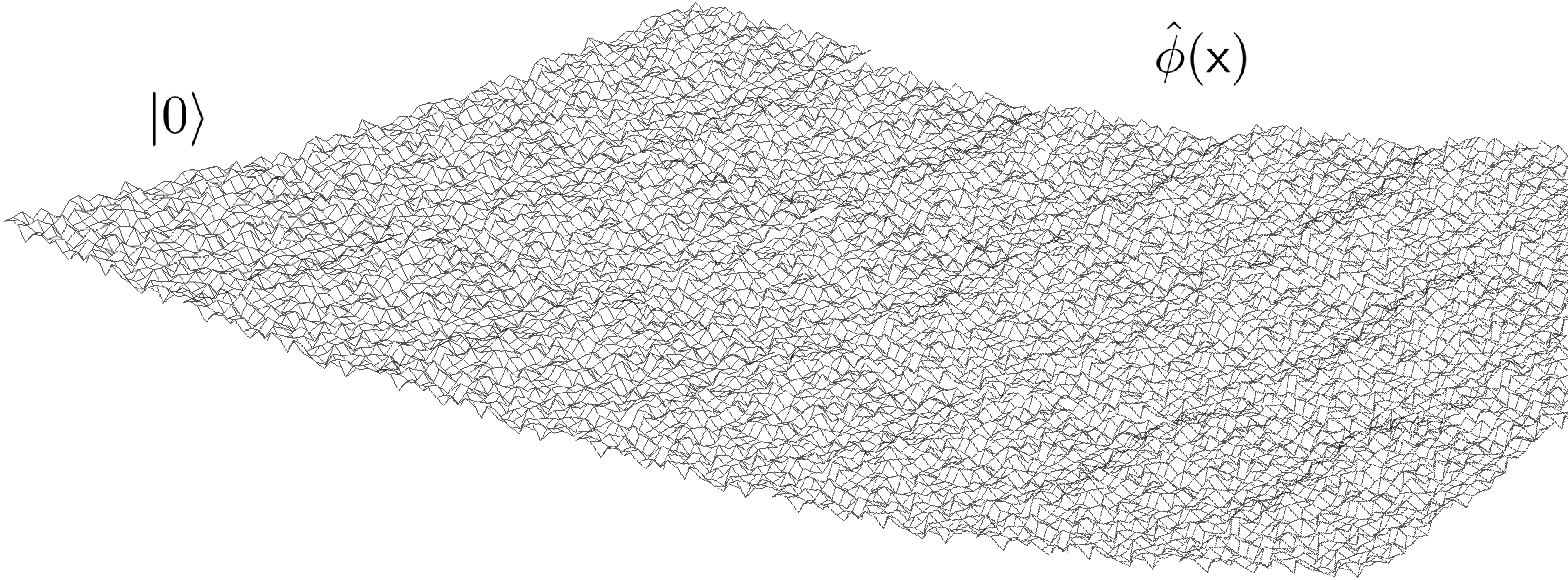

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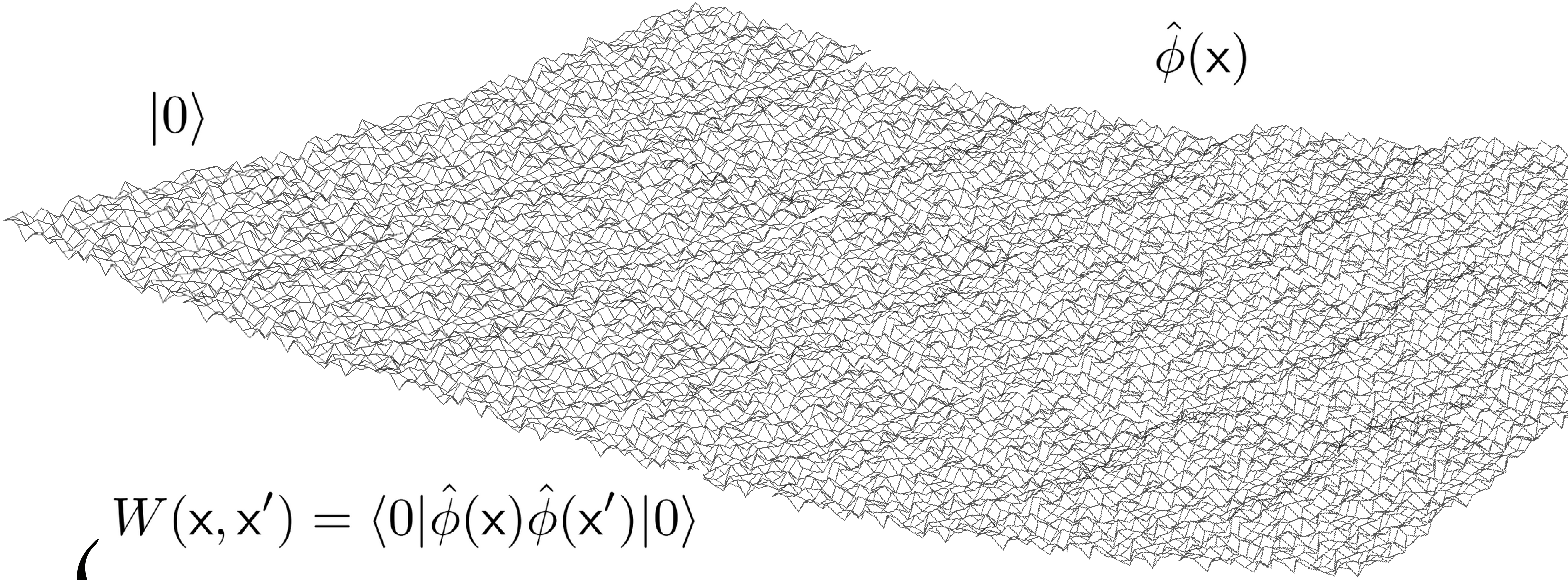

$$W(\mathrm{x},\mathrm{x}') = \langle 0|\hat{\phi}(\mathrm{x})\hat{\phi}(\mathrm{x}')|0\rangle$$

The Wightman function

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This can be used to define spacetime separations in scales where QFT is valid, but general relativity is not.

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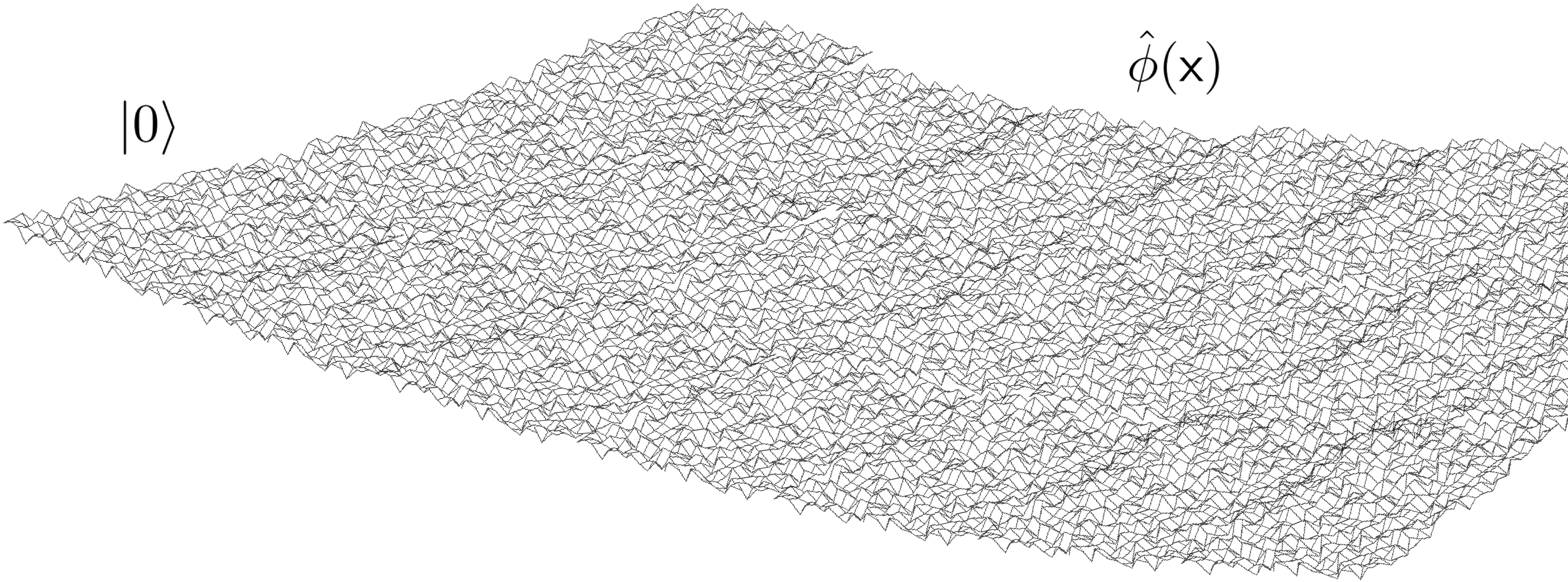
Measuring the Correlation Function



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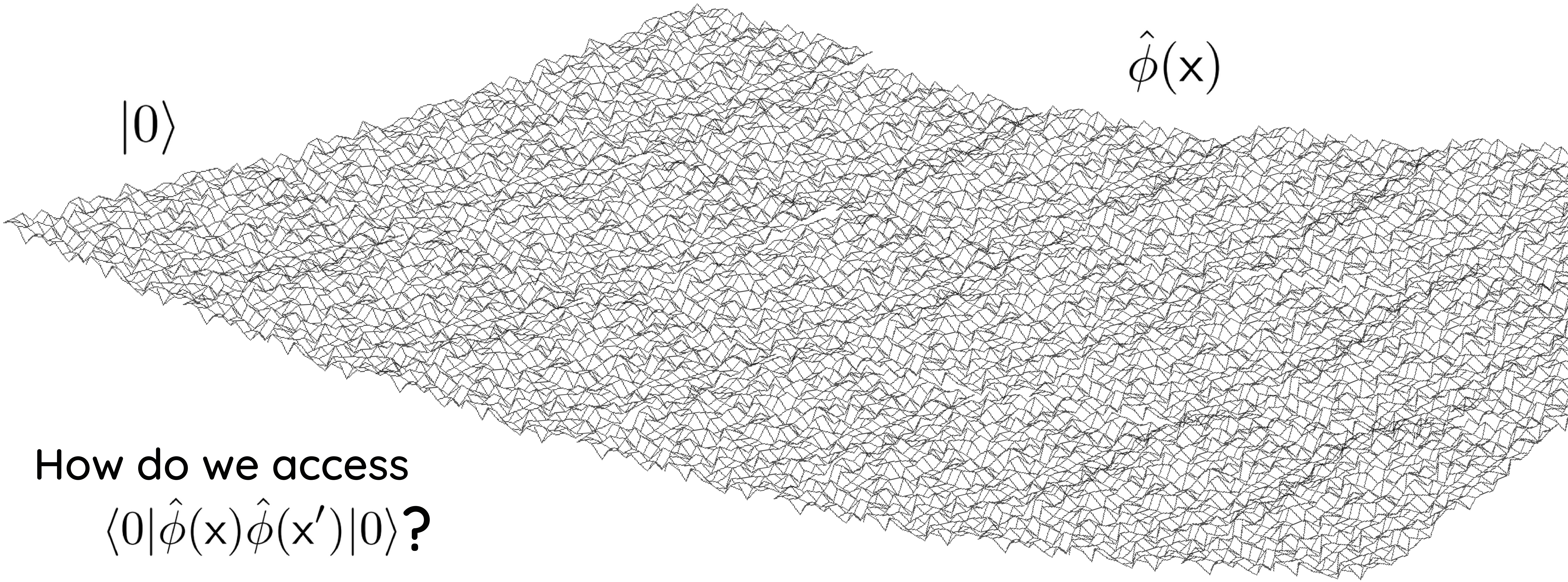

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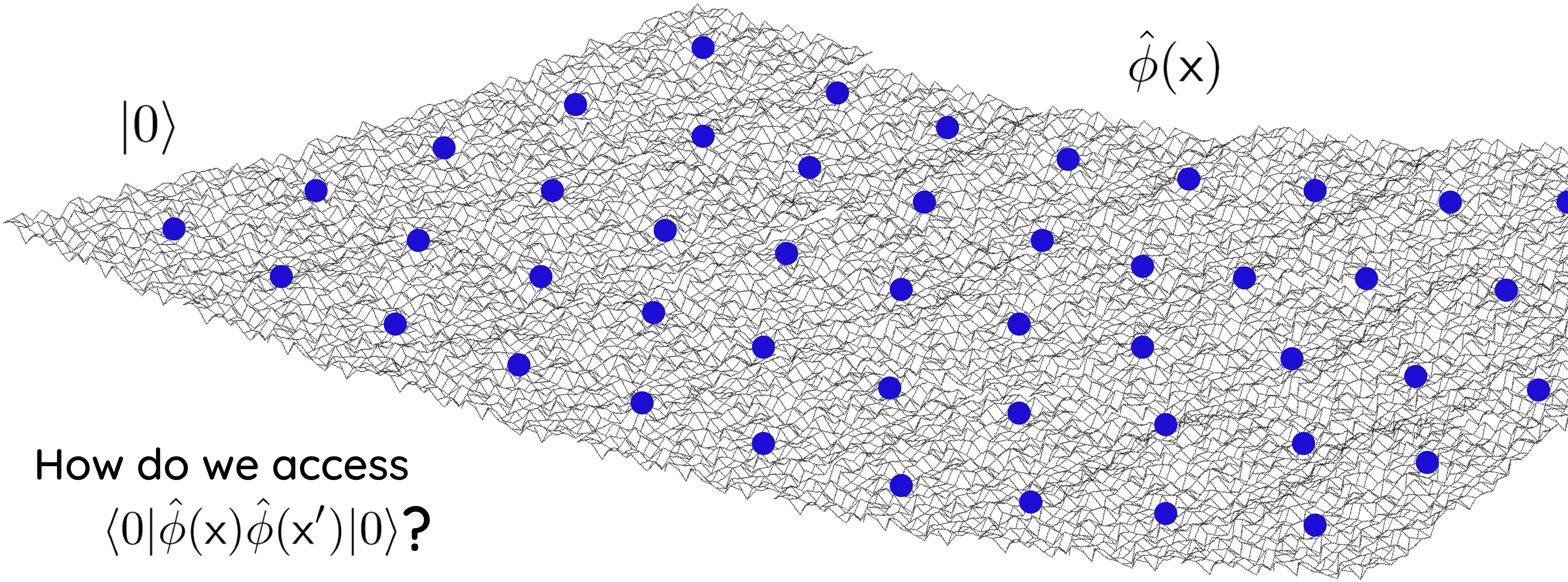

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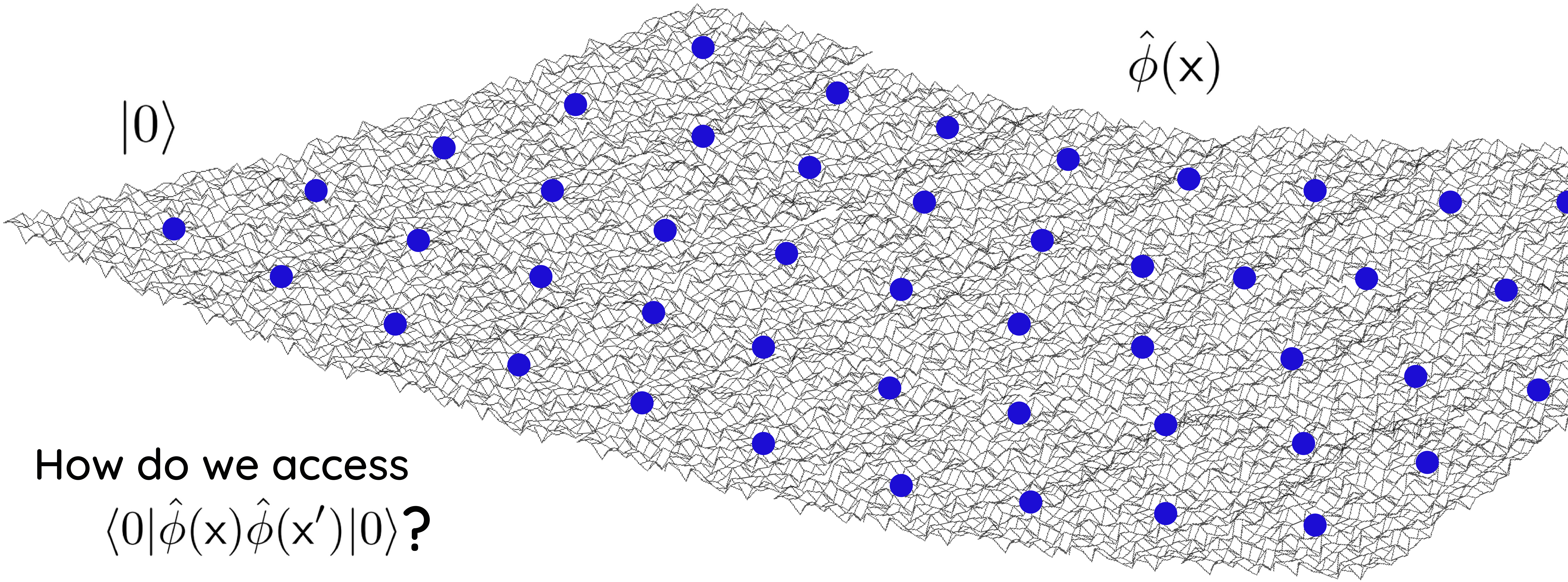

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→ Particle Detectors

Particle Detector Models

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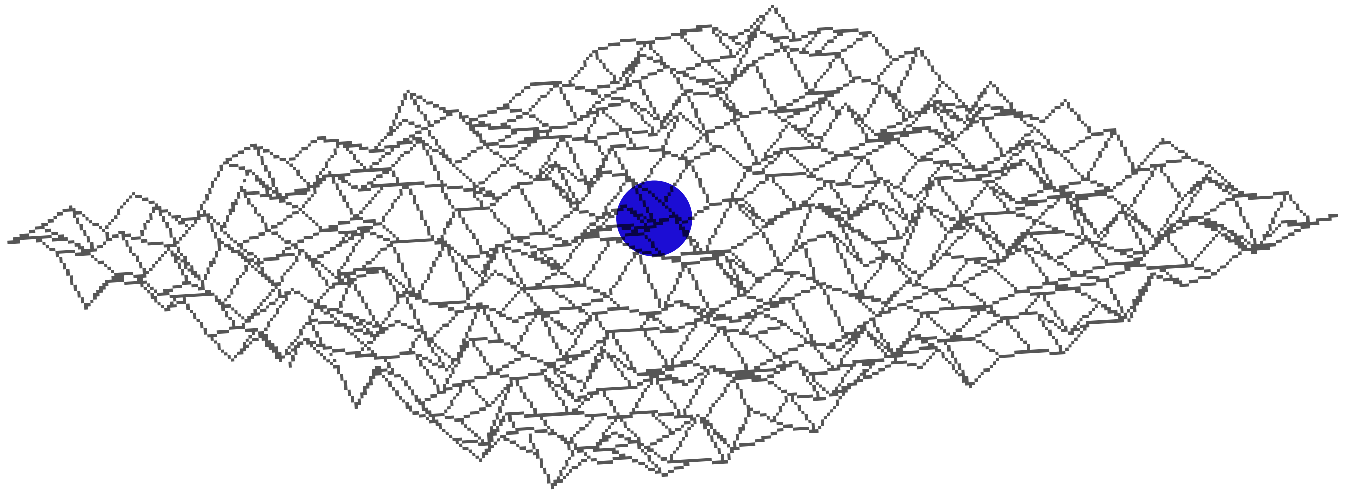

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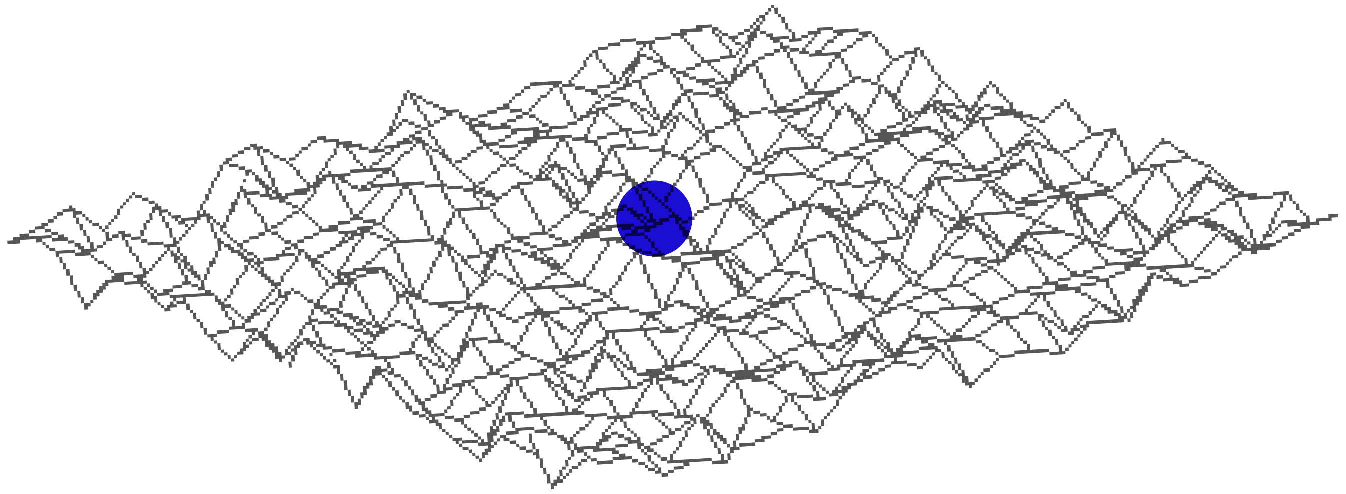

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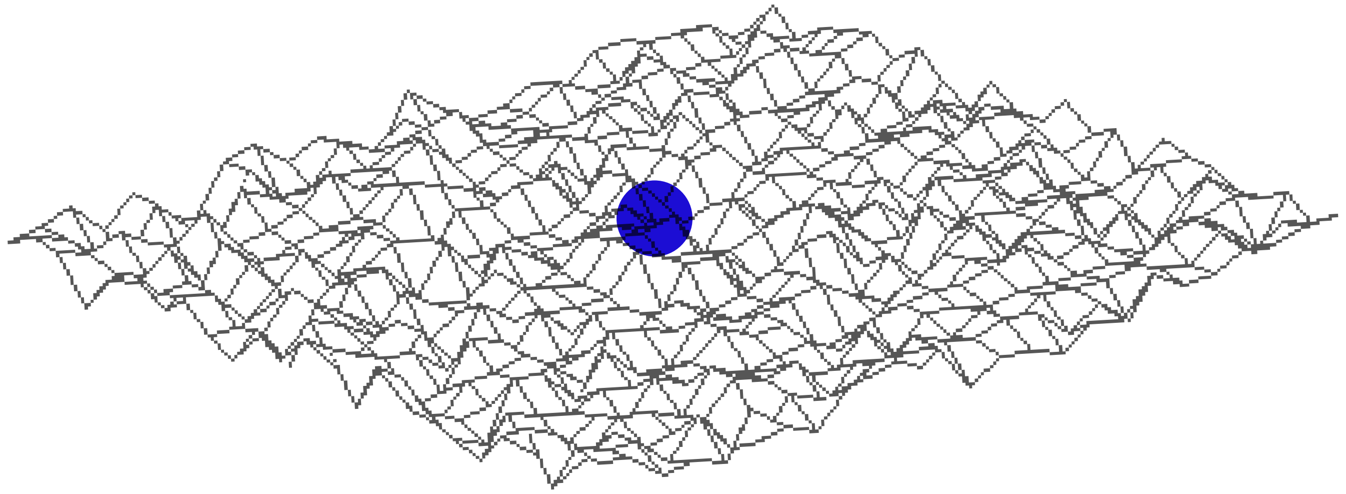

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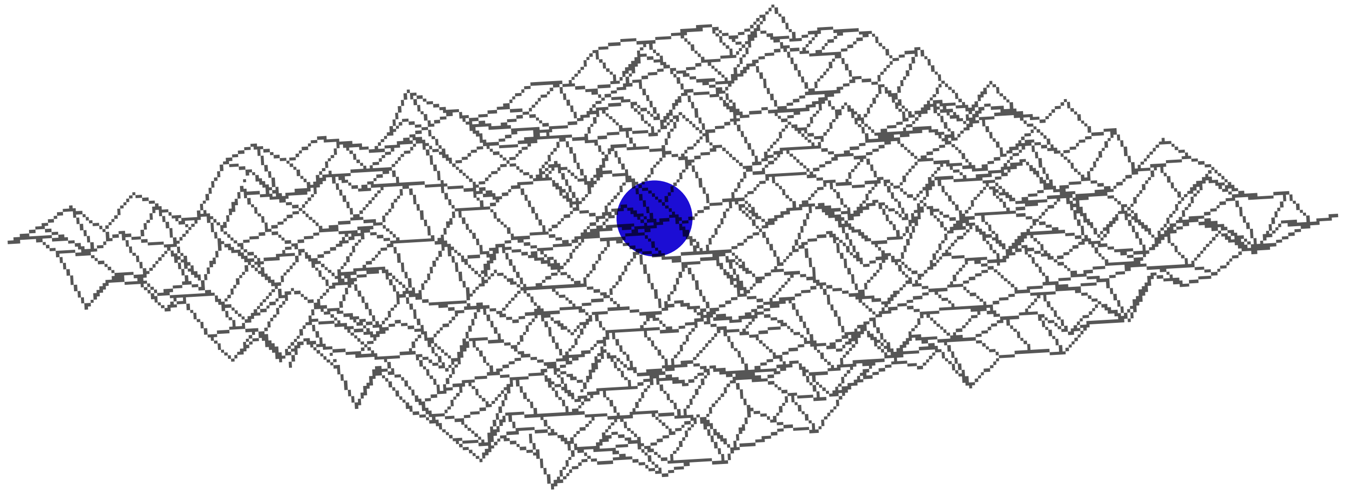

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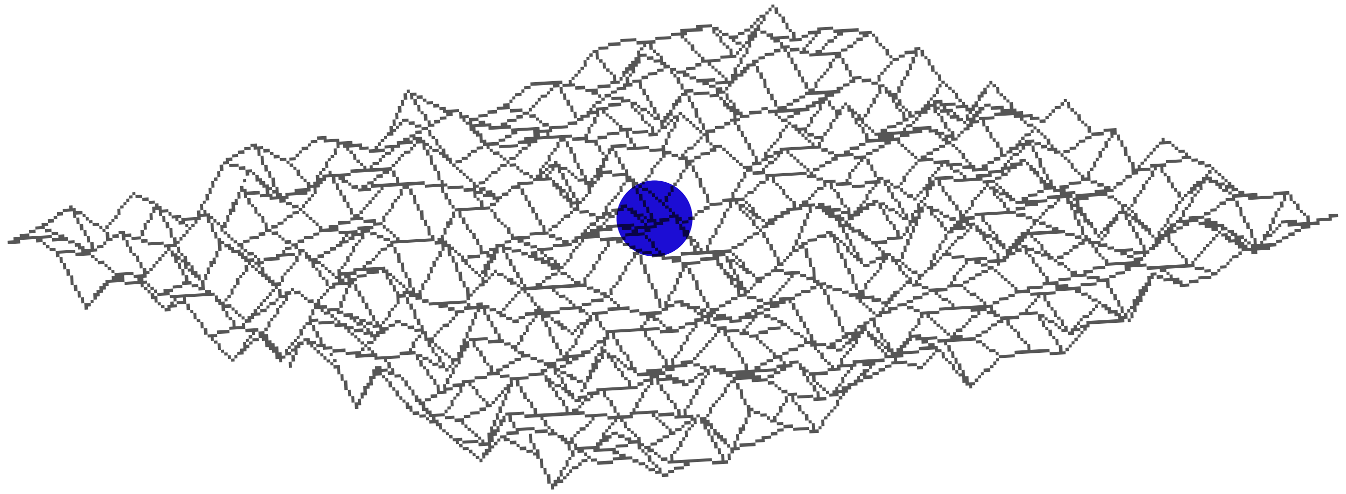

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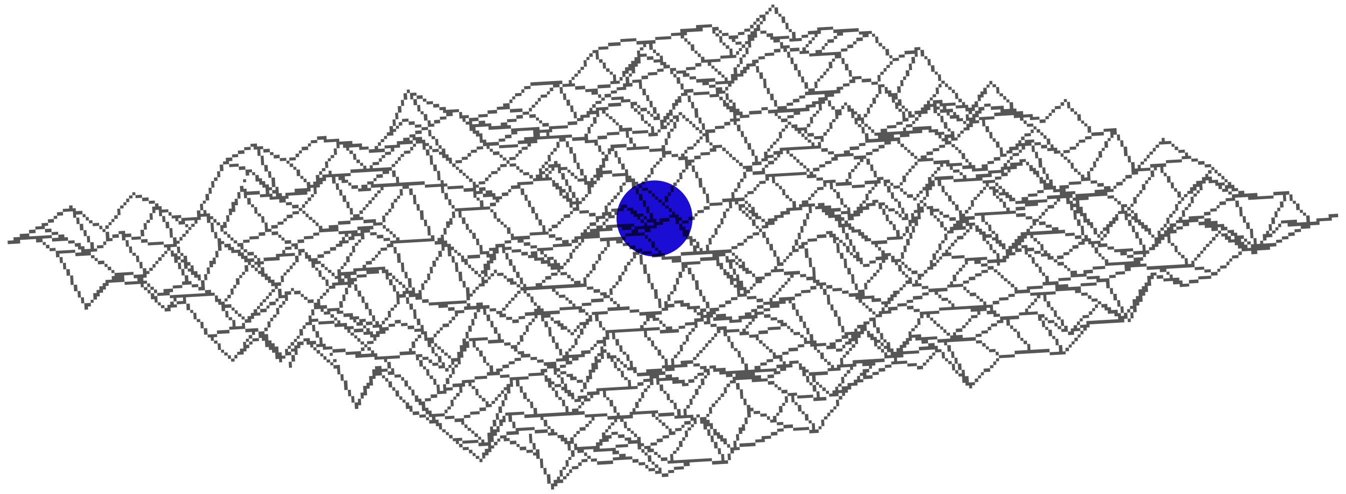

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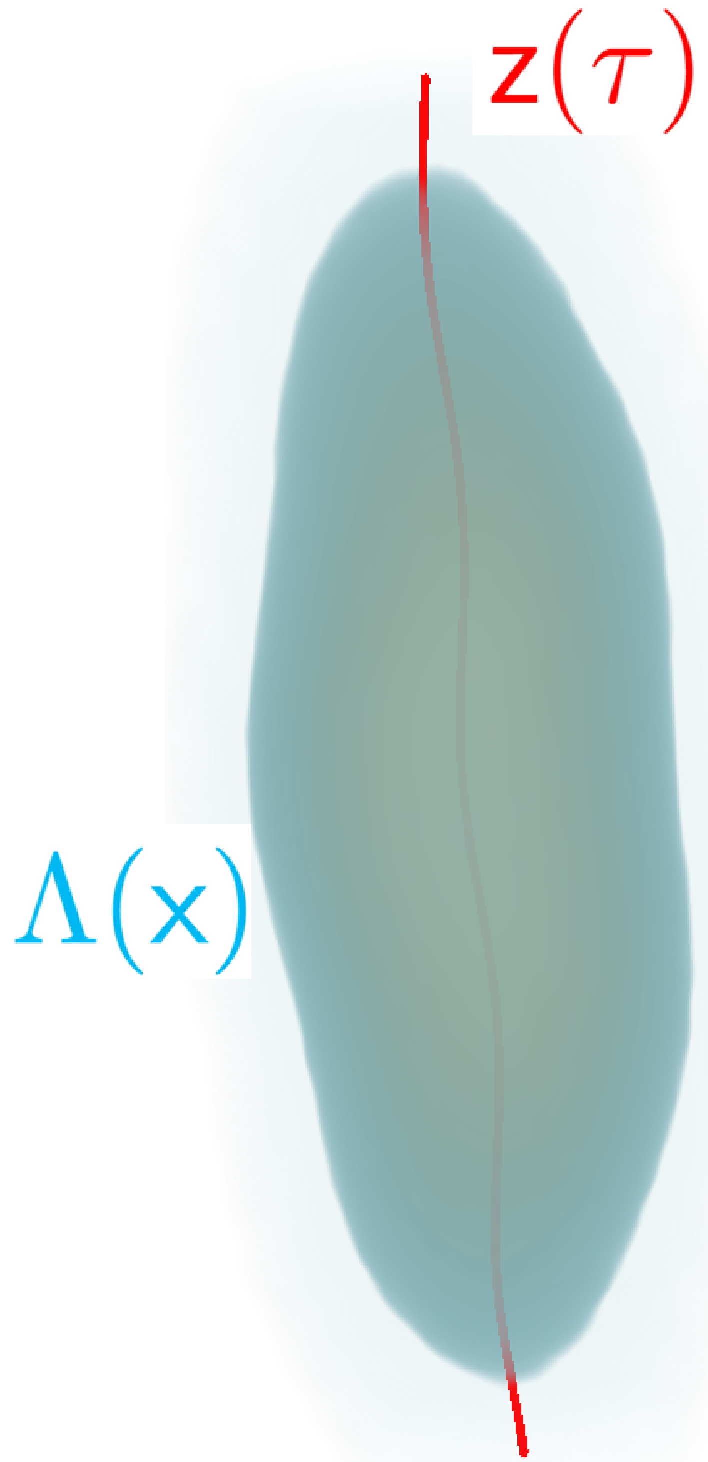

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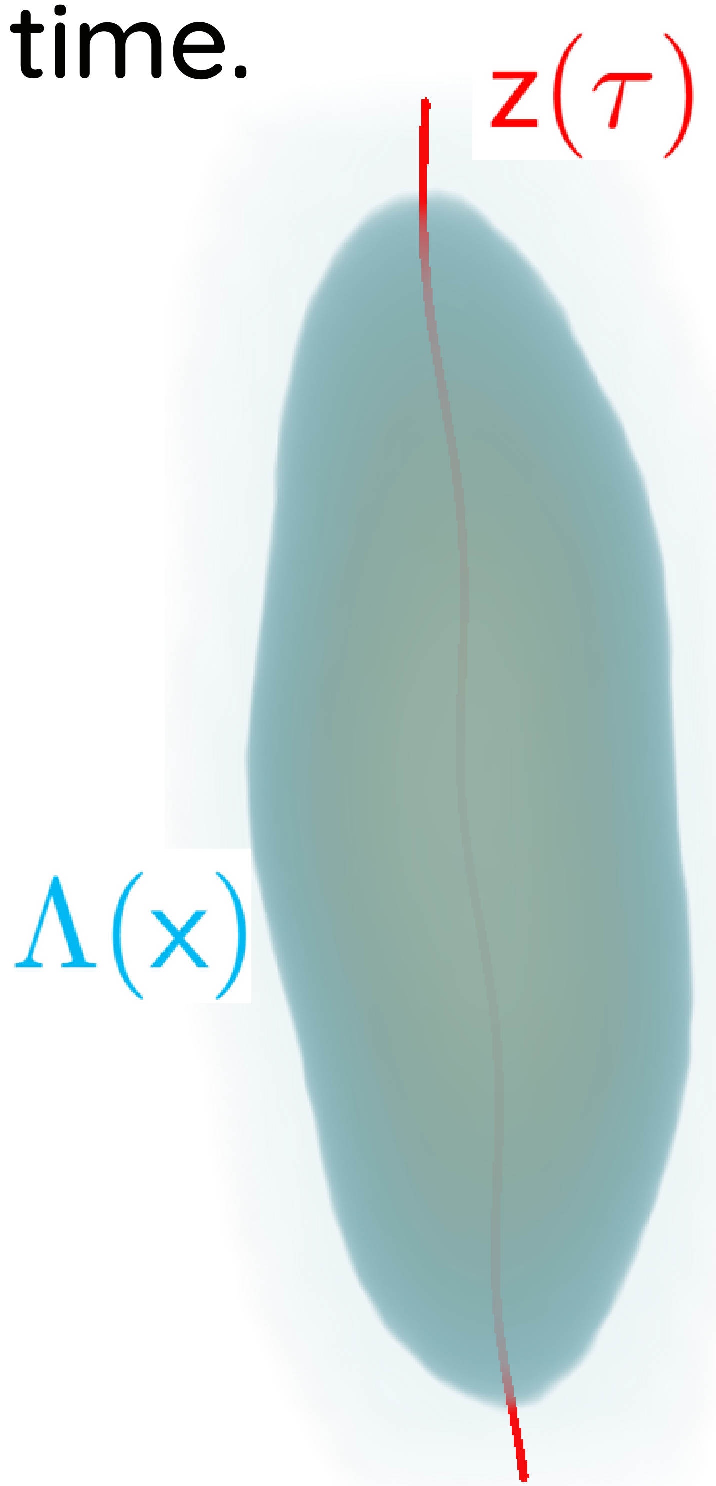

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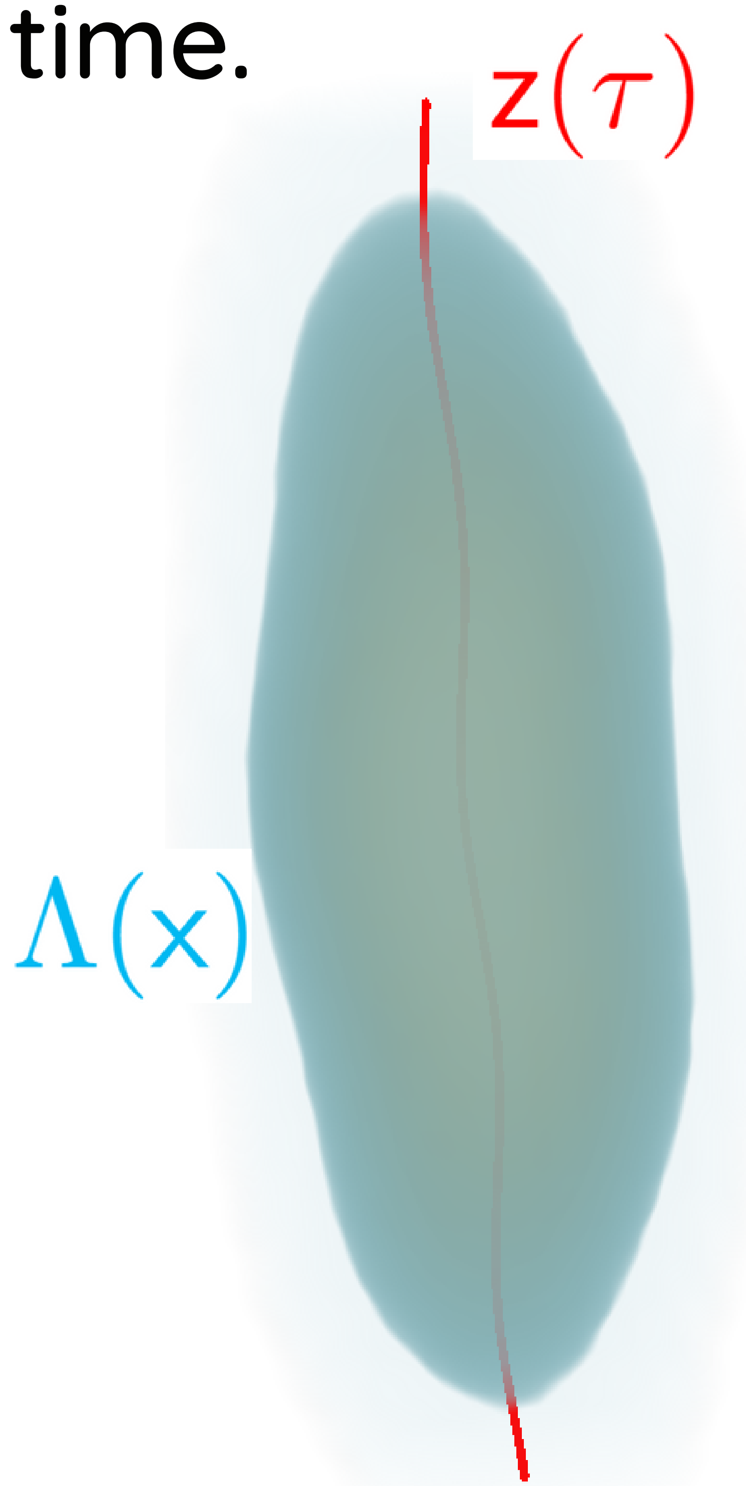

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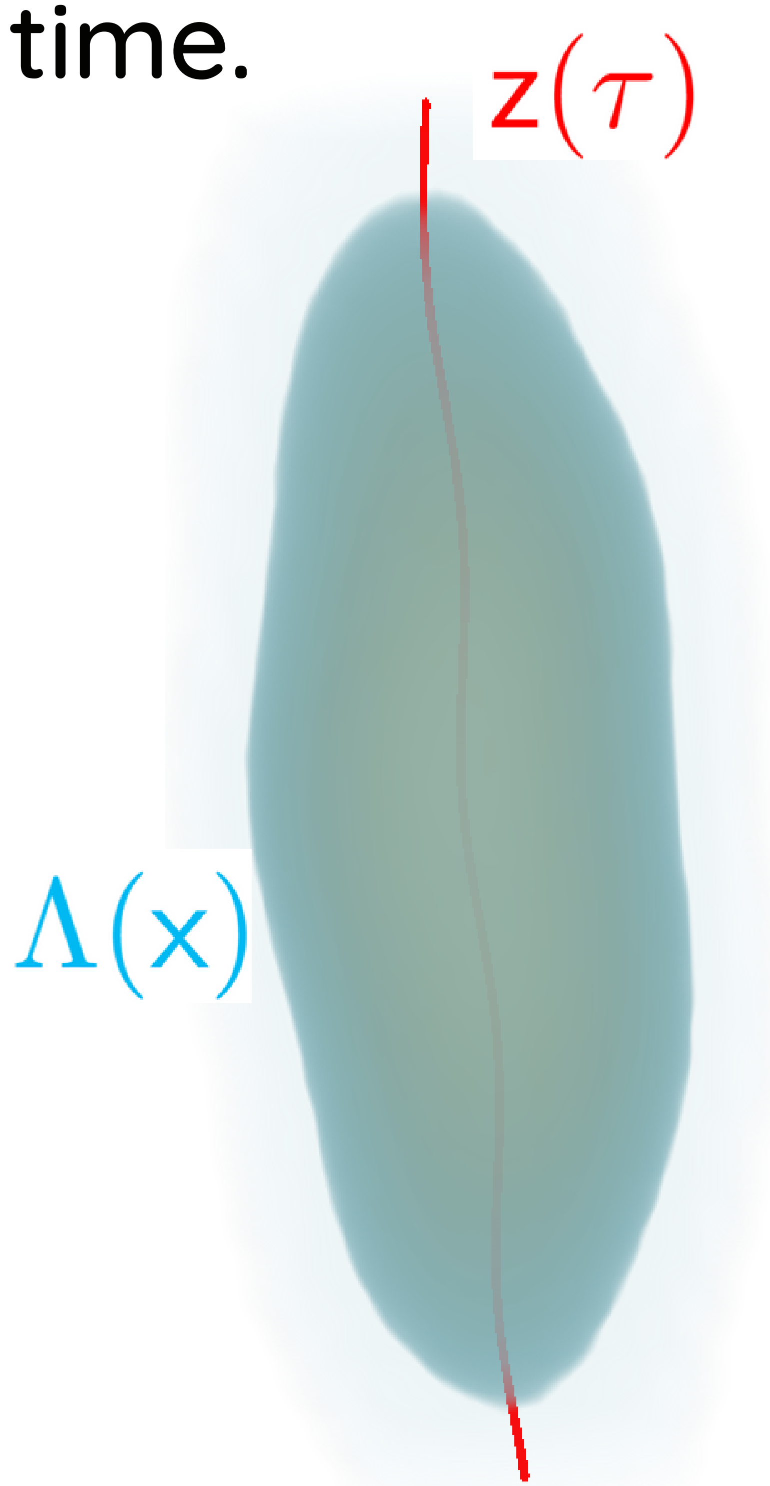

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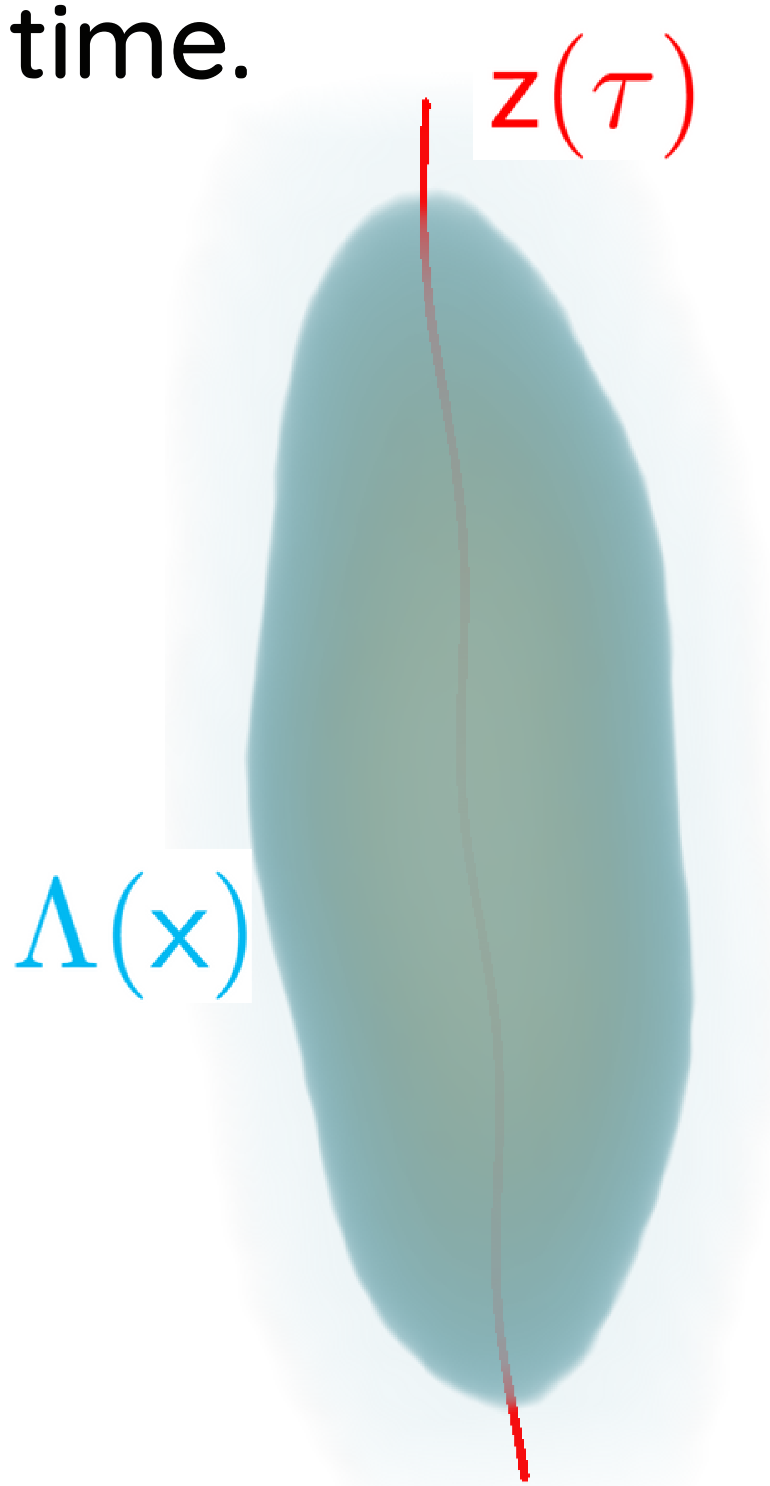

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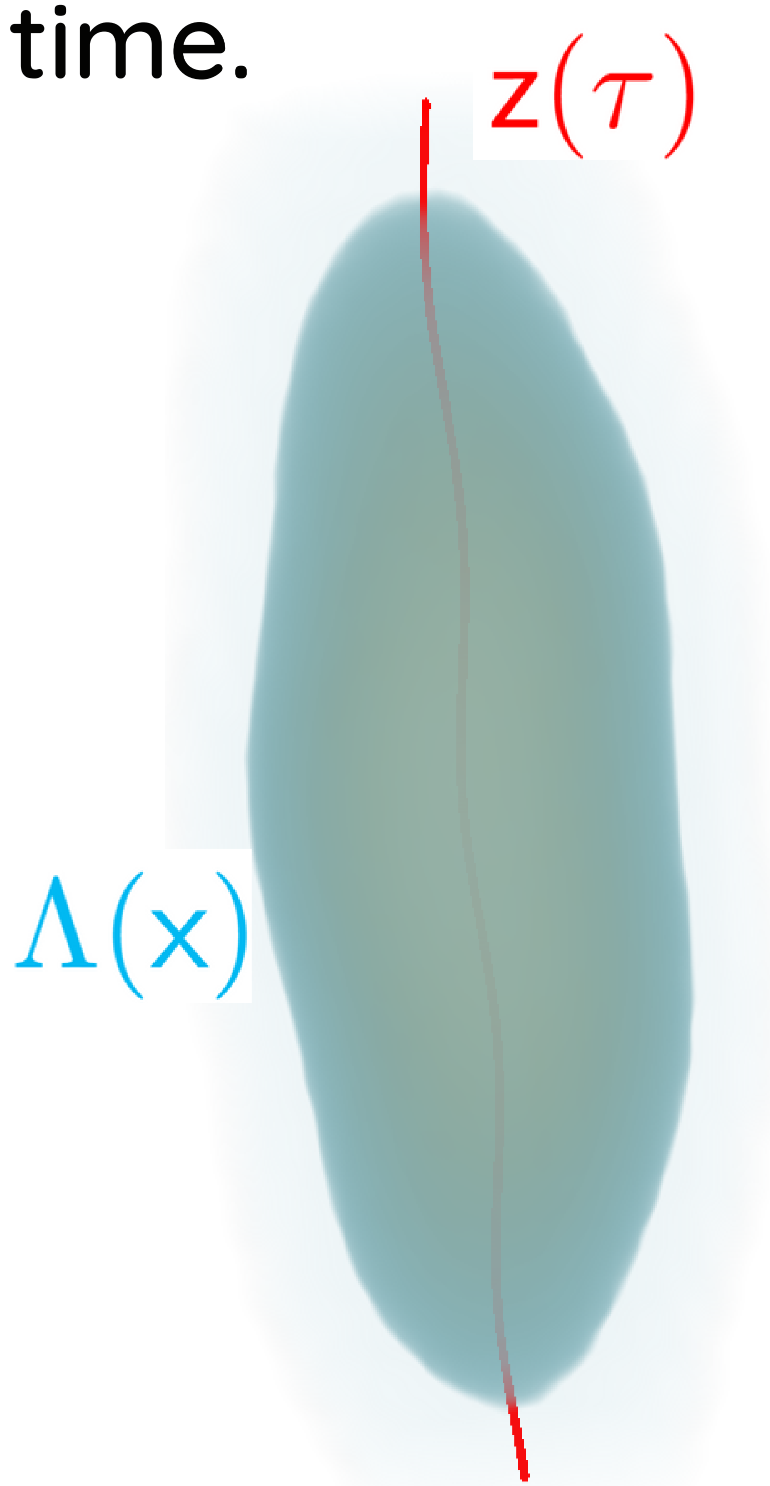

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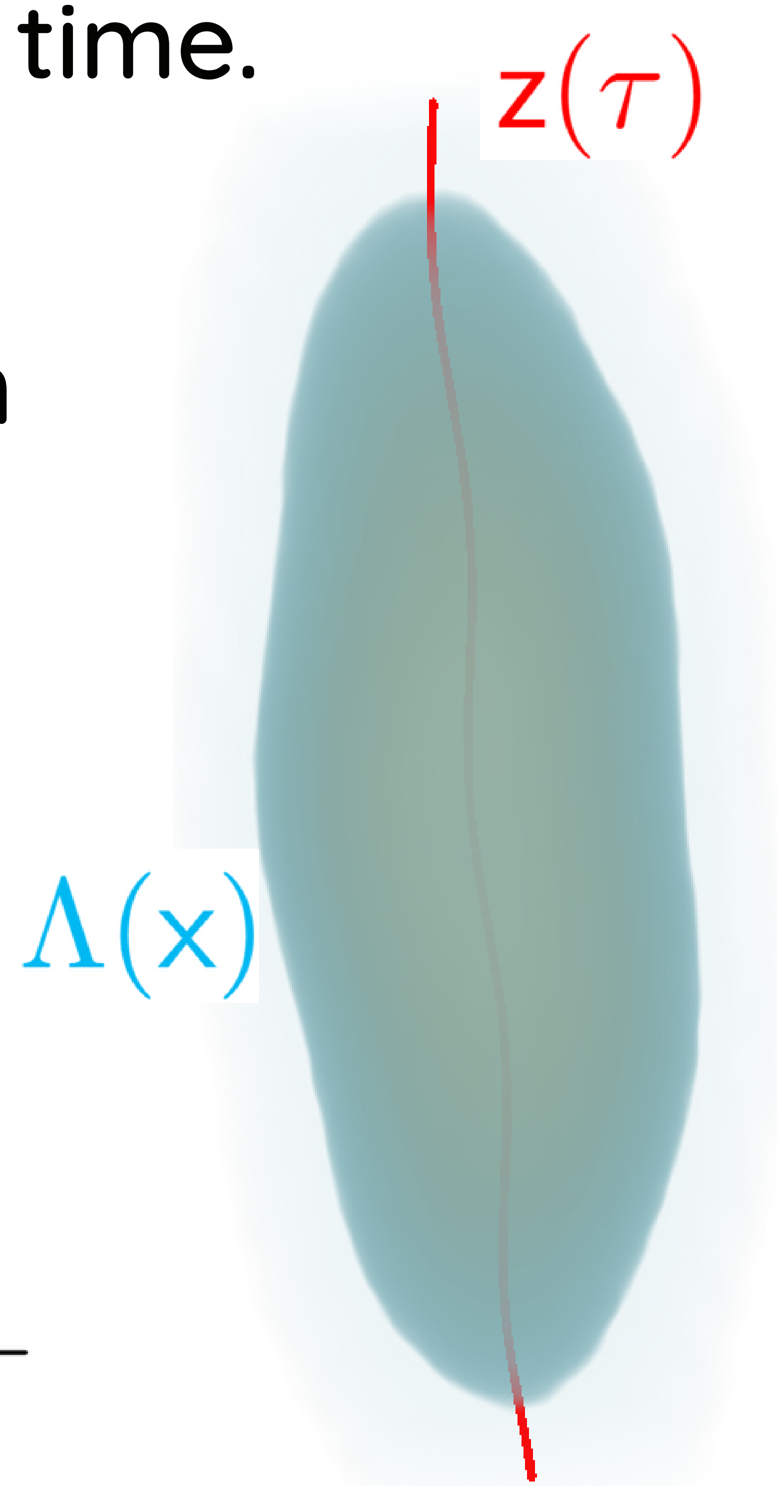

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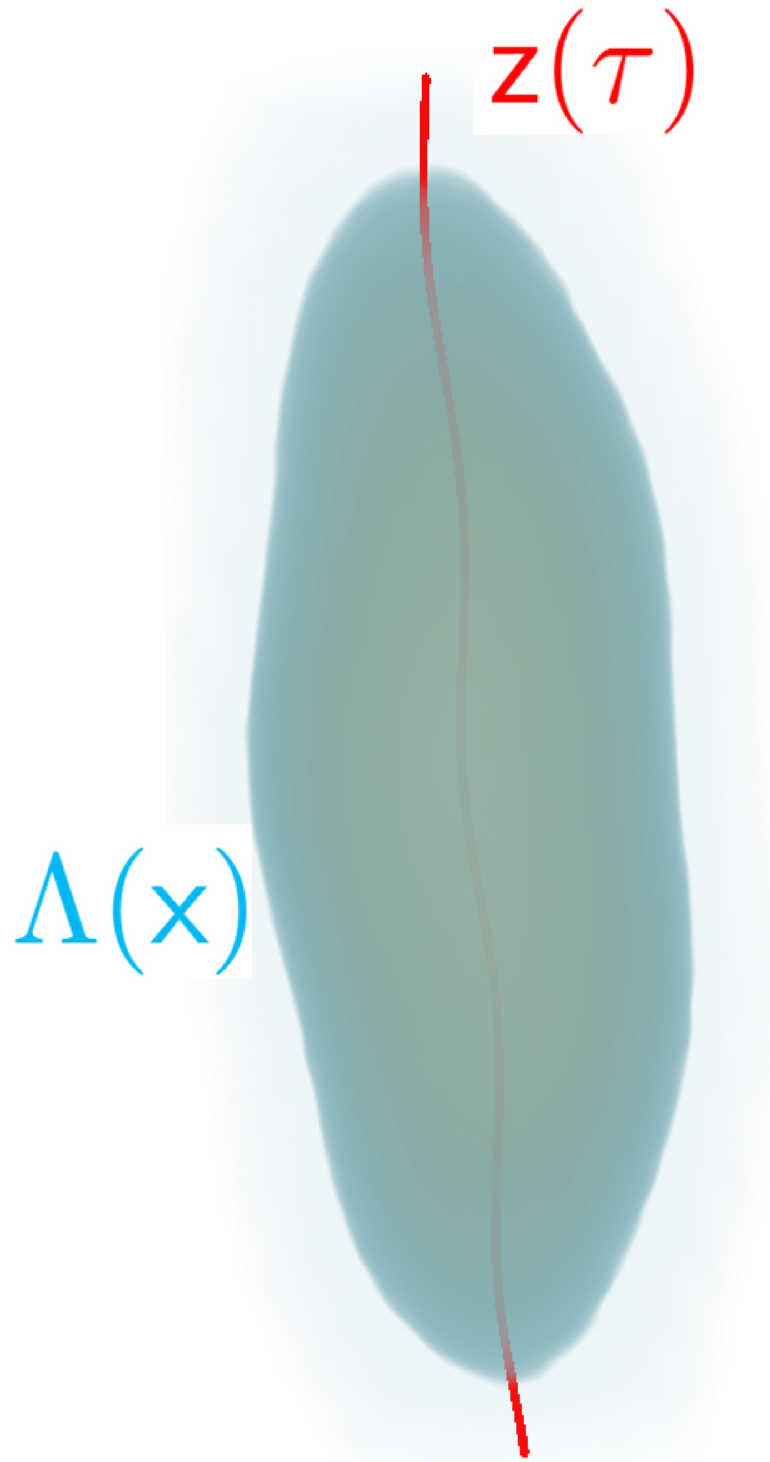

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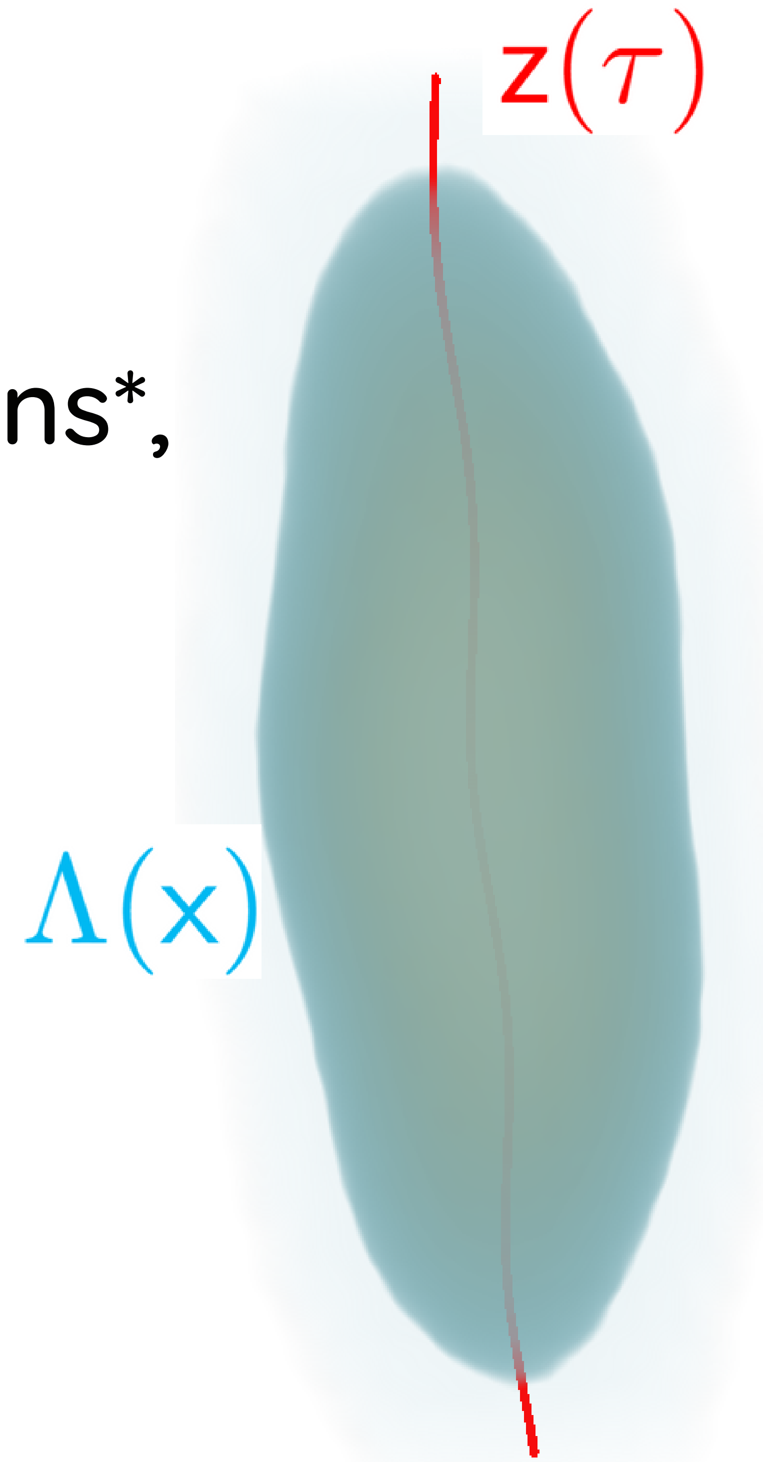

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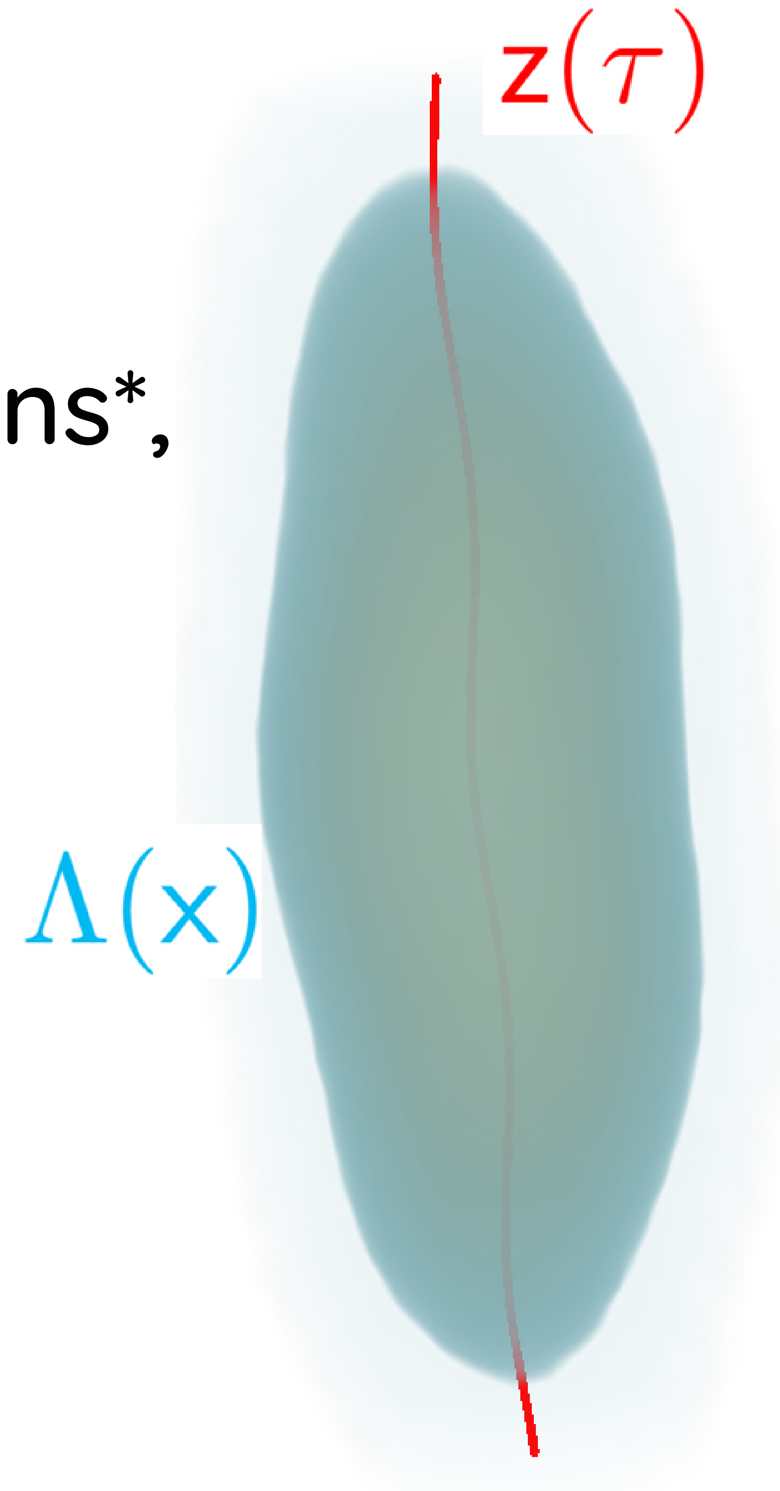

[5] Alejandro Pozas-Kerstjens and Eduardo Martín-Martínez - Phys. Rev. D 94, 064074 (2016)
[6] Nicholas Funai, Jorma Louko, and Eduardo Martín-Martínez - Phys. Rev. D 99, 065014 (2019)
[7] Richard Lopp and Eduardo Martín-Martínez - Phys. Rev. A 103, 013703 (2021)

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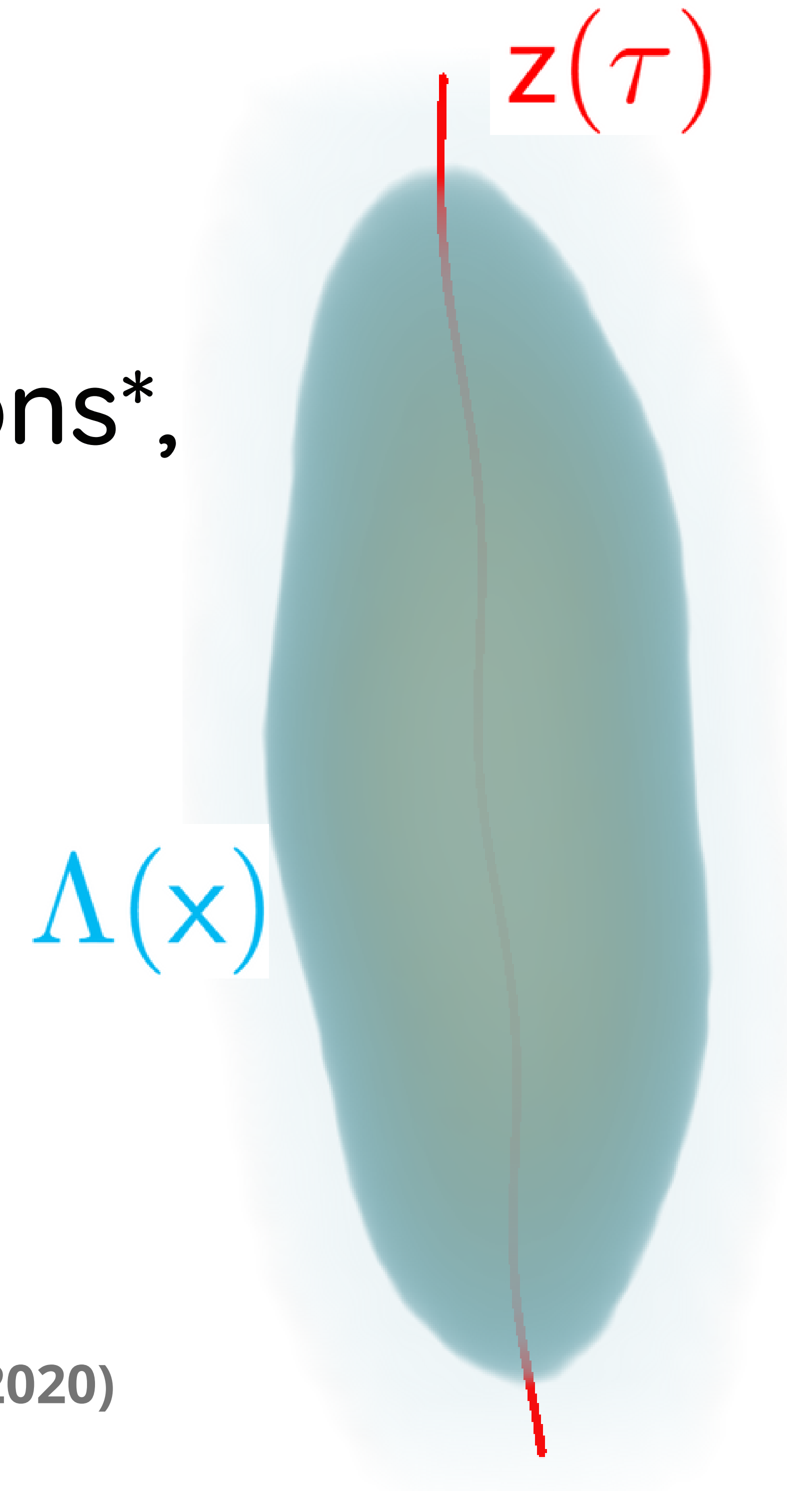
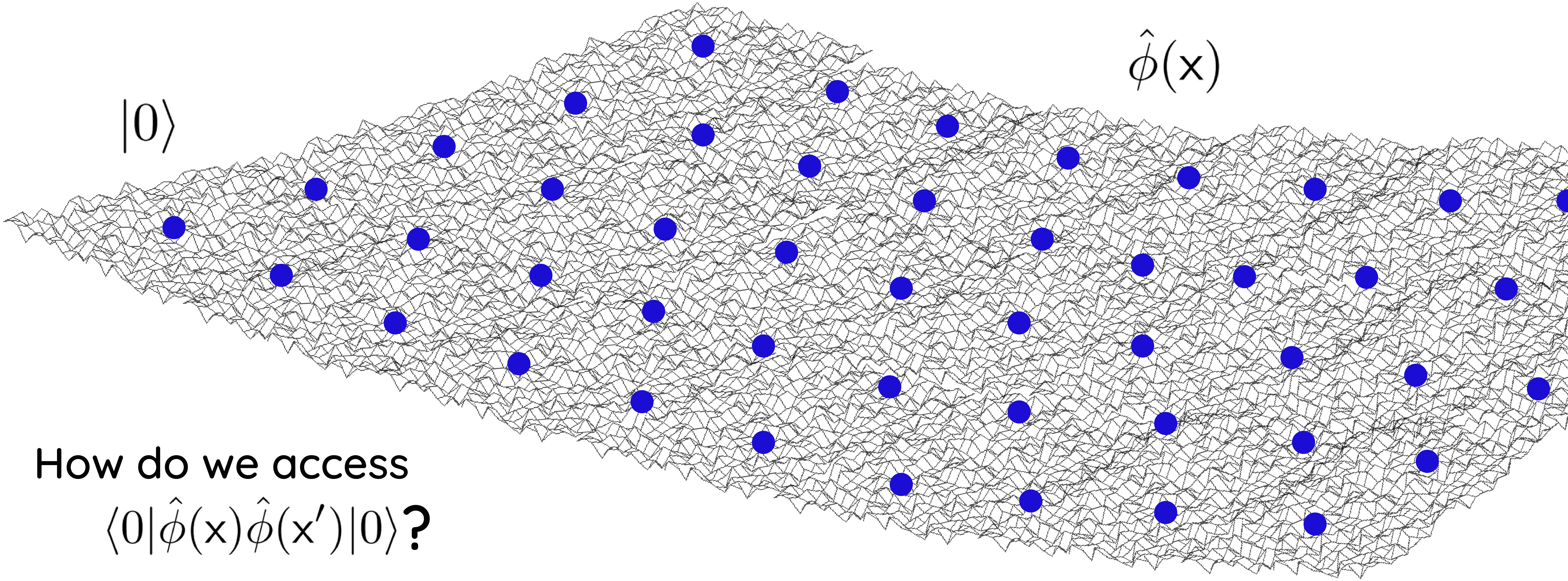
→ Light-Matter interaction.[5-7]

→ Interactions of nucleons with neutrinos.[8,9]



[5] Alejandro Pozas-Kerstjens and Eduardo Martín-Martínez - Phys. Rev. D 94, 064074 (2016)
[6] Nicholas Funai, Jorma Louko, and Eduardo Martín-Martínez - Phys. Rev. D 99, 065014 (2019)
[7] Richard Lopp and Eduardo Martín-Martínez - Phys. Rev. A 103, 013703 (2021)
[8] Bruno de S. L. Torres, T. Rick Perche, André G. S. Landulfo, and George E. A. Matsas - Phys. Rev. D 102, 093003 (2020)
[9] T. Rick Perche and Eduardo Martín-Martínez -Phys. Rev. D 104, 105021 (2021)

Measuring the Correlation Function



How do we access

$$\langle 0|\hat{\phi}(x)\hat{\phi}(x')|0\rangle ?$$

Probes!

→ **Particle Detectors!**

Recovering the Correlation Functions

UDW detectors with sufficiently localized interactions can be used to recover the correlation functions of quantum fields!

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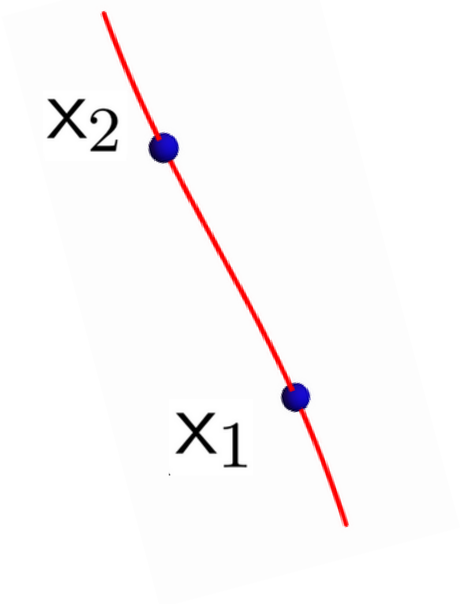

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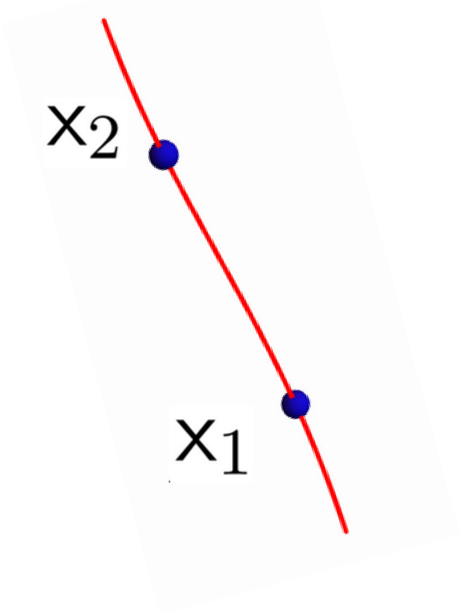

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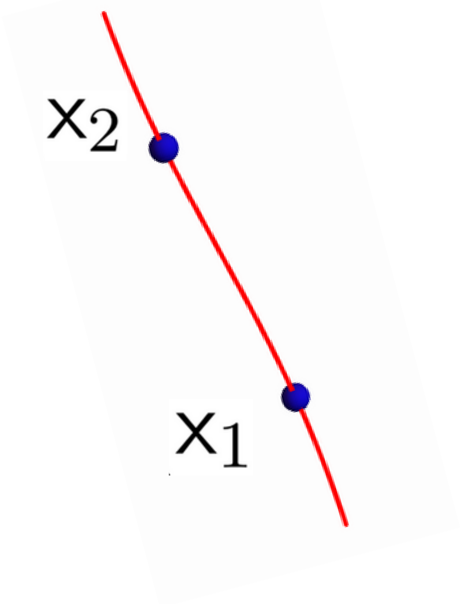

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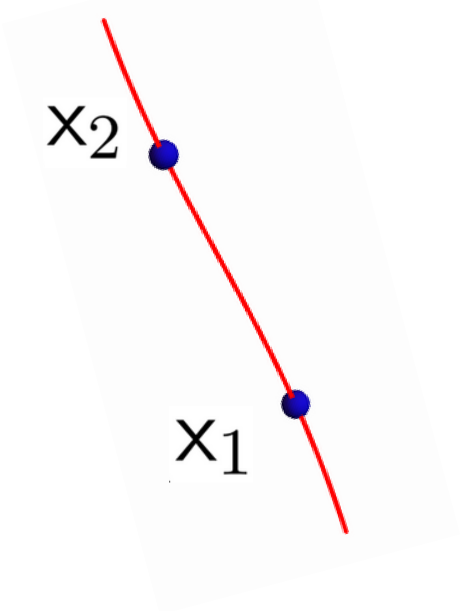

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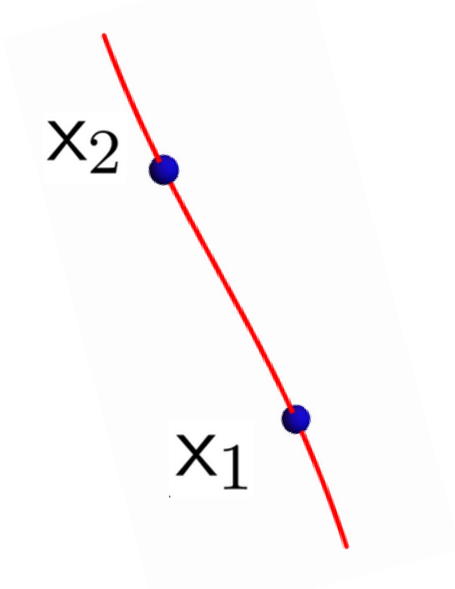

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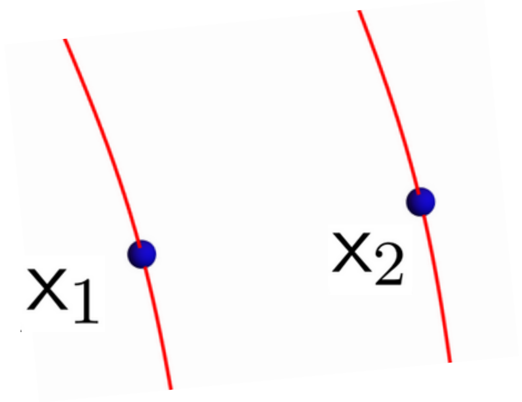

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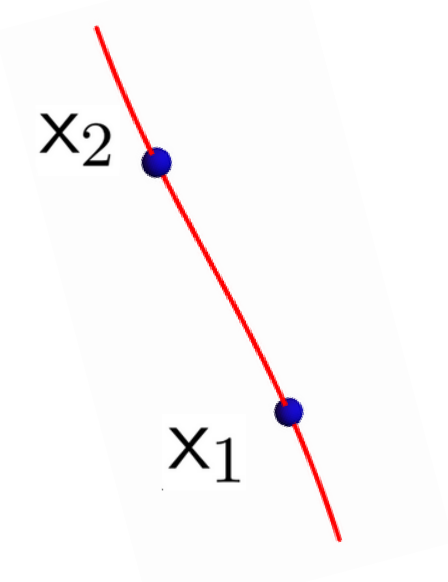

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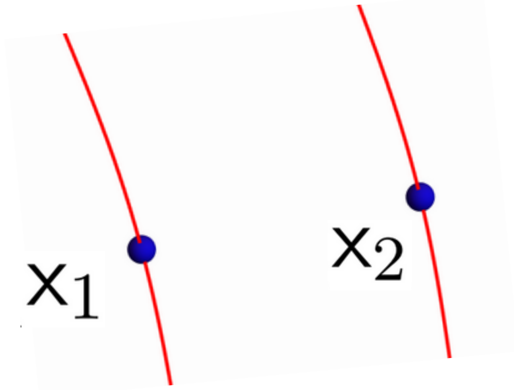

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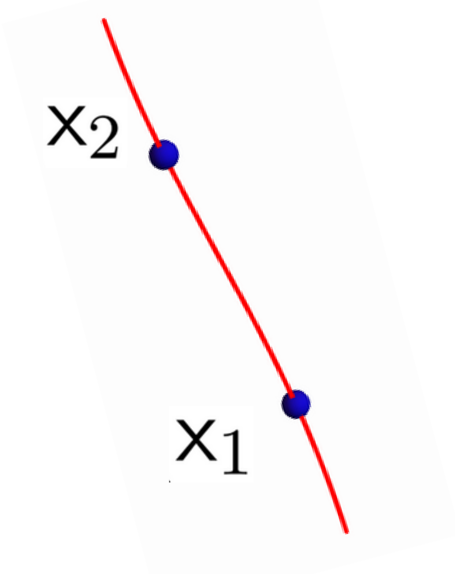

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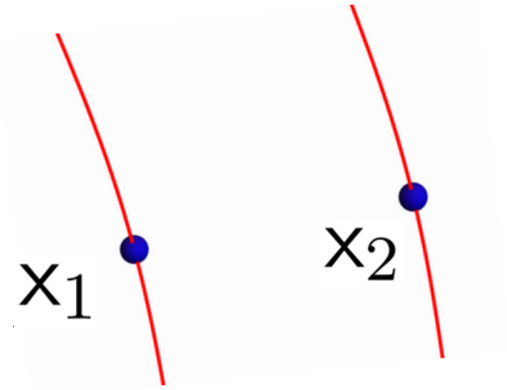

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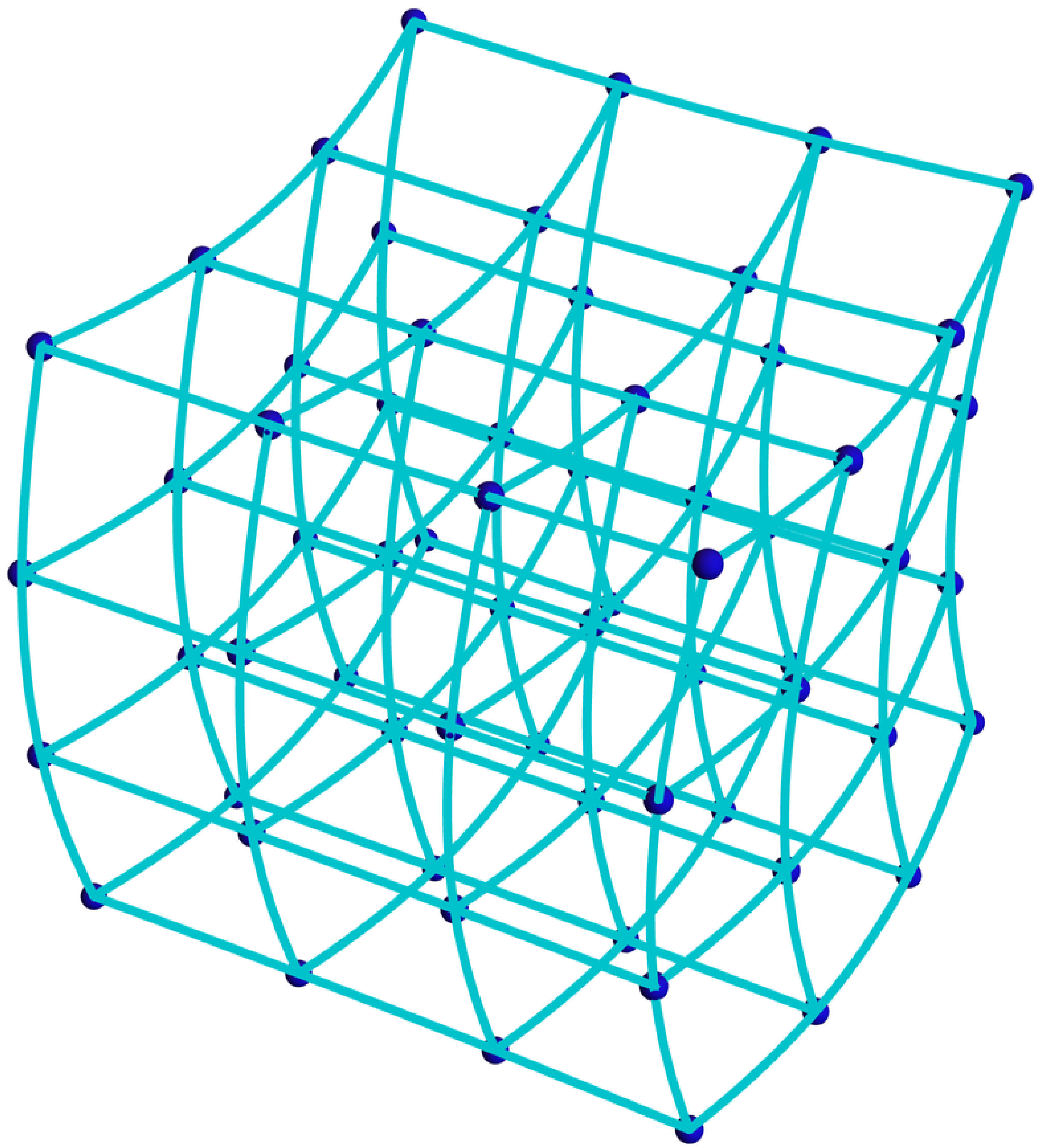
Spacetime Geometry from Quantum Measurements

If detectors can recover **correlation functions**, they also can recover the **geometry of spacetime**.



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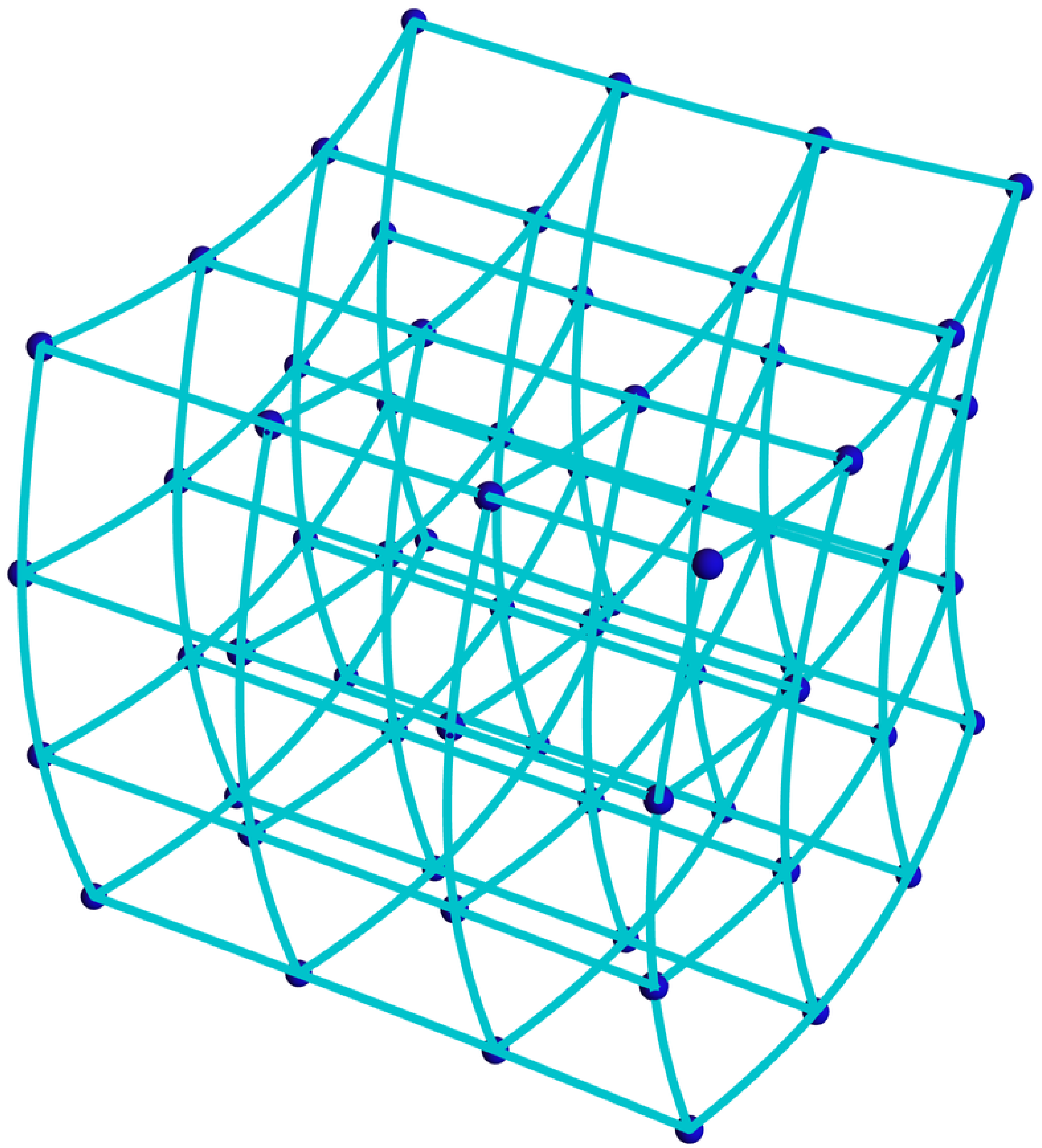

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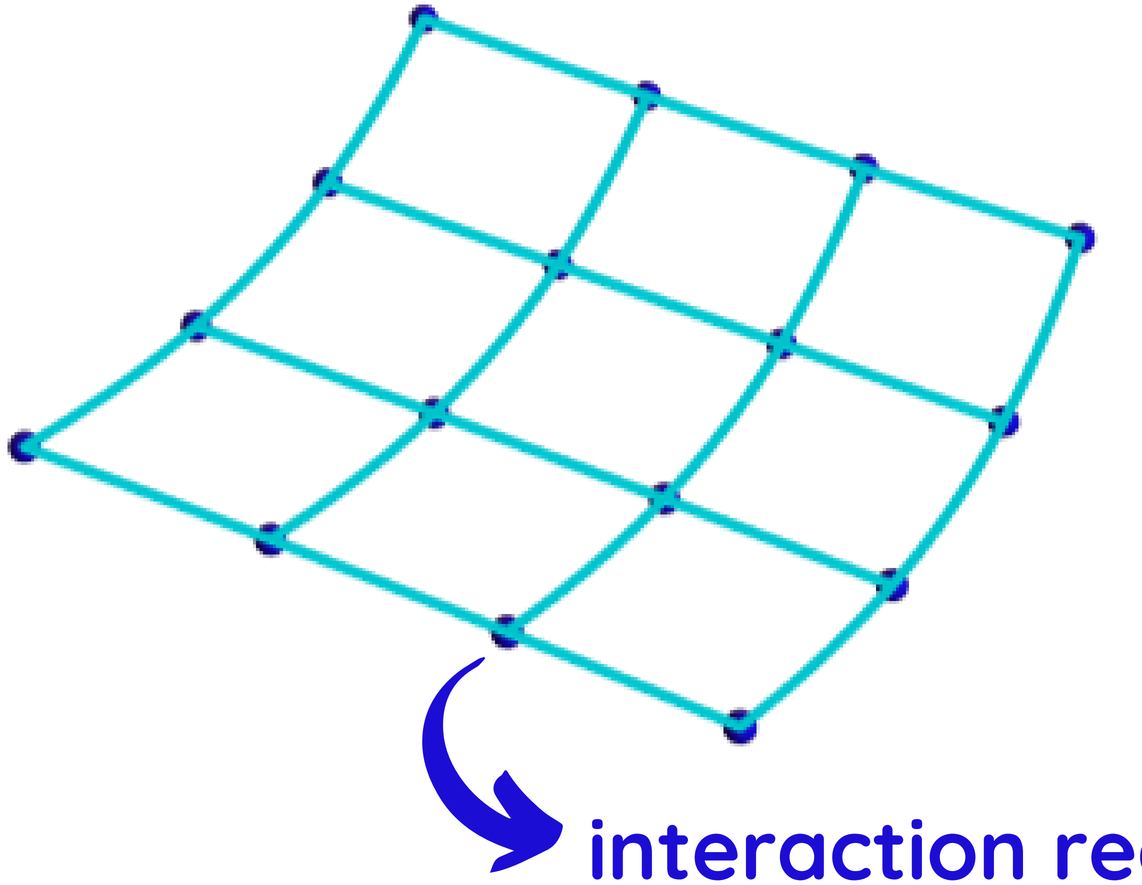

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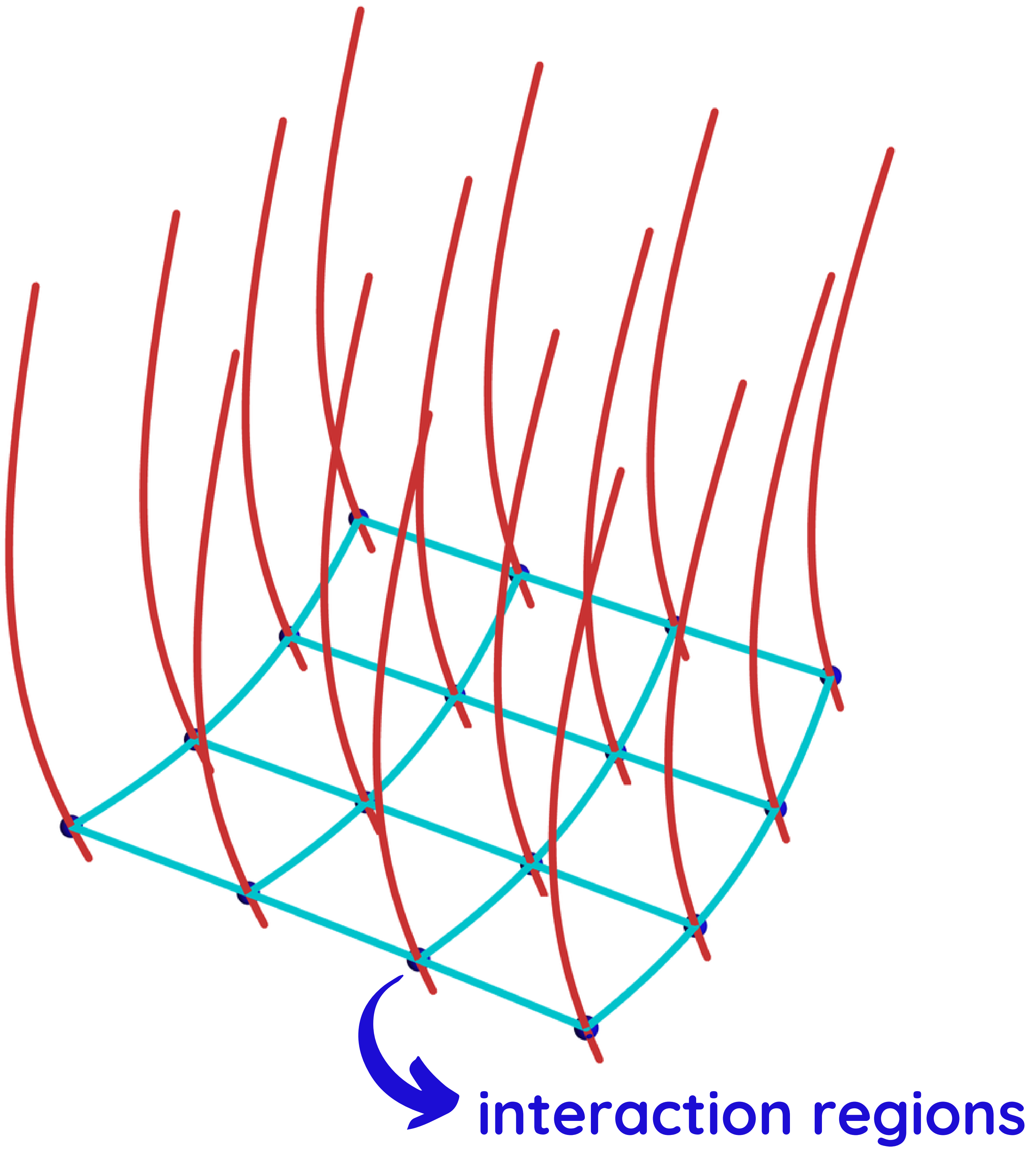

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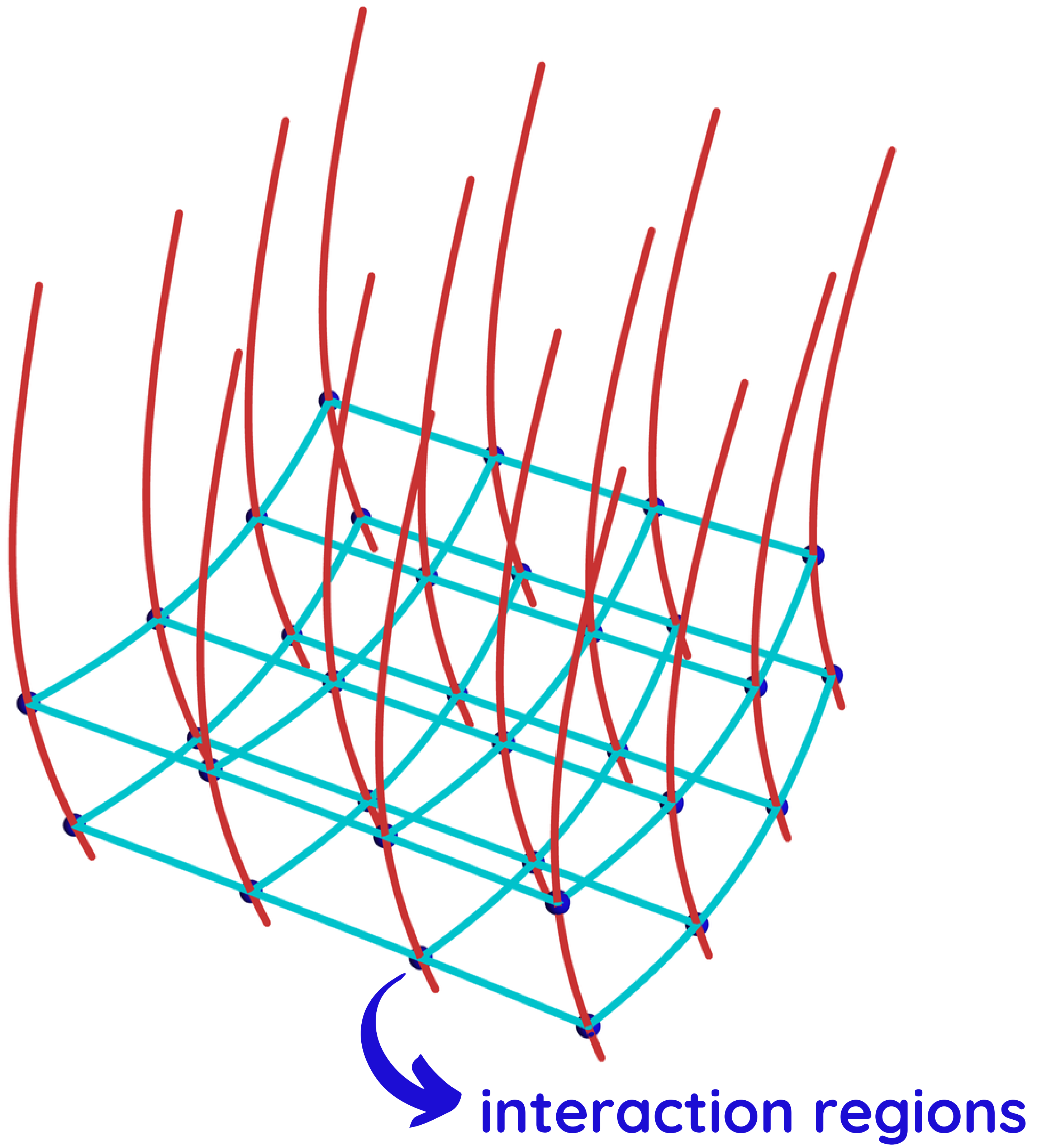

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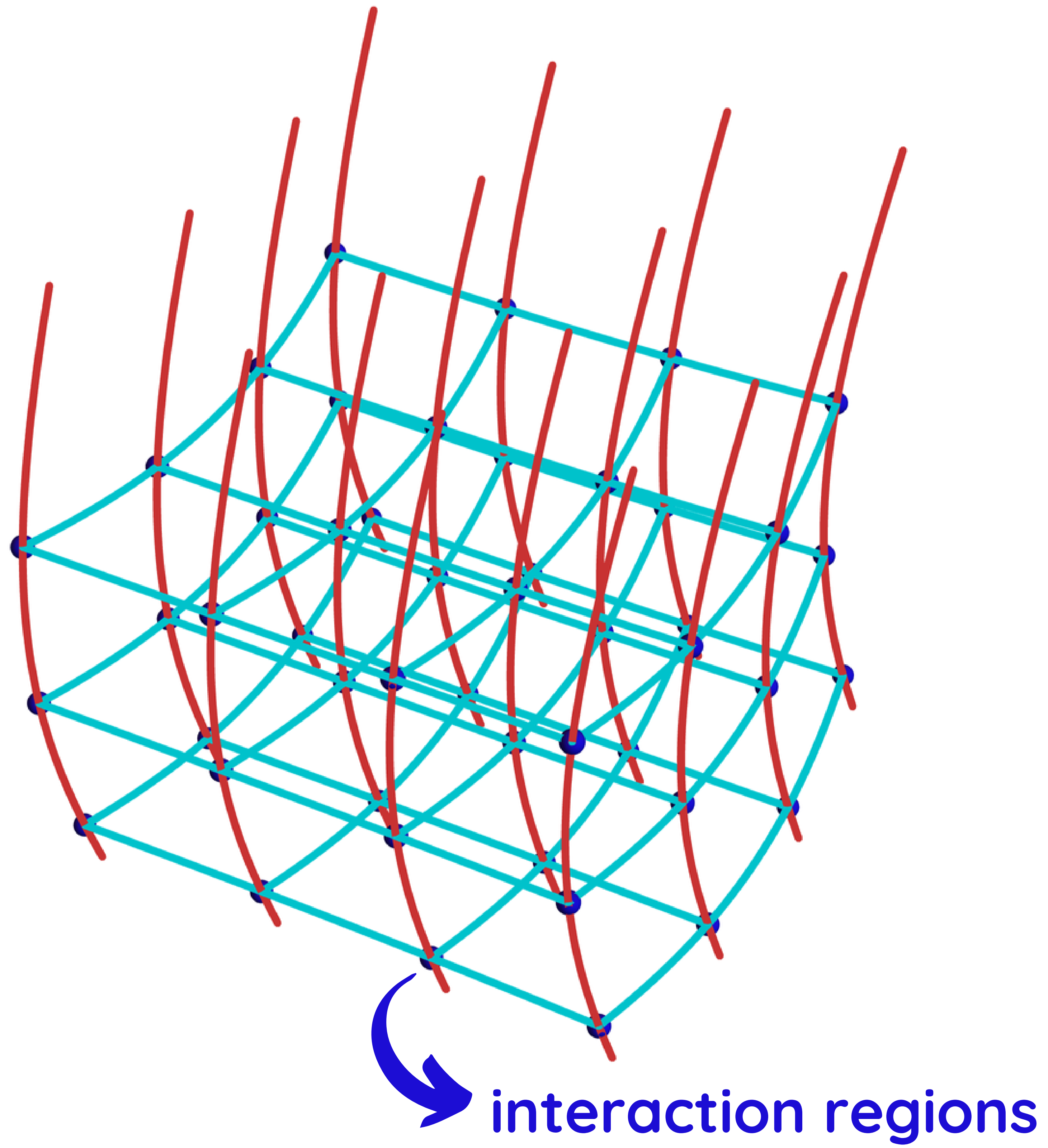

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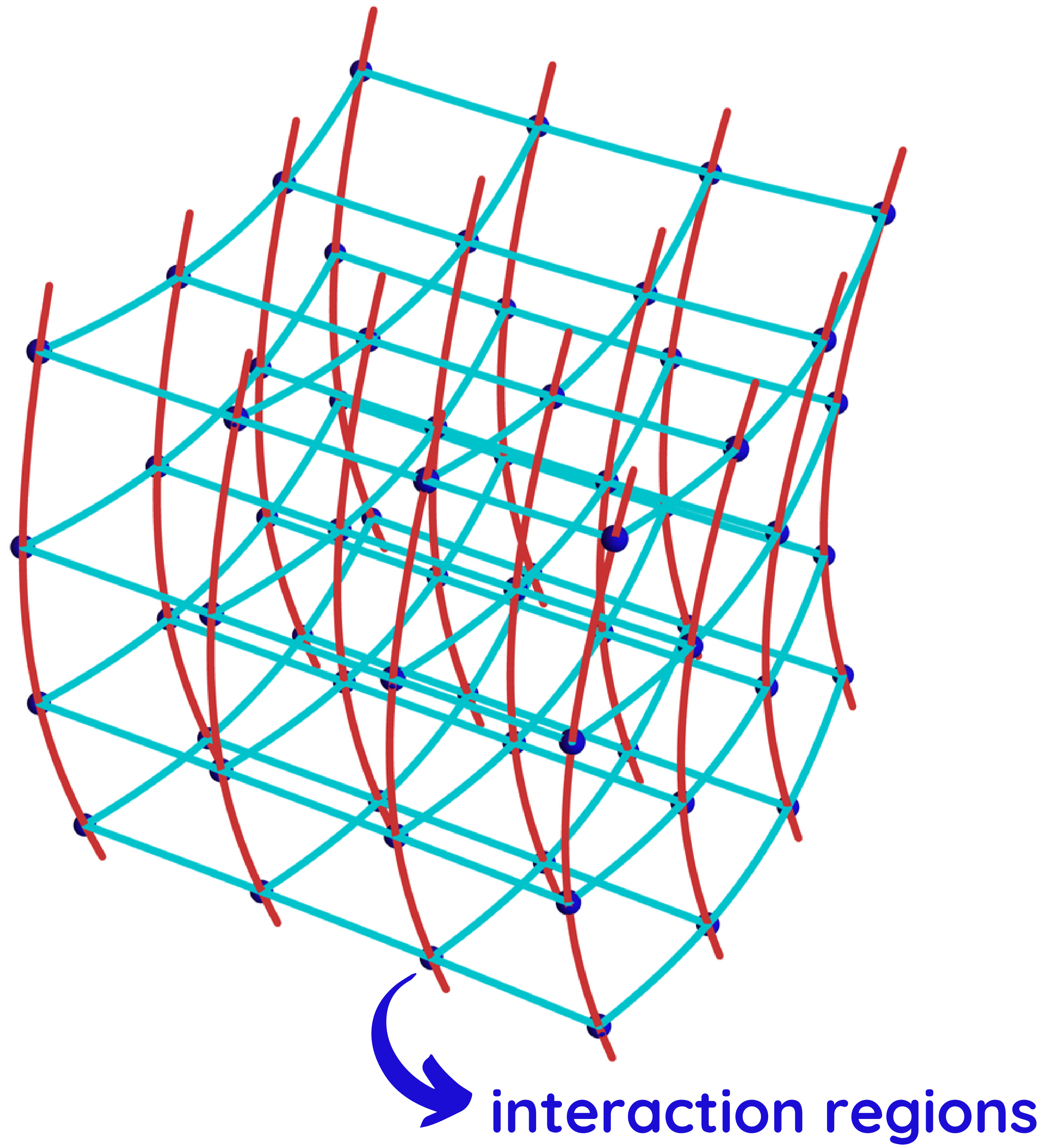

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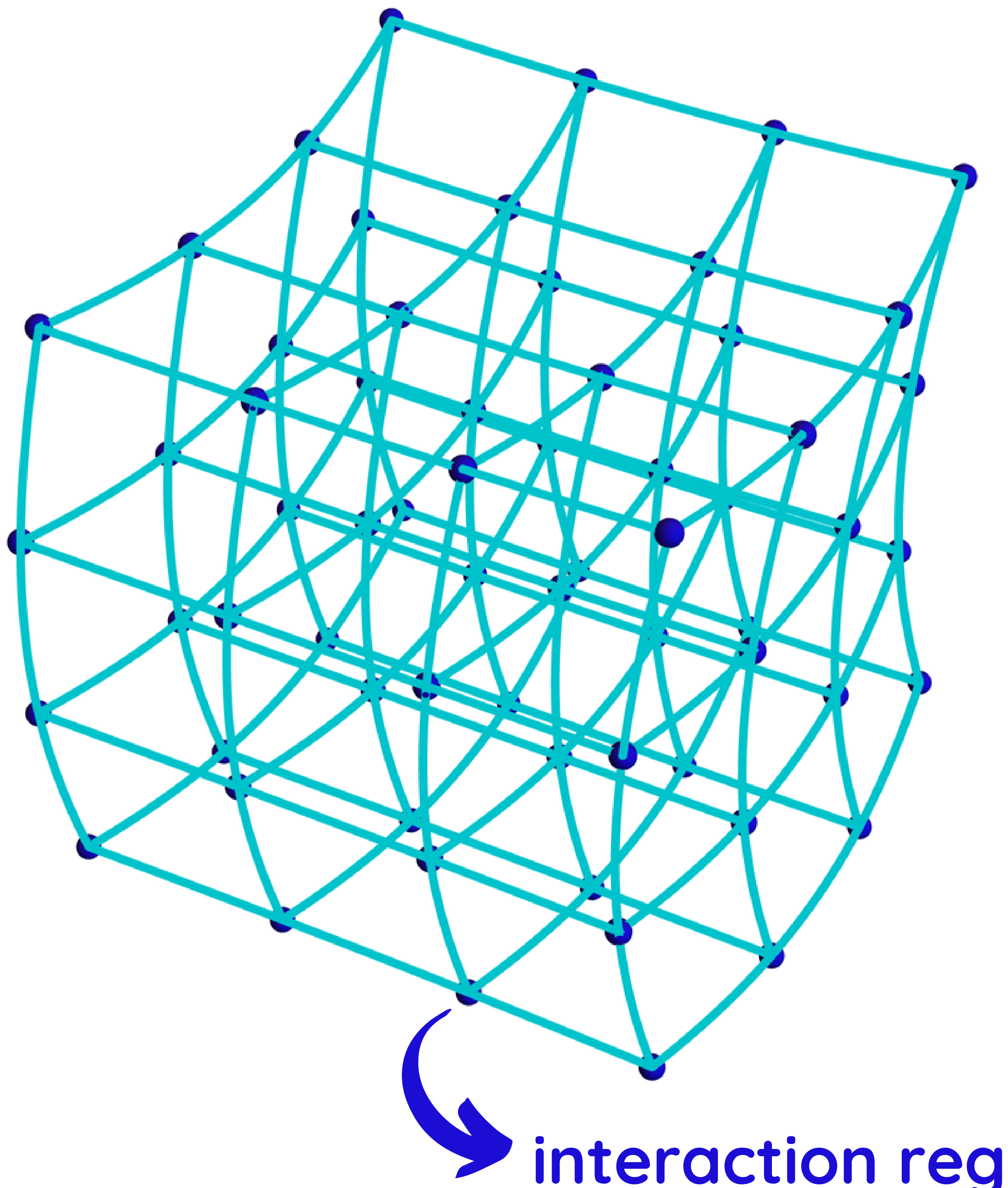

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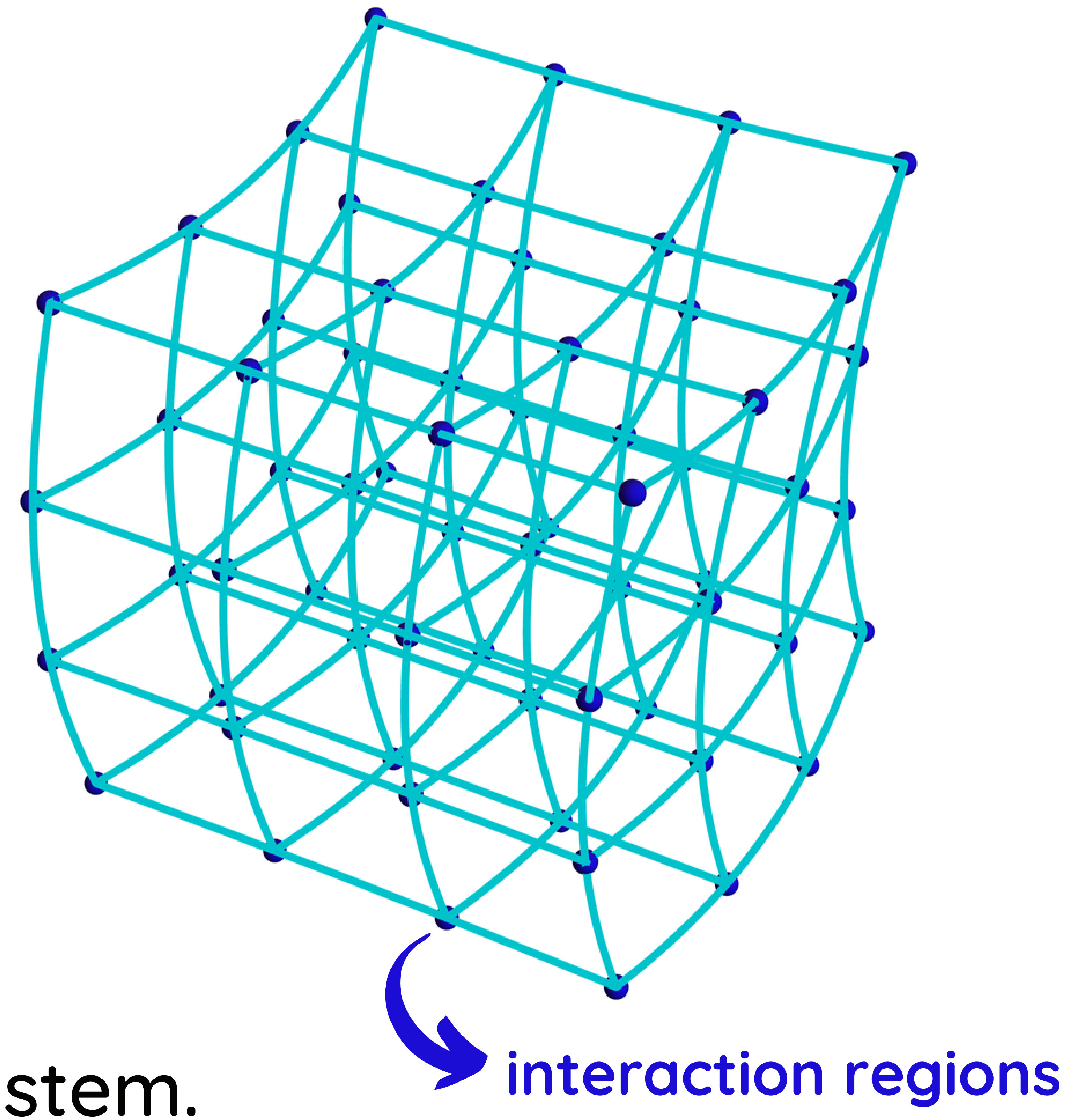
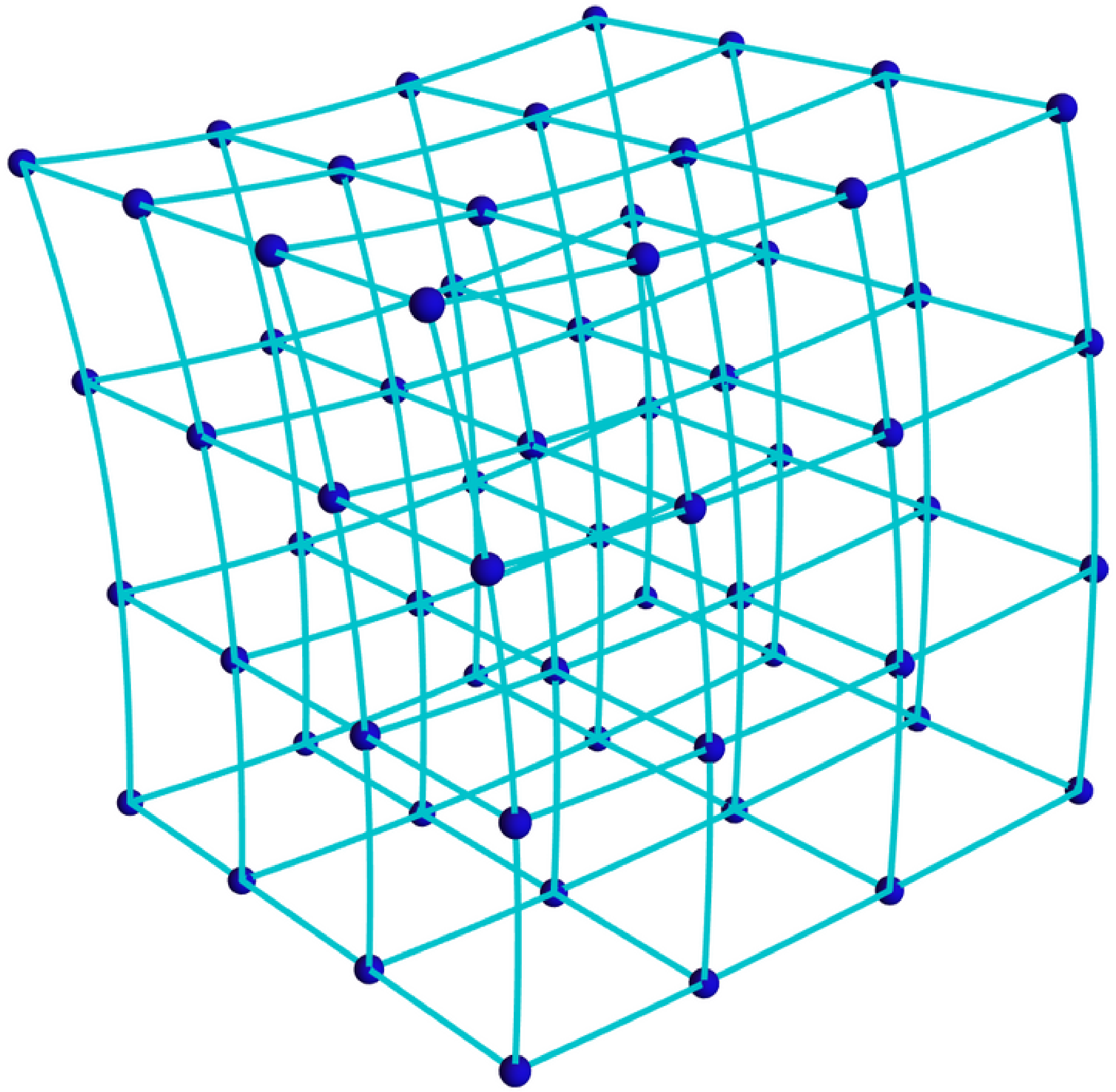


The lattice can induce a coordinate system.

Spacetime Geometry from Quantum Measurements

Using the expression for the metric in terms of the Wightman:

$$g_{\mu\nu}(\mathrm{x}) = -\lim_{\mathrm{x}'\to\mathrm{x}} \frac{1}{8\pi^2} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^{\nu'}} W^{-1}(\mathrm{x},\mathrm{x}').$$



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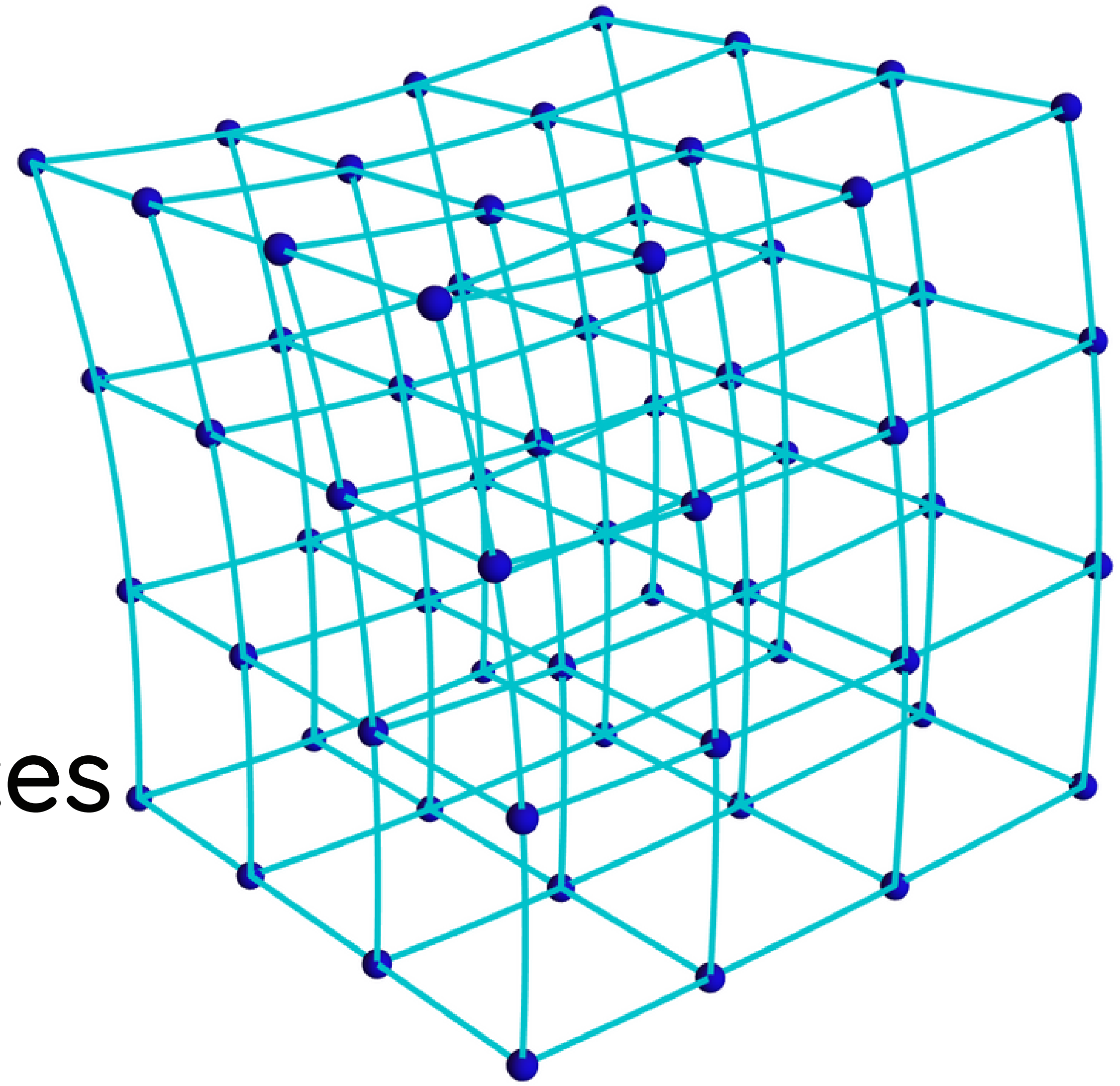

We can approximate the derivatives by finite distances in the lattice:

$$g_{\mu\nu}(\mathsf{x}_\mathsf{j}) \approx -\frac{W^{-1}(\mathsf{x}_{\mathsf{j}+\mathbf{1}_\nu},\mathsf{x}_{\mathsf{j}+\mathbf{1}_\mu}) - W^{-1}(\mathsf{x}_\mathsf{j},\mathsf{x}_{\mathsf{j}+\mathbf{1}_\mu}) - W^{-1}(\mathsf{x}_{\mathsf{j}+\mathbf{1}_\nu},\mathsf{x}_\mathsf{j}) + W^{-1}(\mathsf{x}_\mathsf{j},\mathsf{x}_\mathsf{j})}{8\pi^2(x^\mu_{\mathsf{j}+\mathbf{1}_\mu} - x^\mu_\mathsf{j})(x^\nu_{\mathsf{j}+\mathbf{1}_\nu} - x^\nu_\mathsf{j})},$$

where $\mathbf{1}_\mu = (\overbrace{0,\ldots,0}^{\mu-1},1,0,\ldots,0)$.

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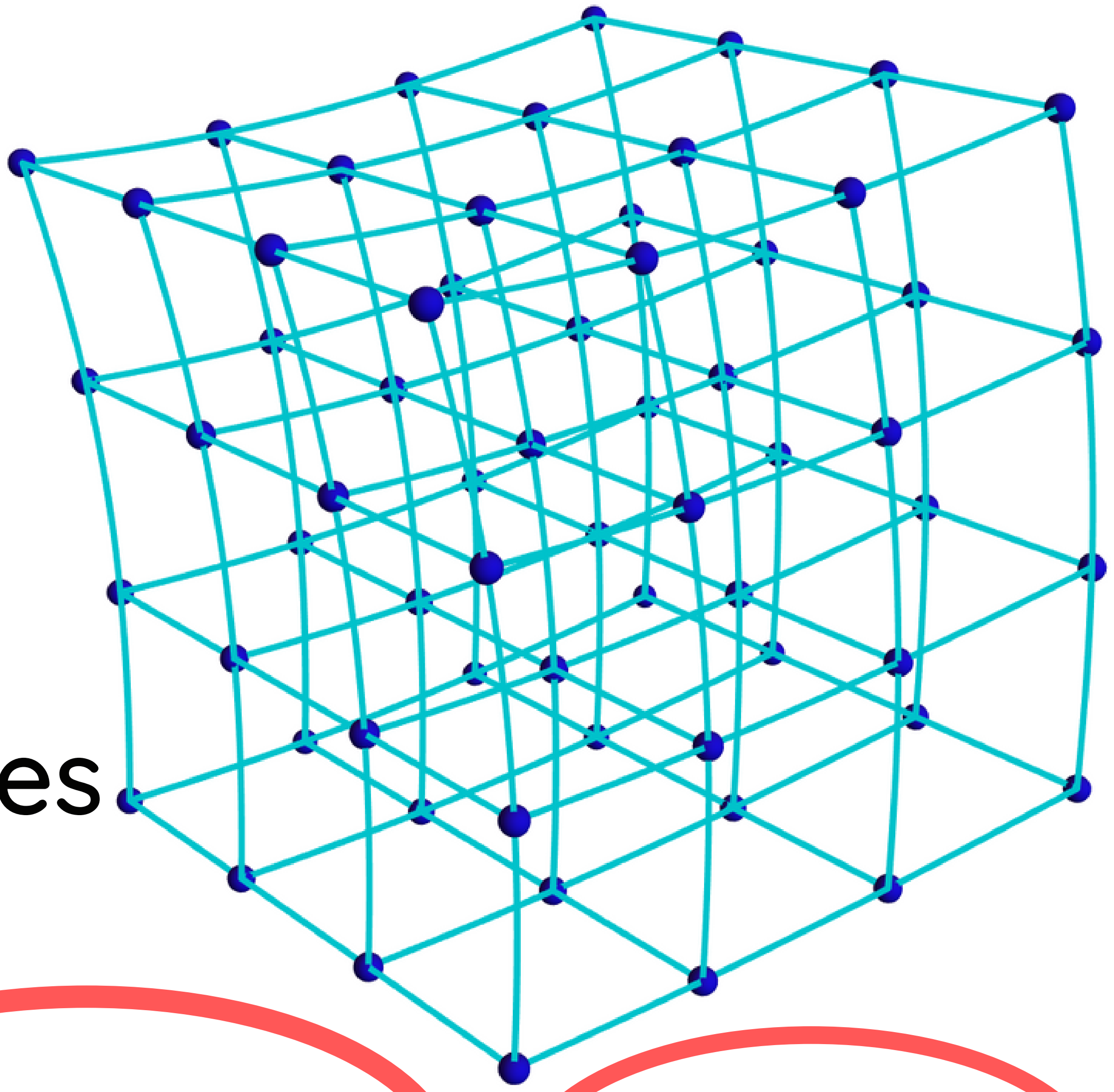

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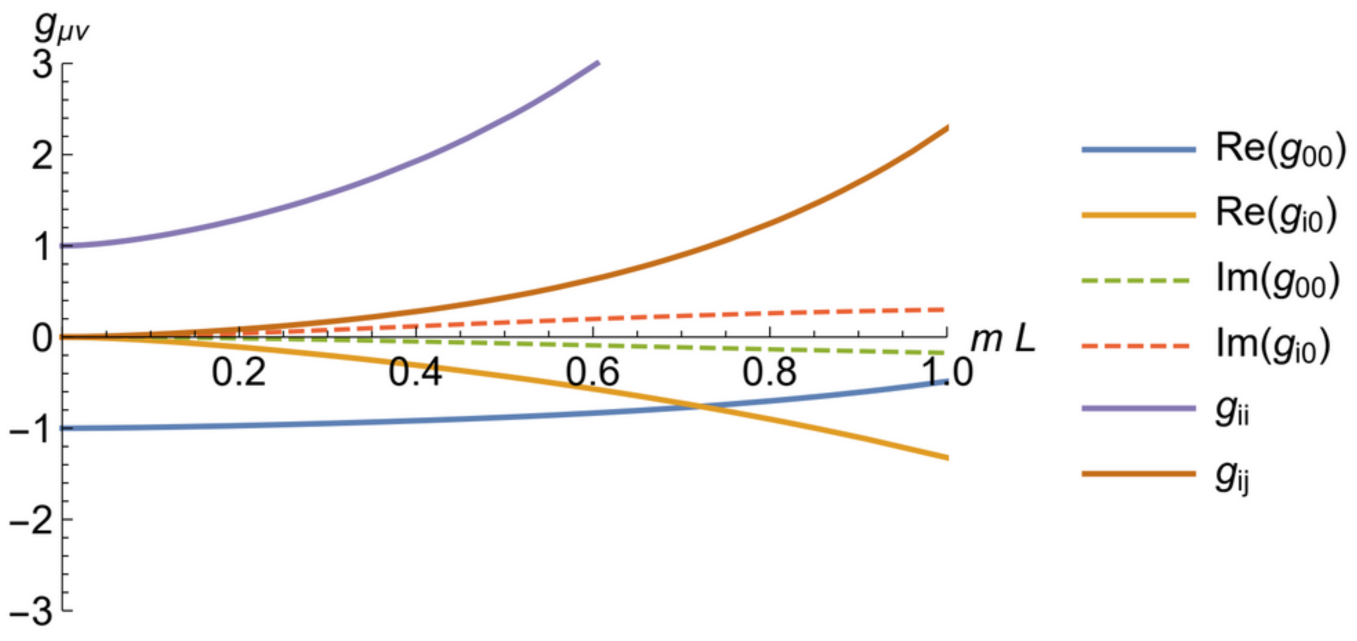
These are obtained from the measurements of the detectors.

Inertial Detectors in Minkowski



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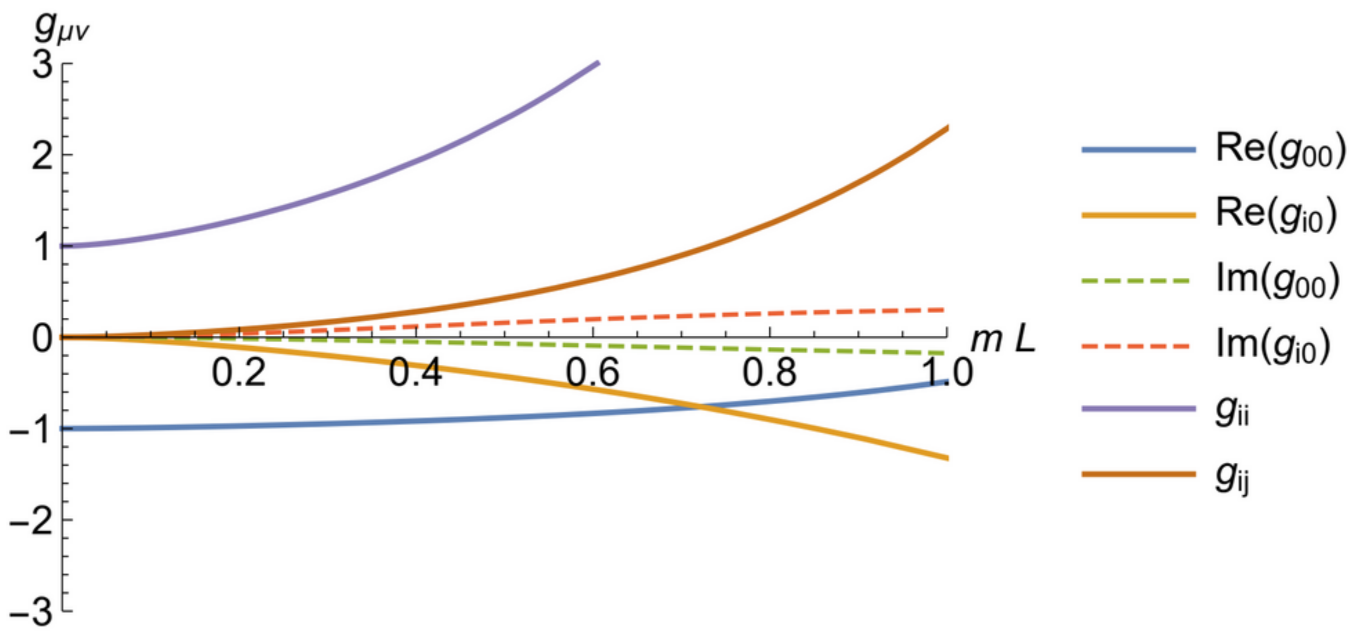

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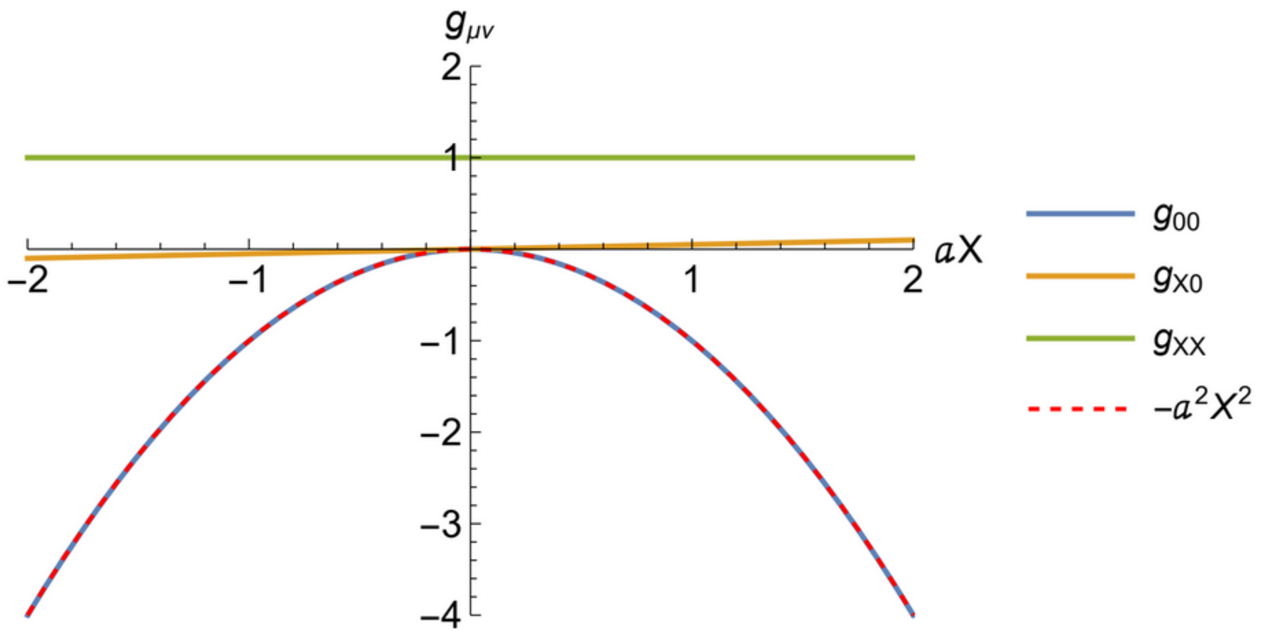

All Examples Work When Coordinate Separation 0

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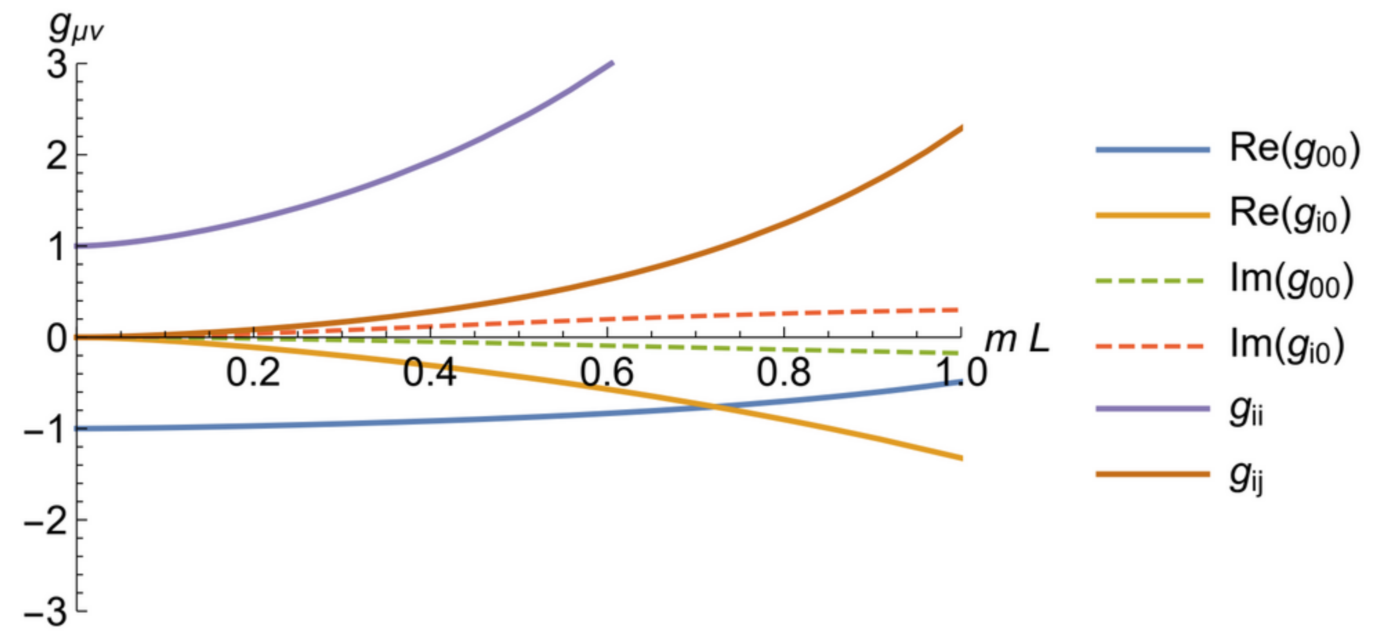

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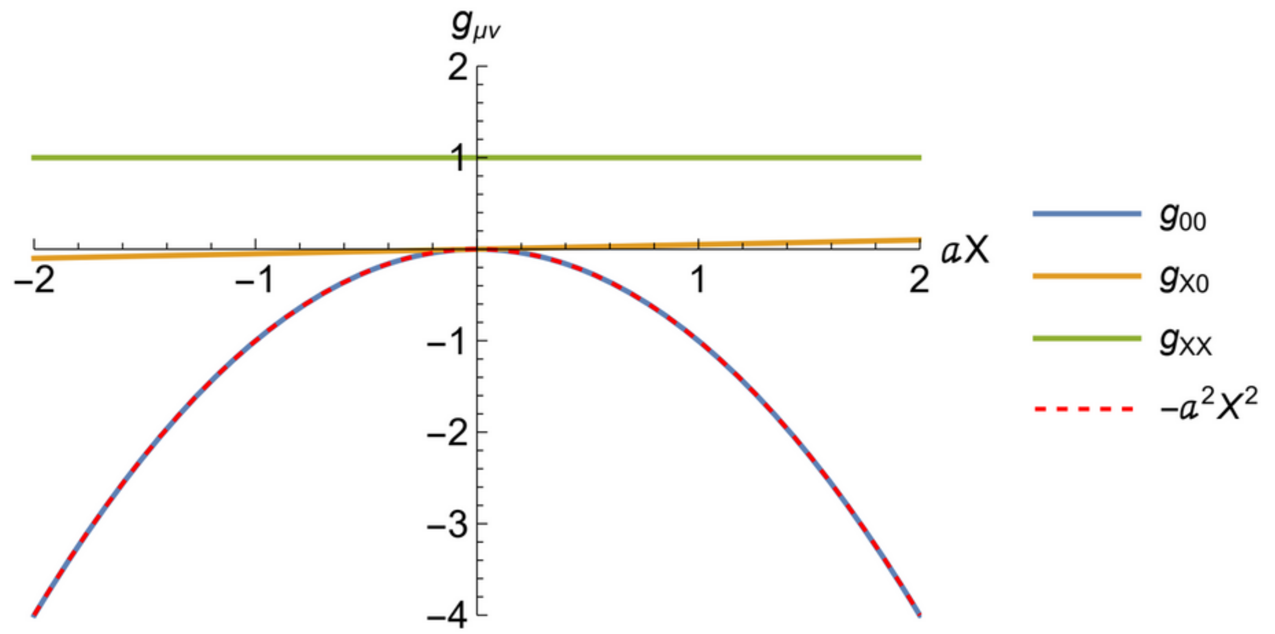
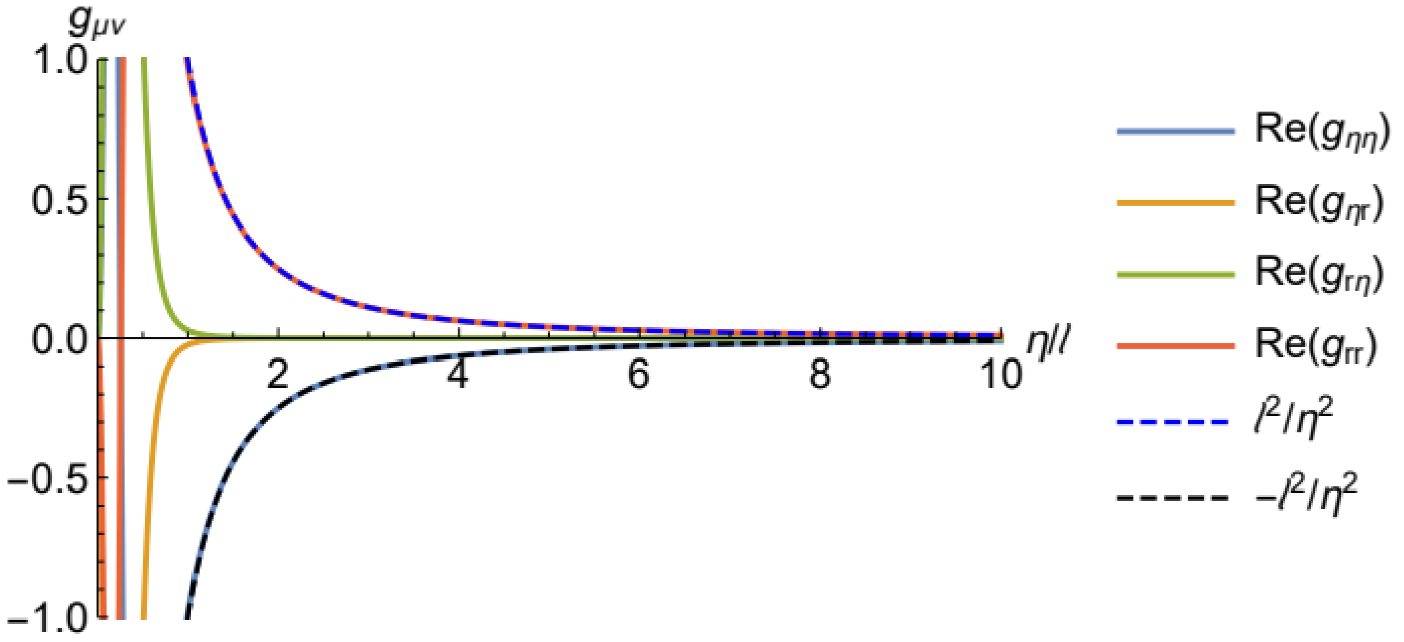


Detectors in deSitter Spacetime



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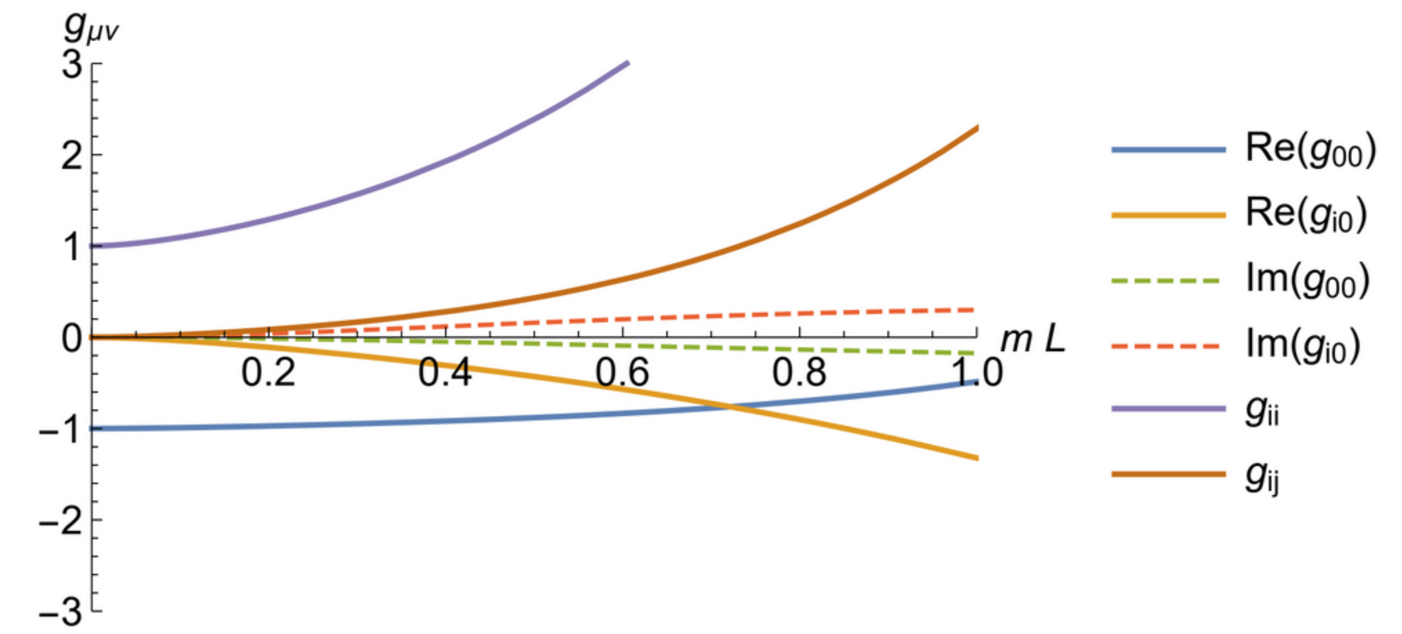

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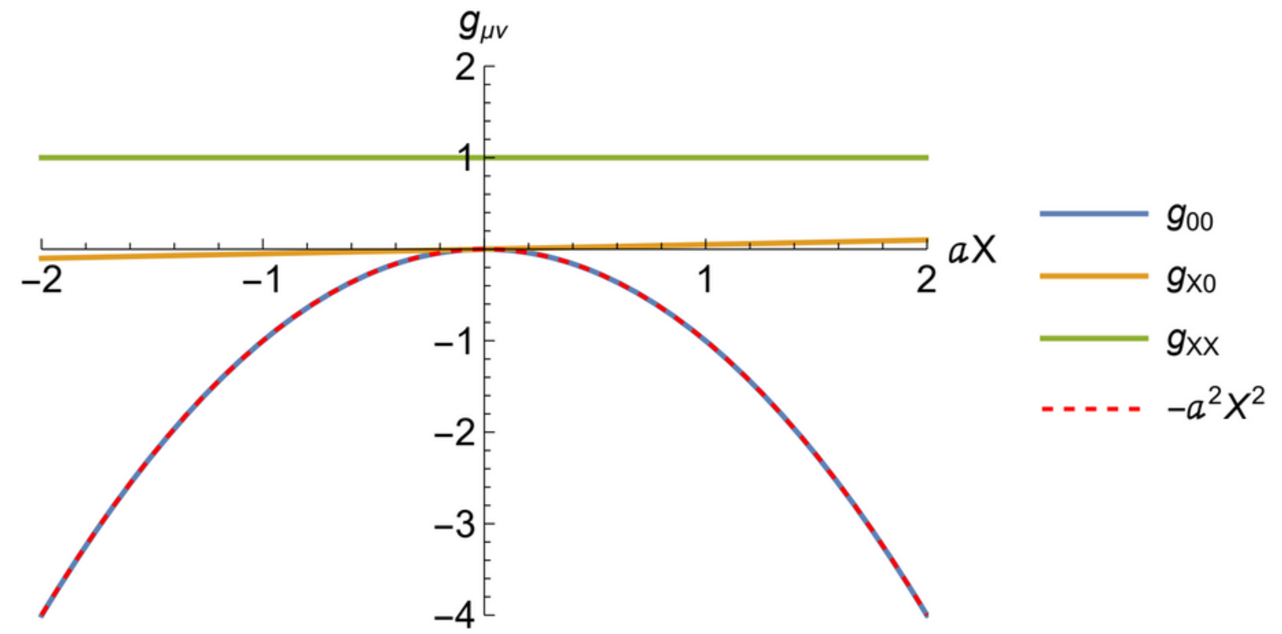

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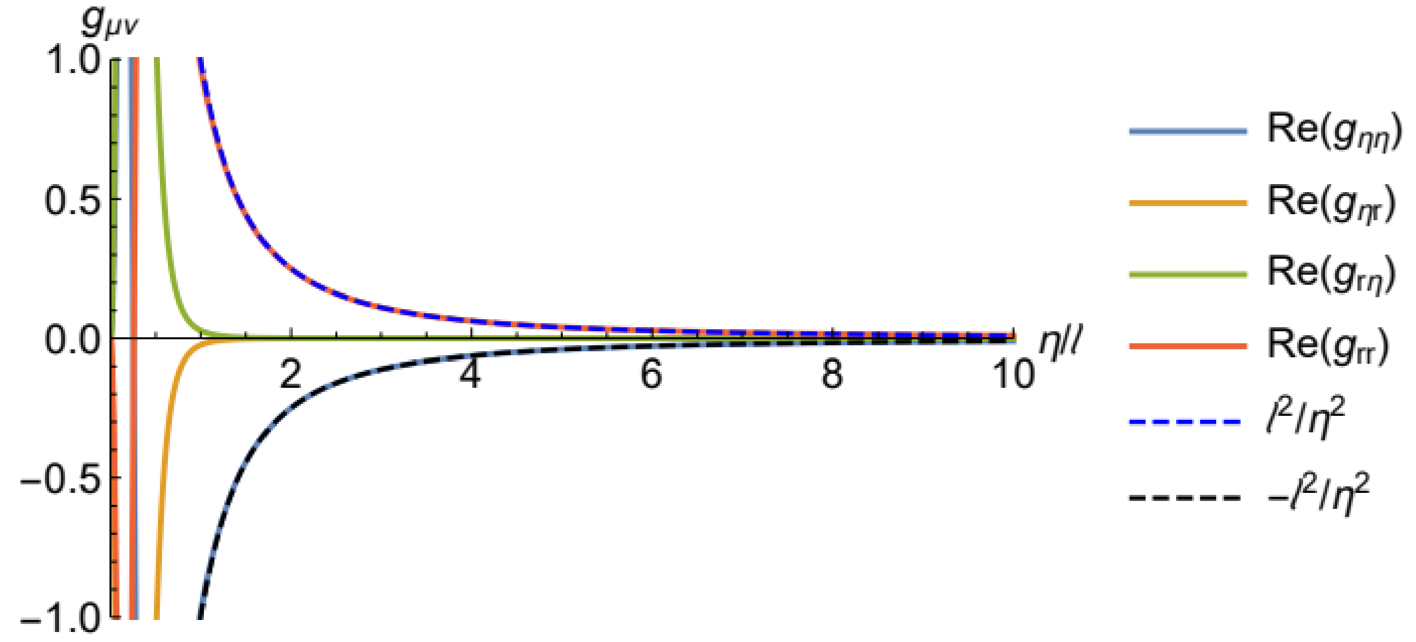
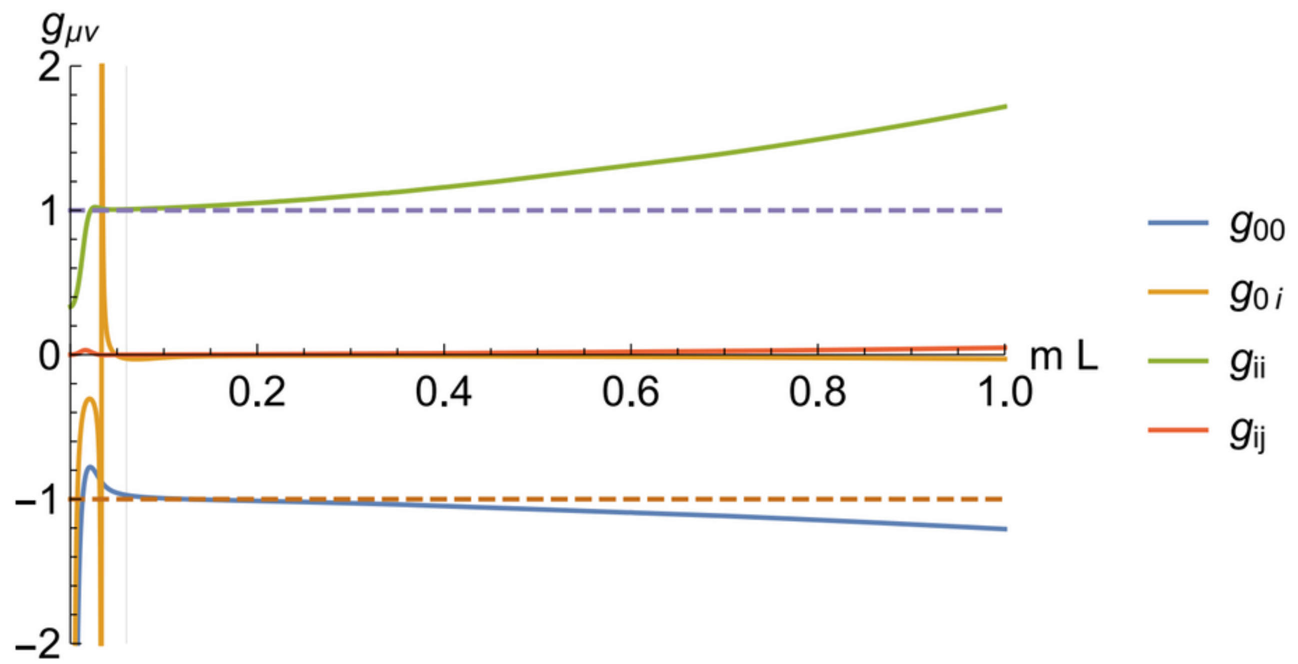


Finite Sized Detectors



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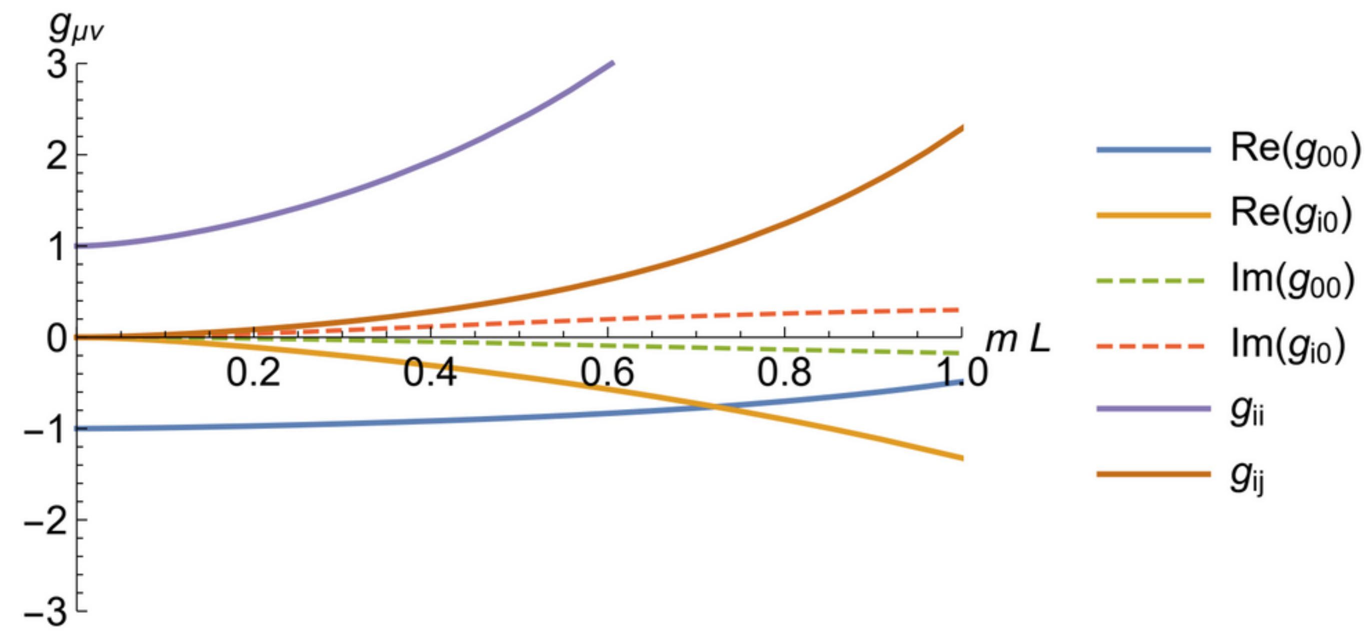

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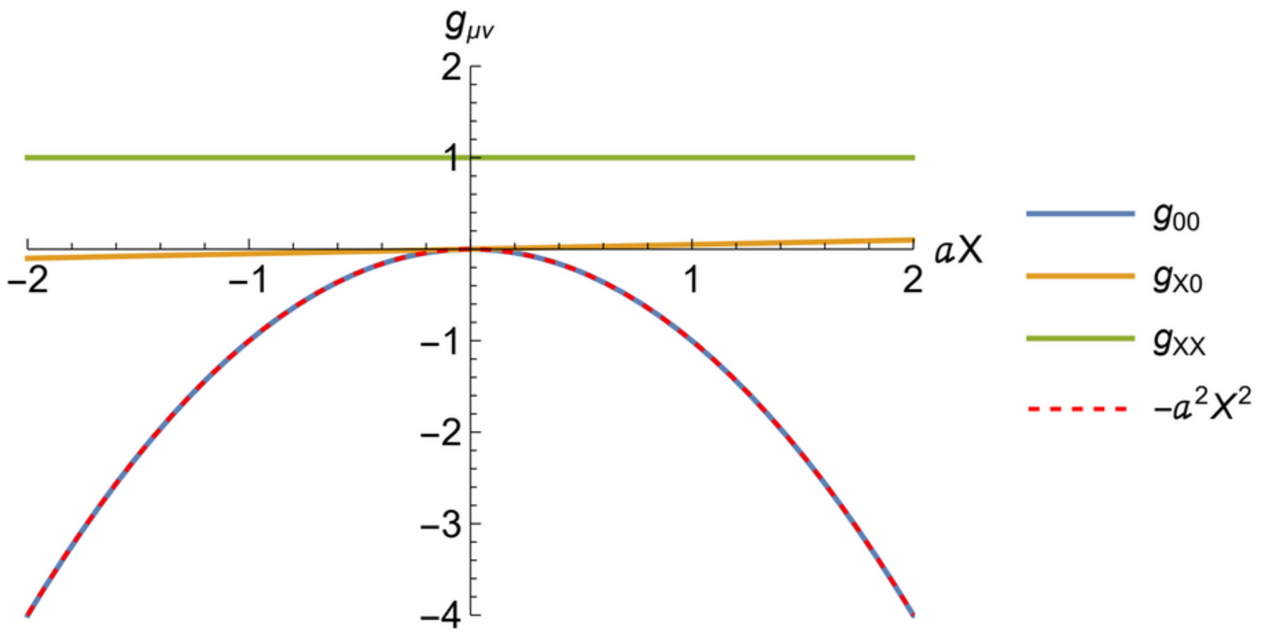

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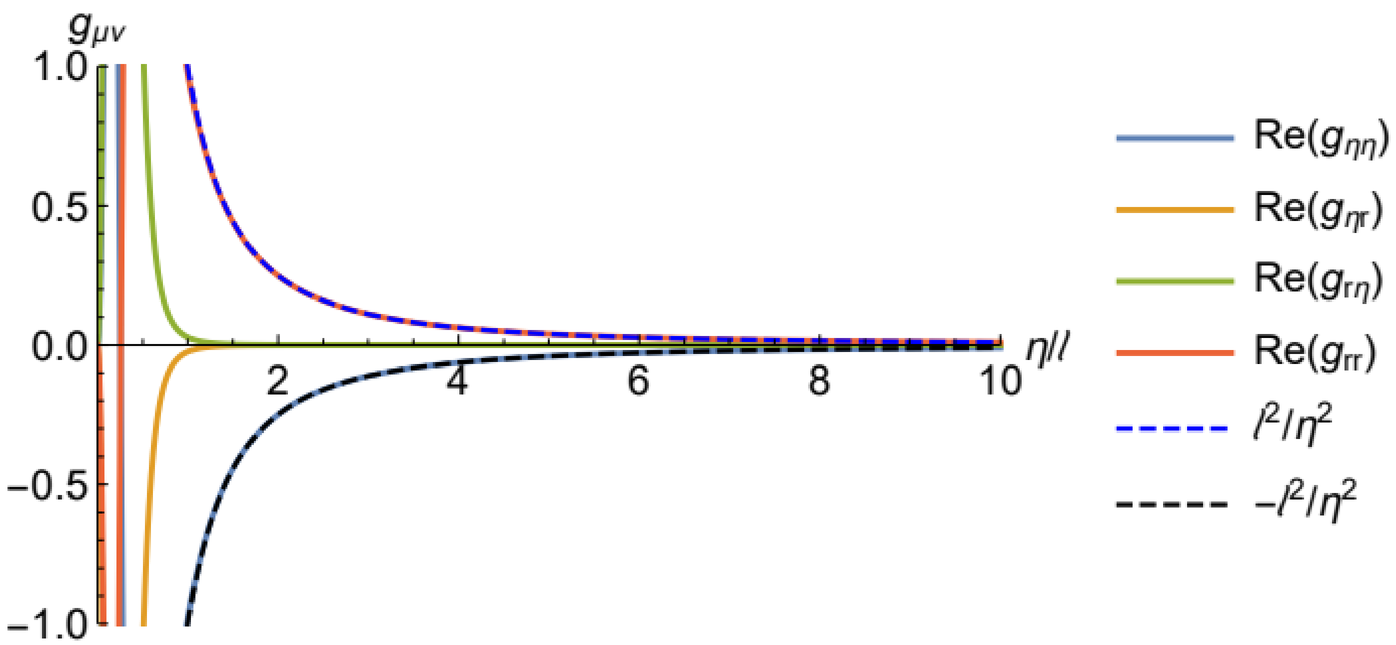

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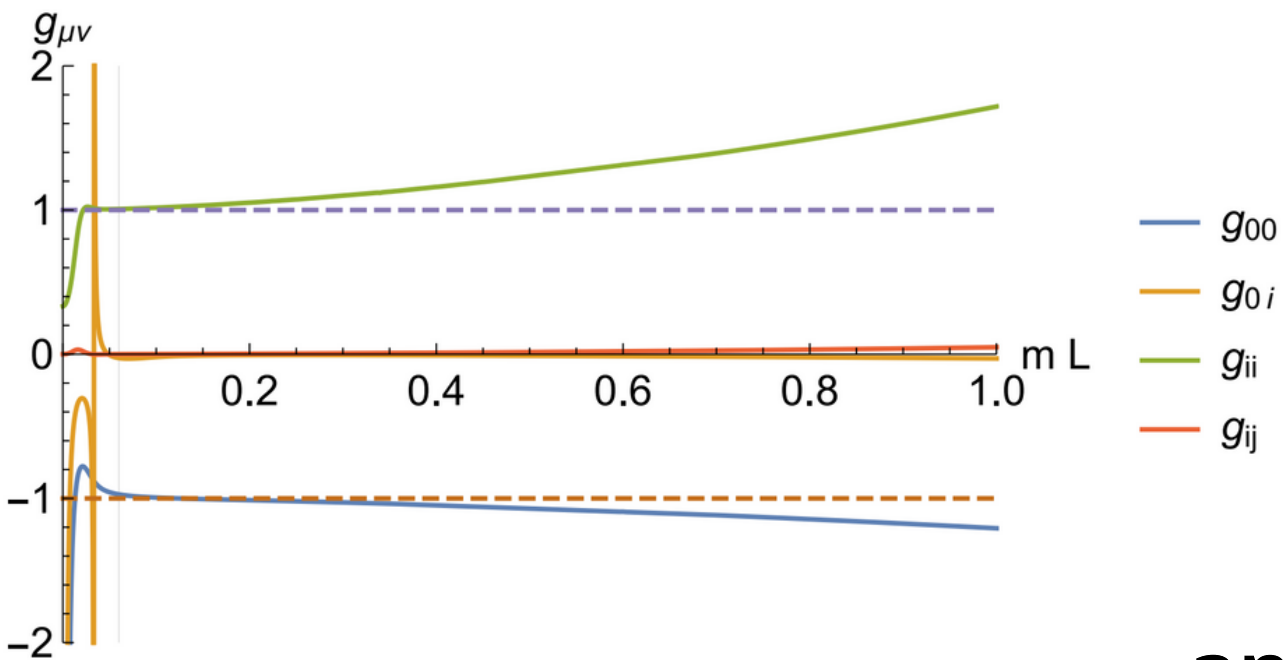

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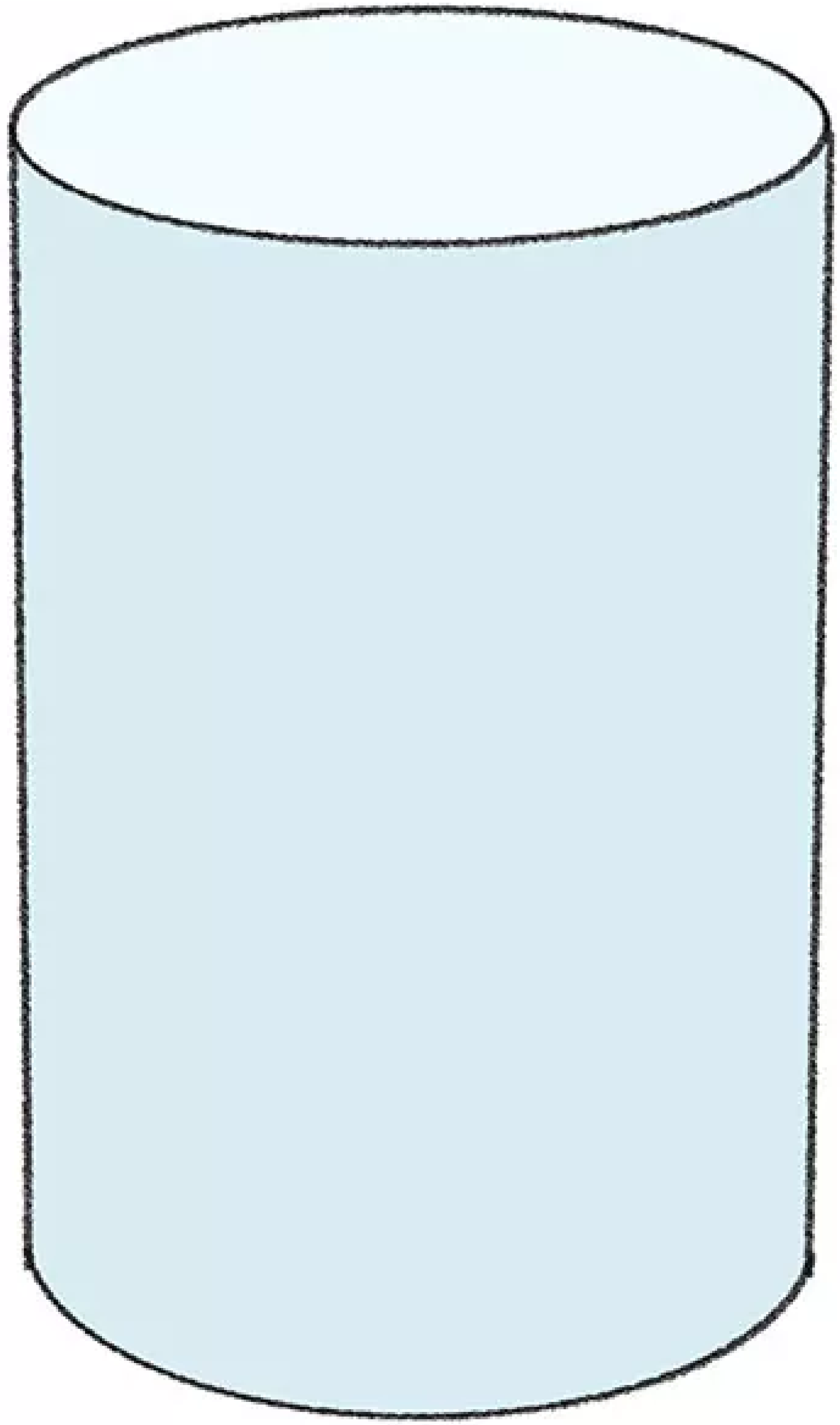
Fairy Tale

Facts



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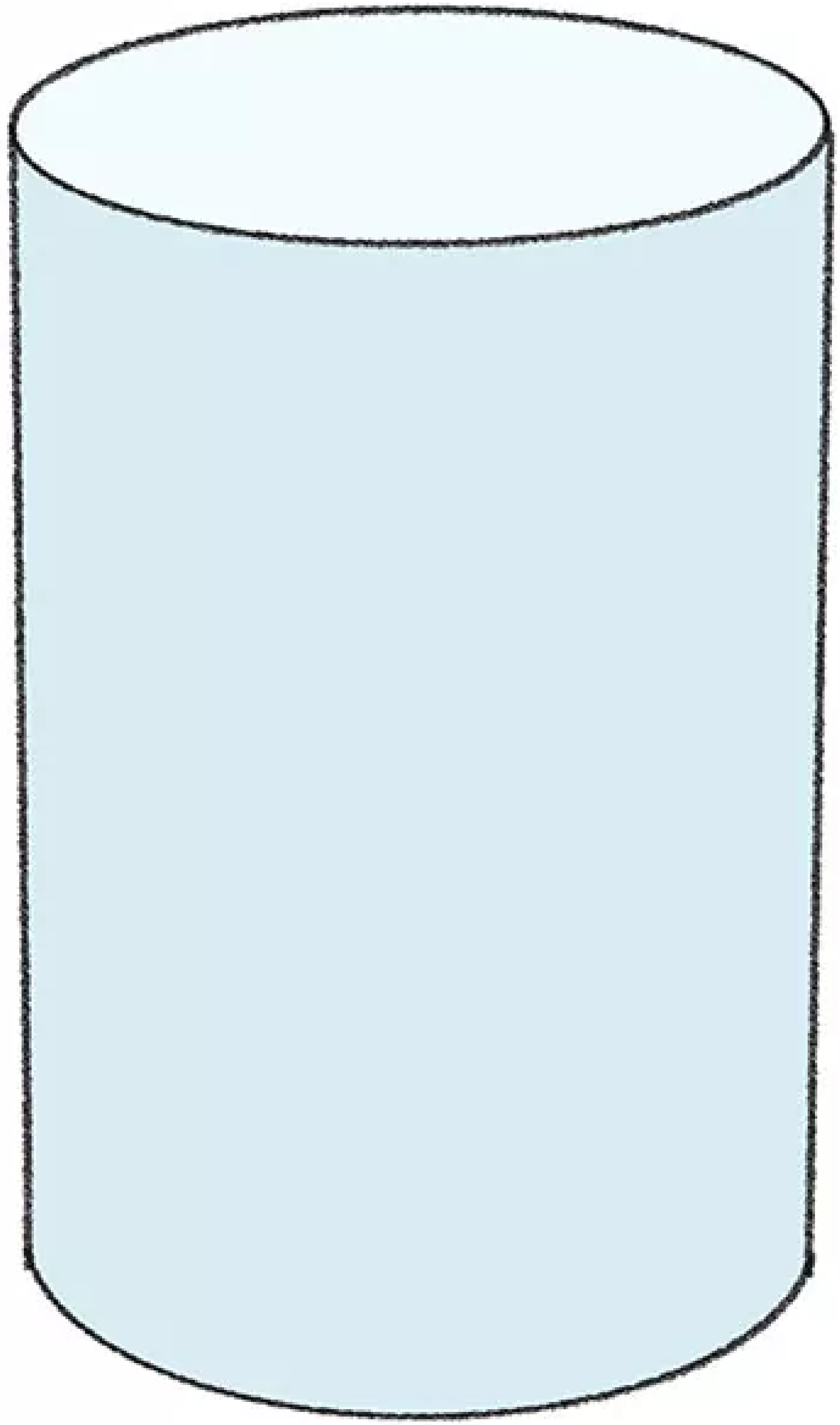

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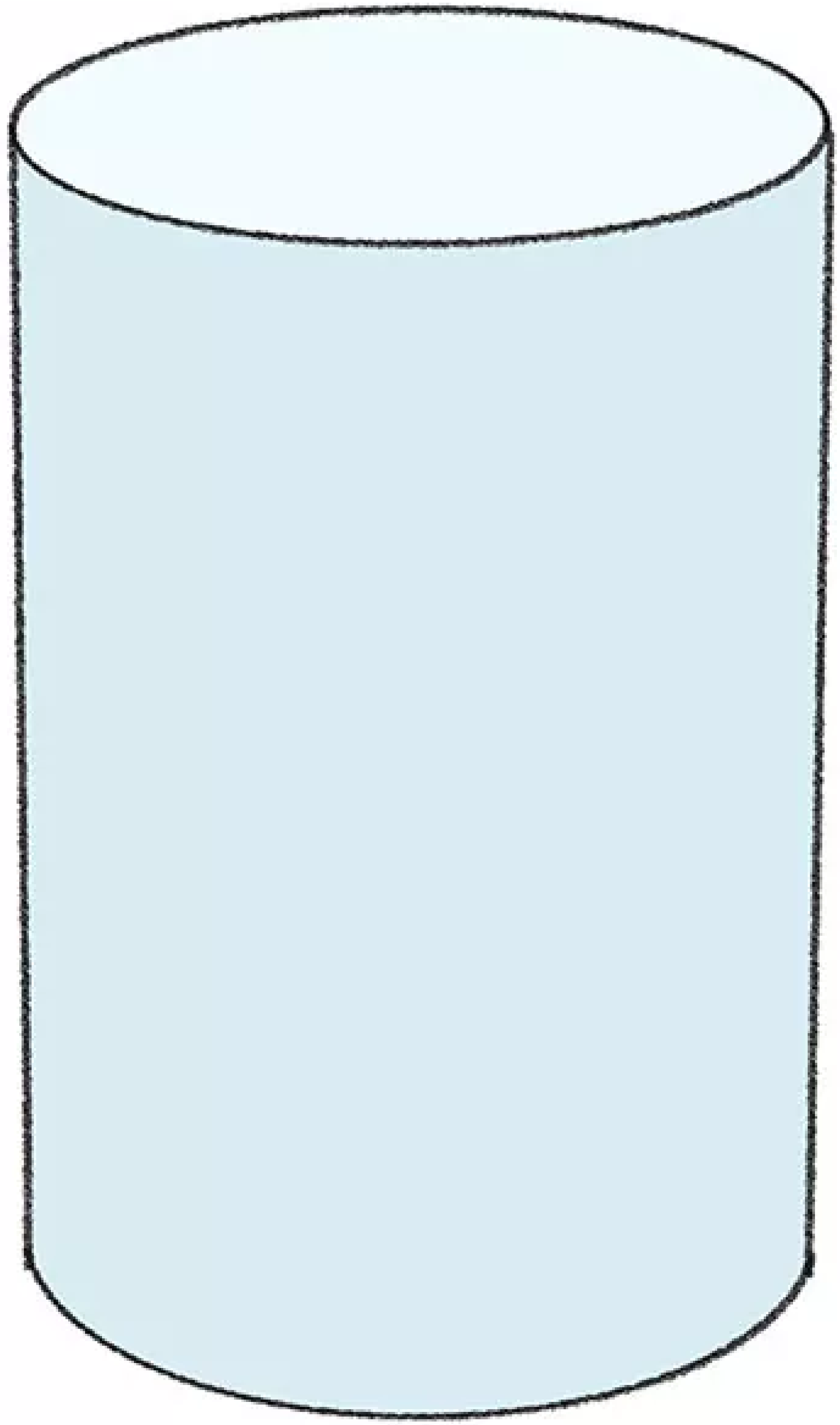

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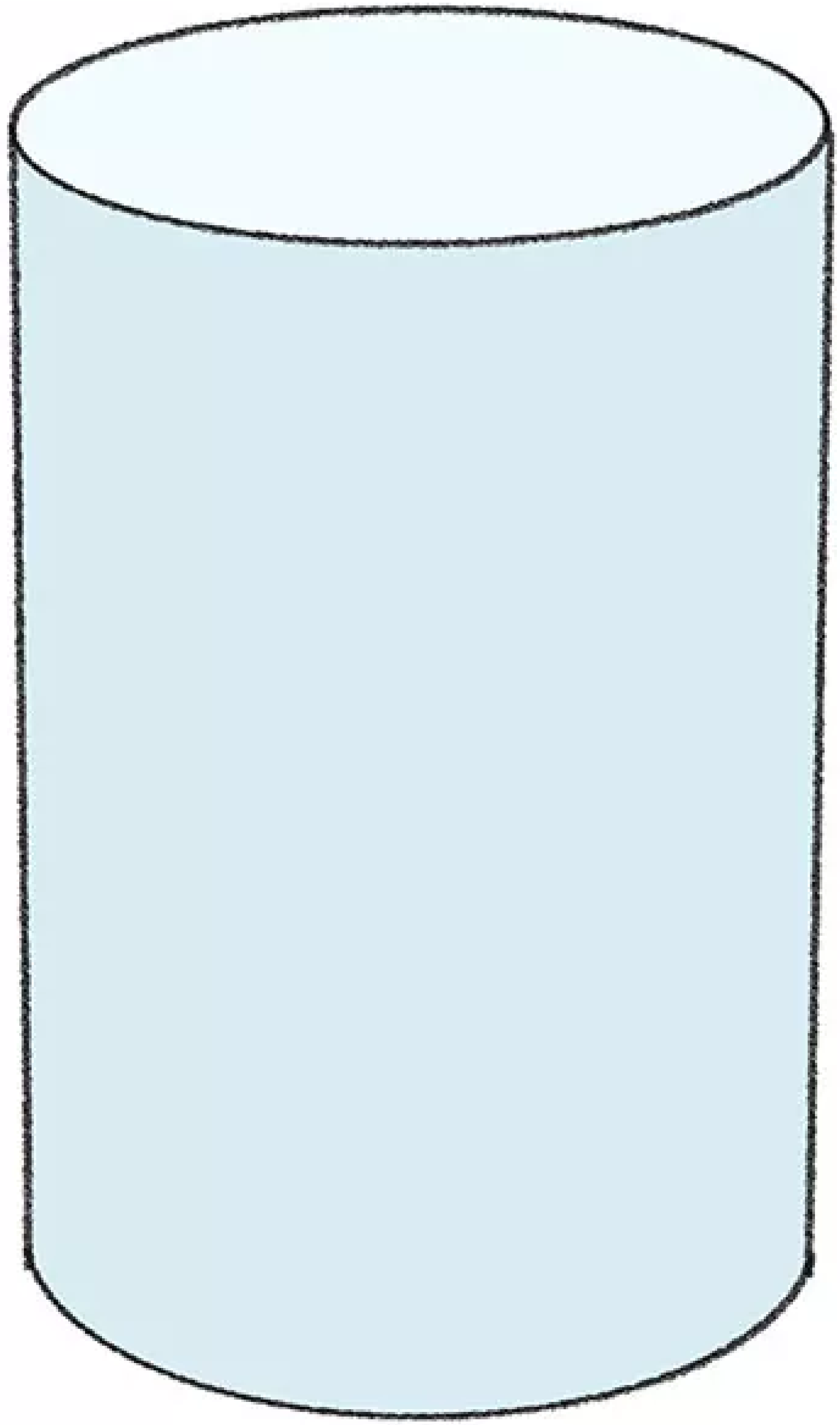

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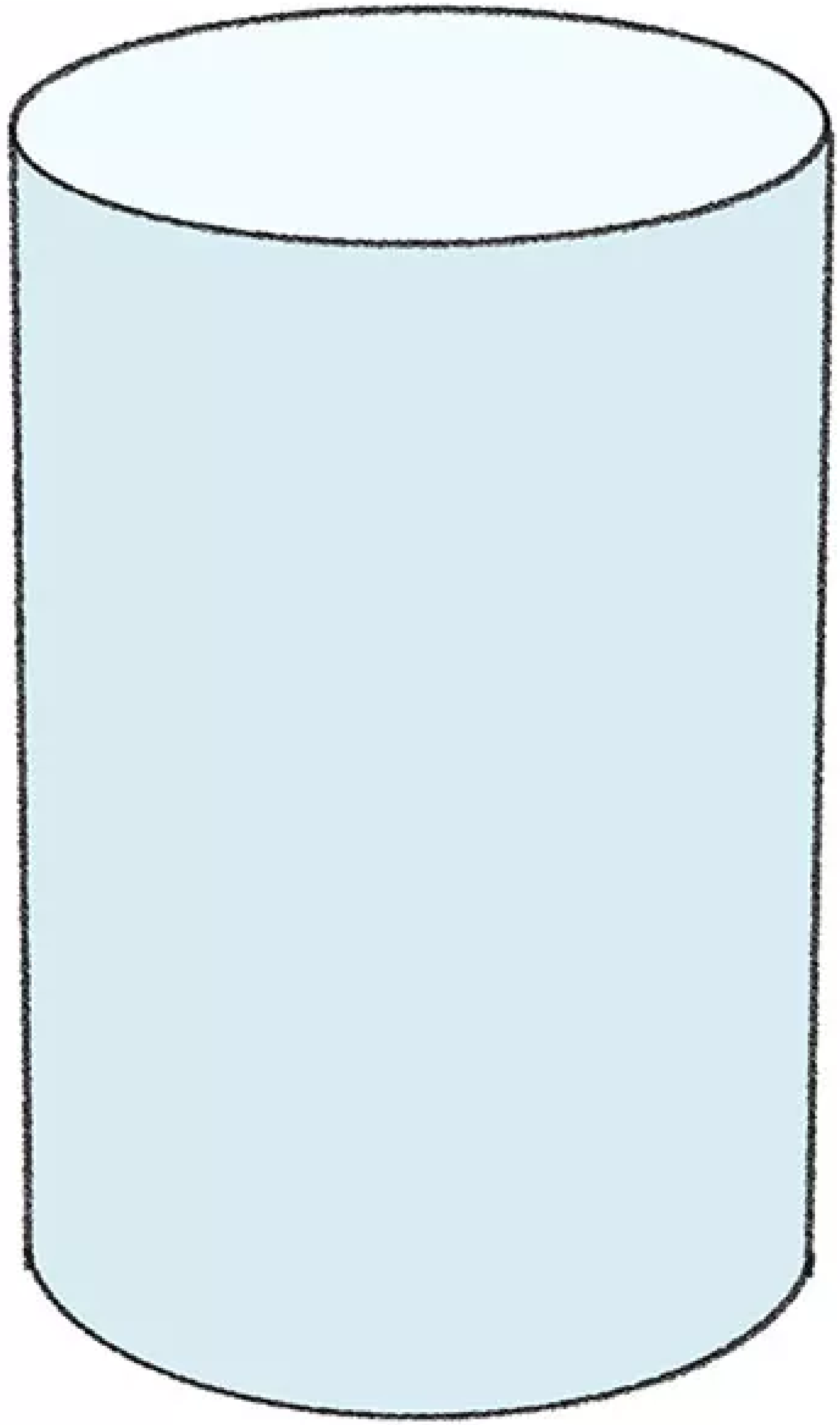

Facts

Einstein's equations are "violated" in the context of QFT in curved spacetimes.

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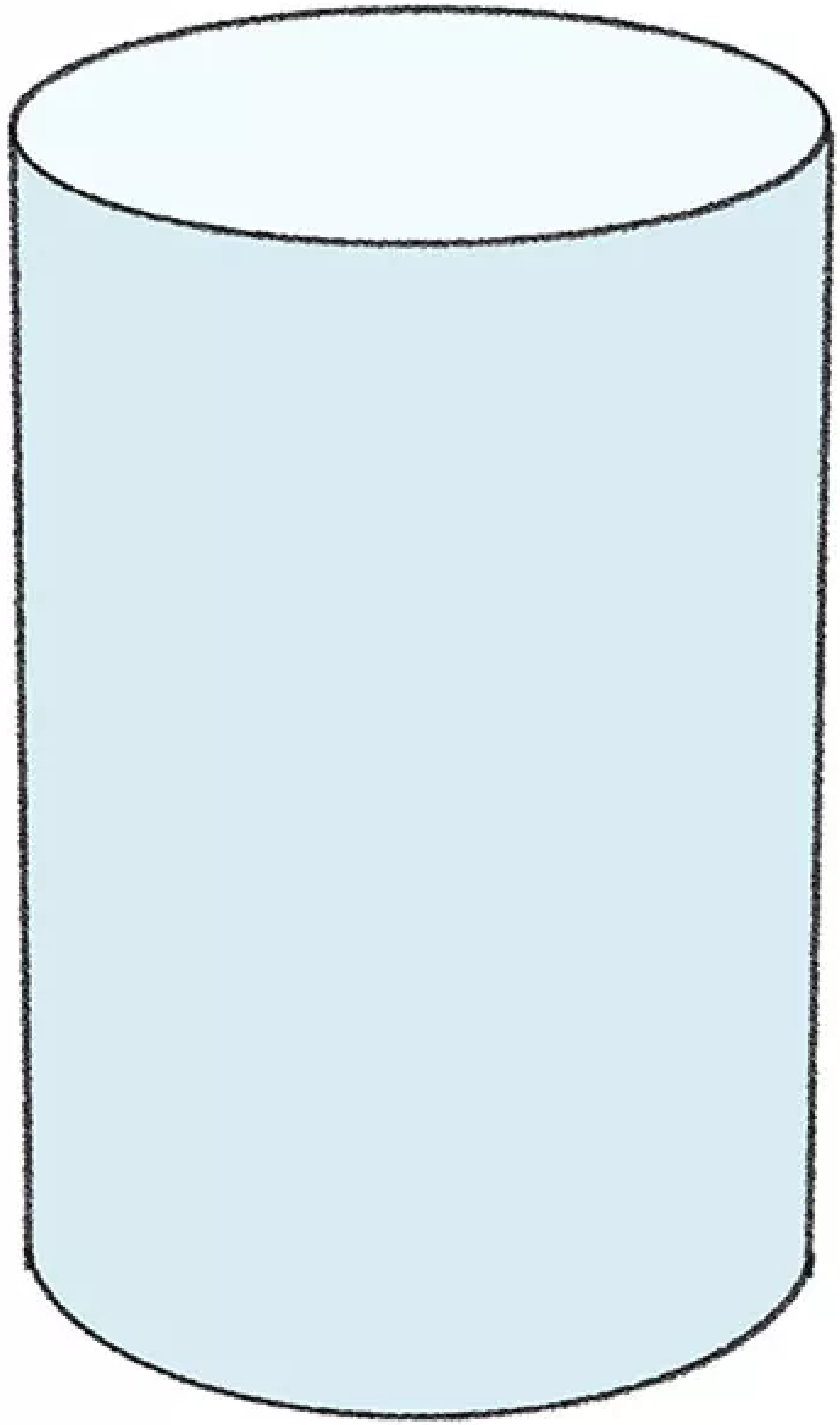
And we would need a theory of quantum gravity in order to write something like

$$\hat{G}_{\mu\nu} = 8\pi\, \hat{T}_{\mu\nu}$$



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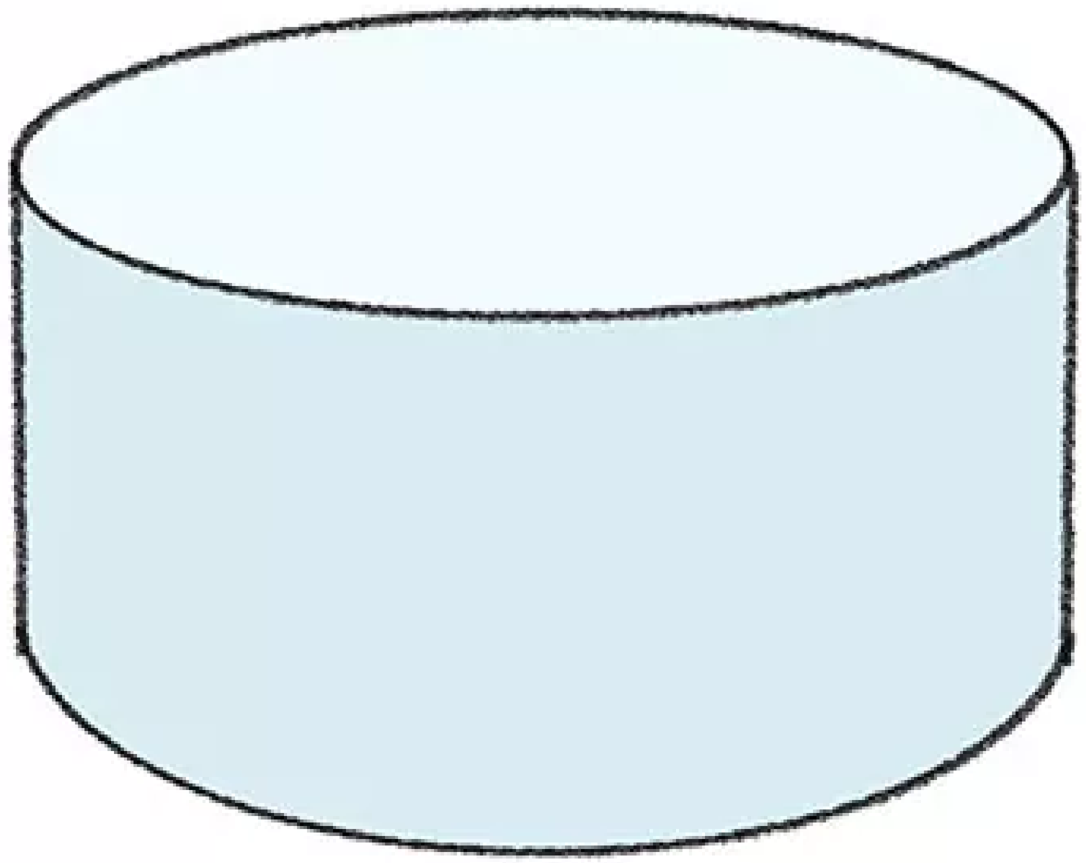
The result $g_{\mu\nu}(\mathrm{x}) = -\lim_{\mathrm{x}' \to \mathrm{x}} \frac{1}{8\pi^2} \frac{\partial}{\partial x^{\mu}} \frac{\partial}{\partial x^{\nu'}} W^{-1}(\mathrm{x}, \mathrm{x}')$ gives us the background spacetime.



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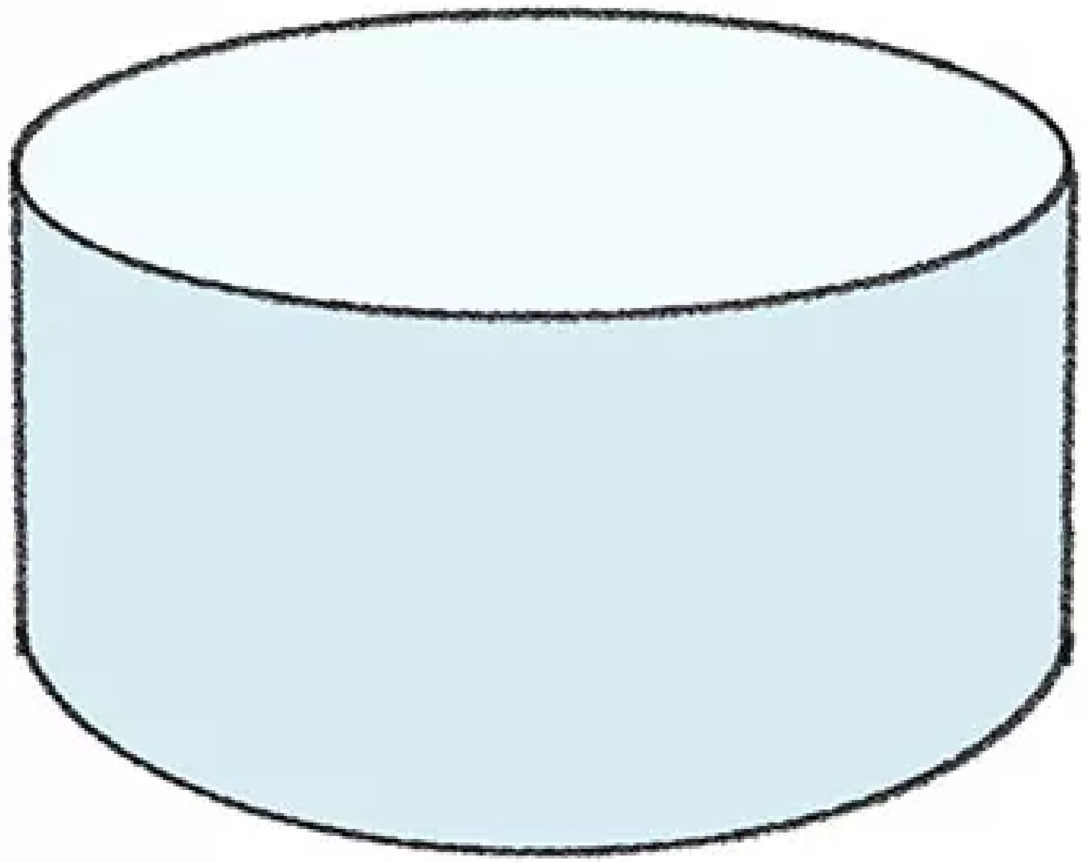

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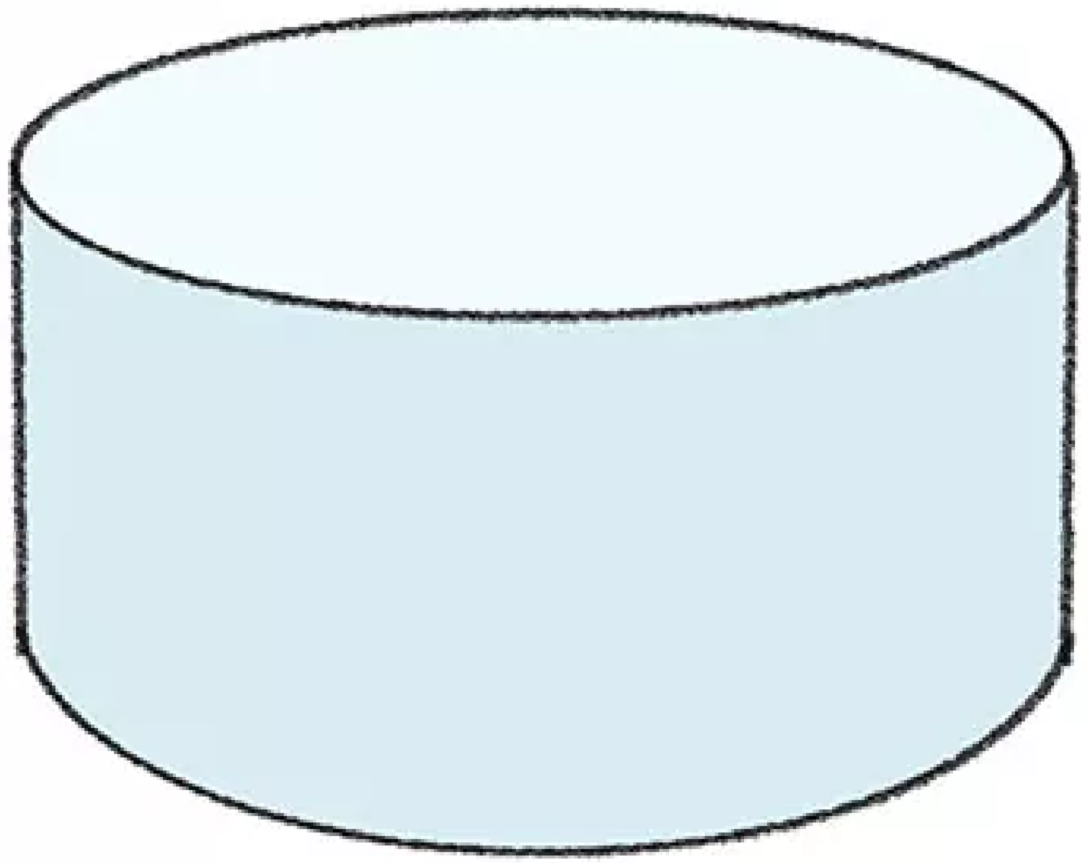
E.g.: In the cylinder spacetime, this prescription would give the flat Minkowski metric.

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Fairy Tale

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However, if spacetime is discrete, and there is still some sense to

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and it might very well be that

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Arguments in Favour

Writing

$$W(\mathrm{x}, \mathrm{x}') = W_0(\mathrm{x}, \mathrm{x}') + h(\mathrm{x}, \mathrm{x}')$$

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↳ Cosmological Constant Problem?

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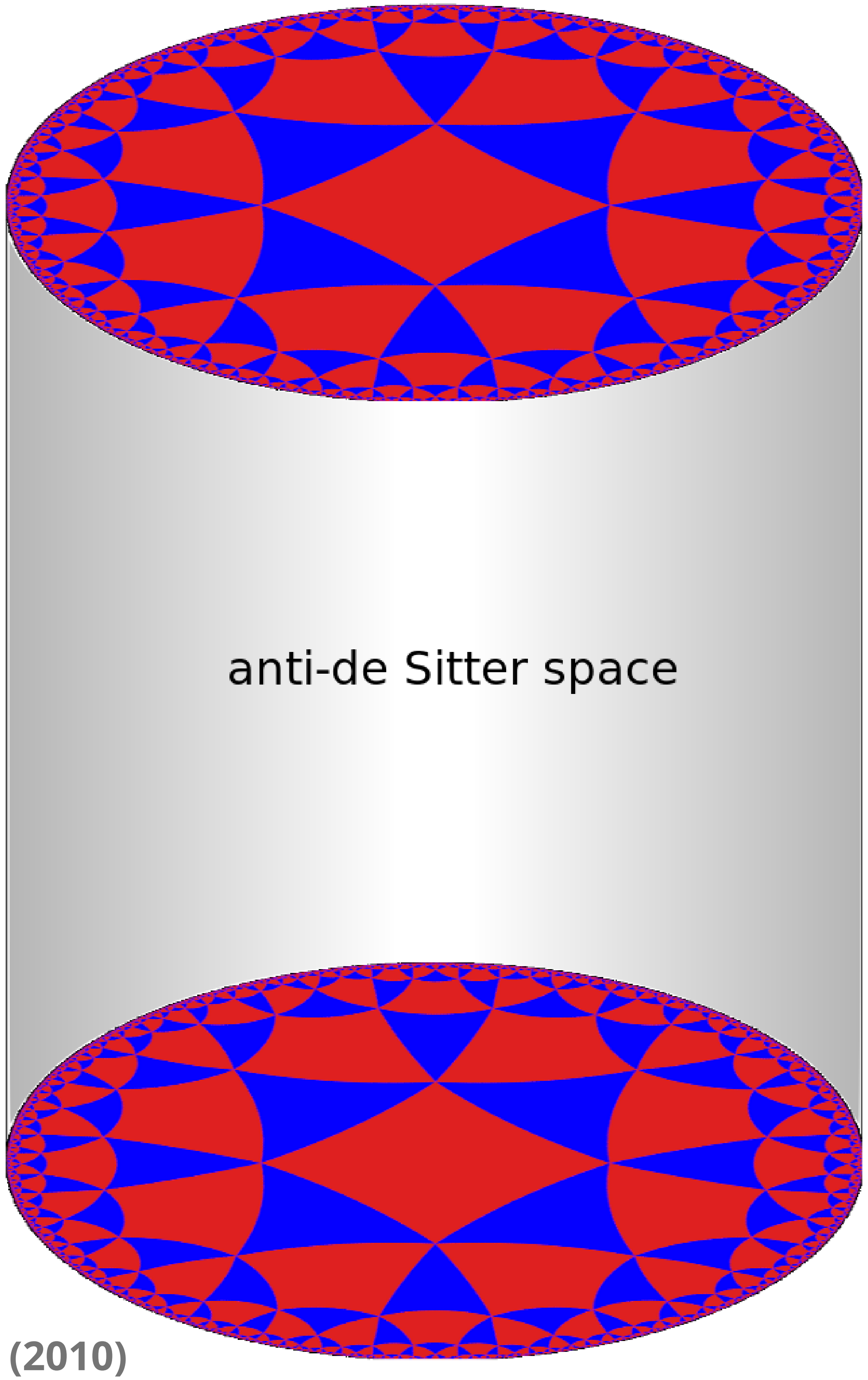

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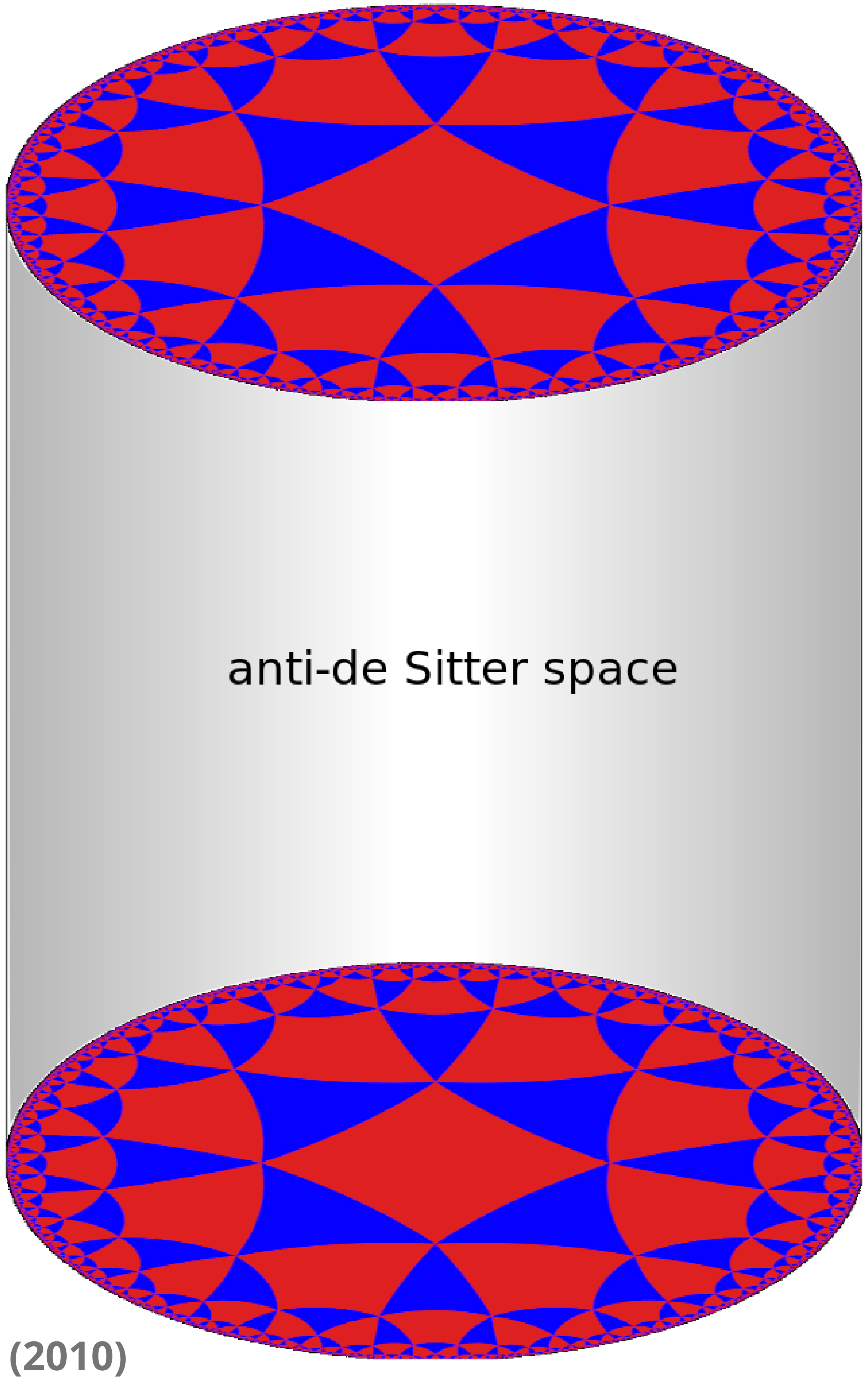

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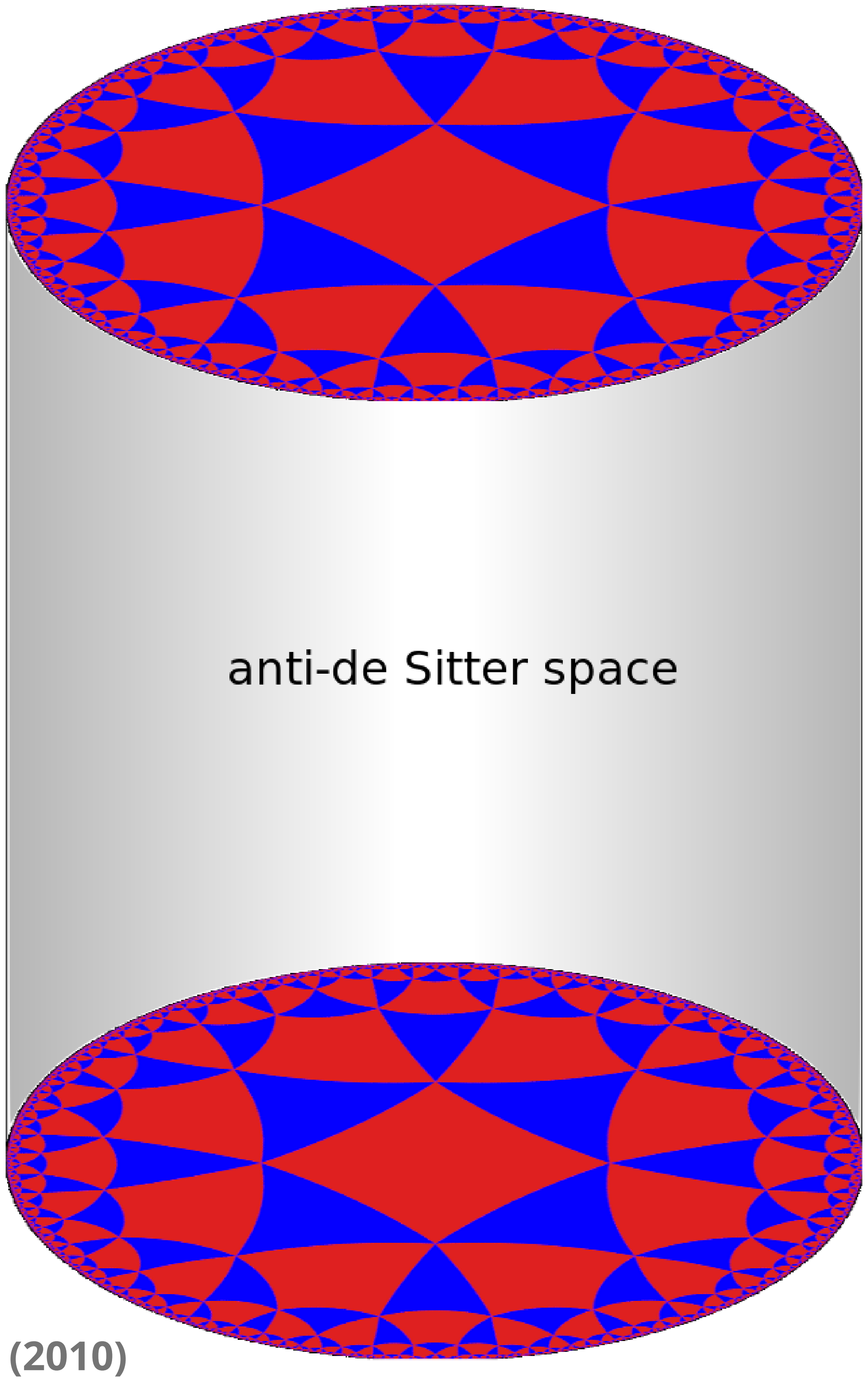

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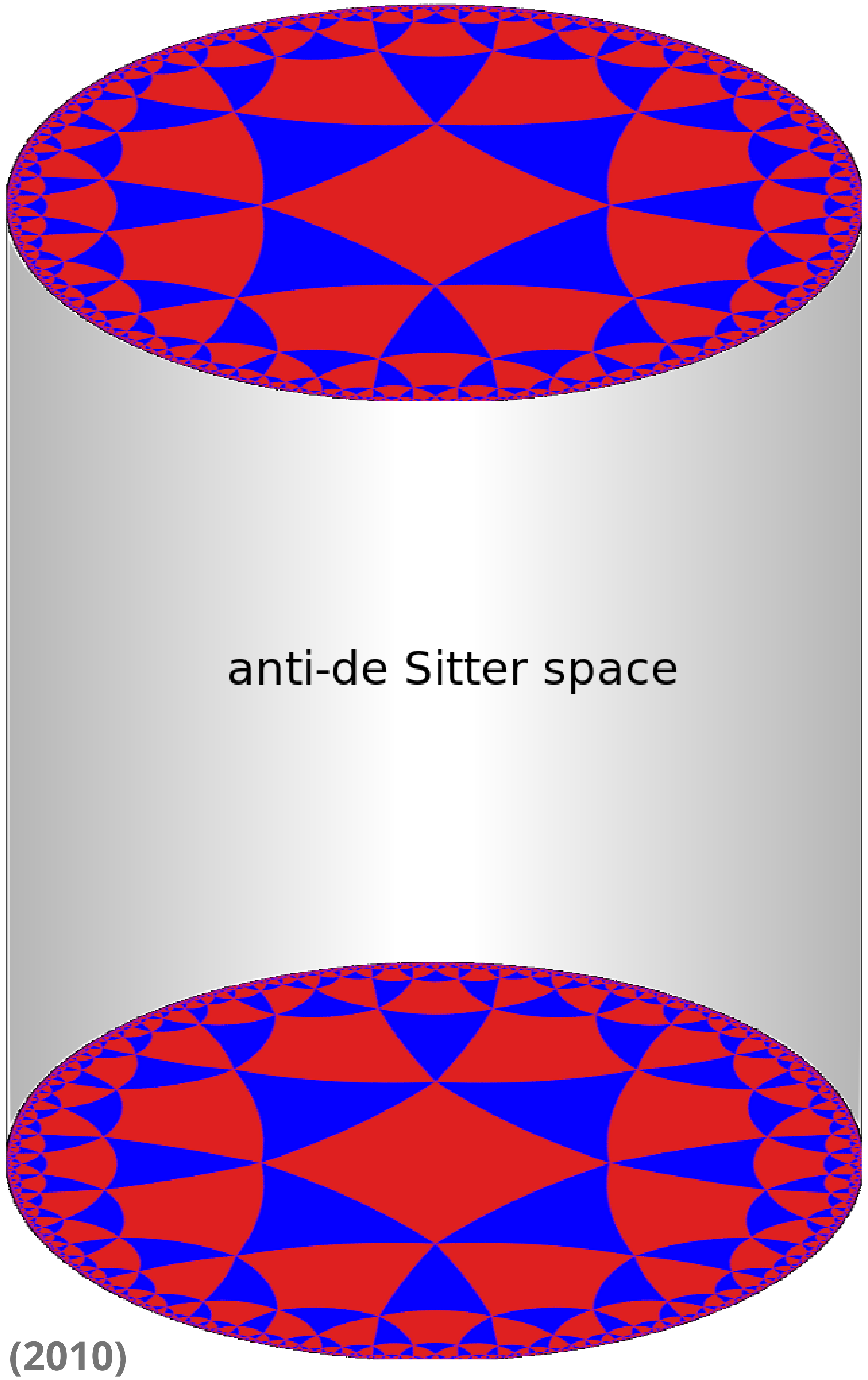
entanglement quantifier



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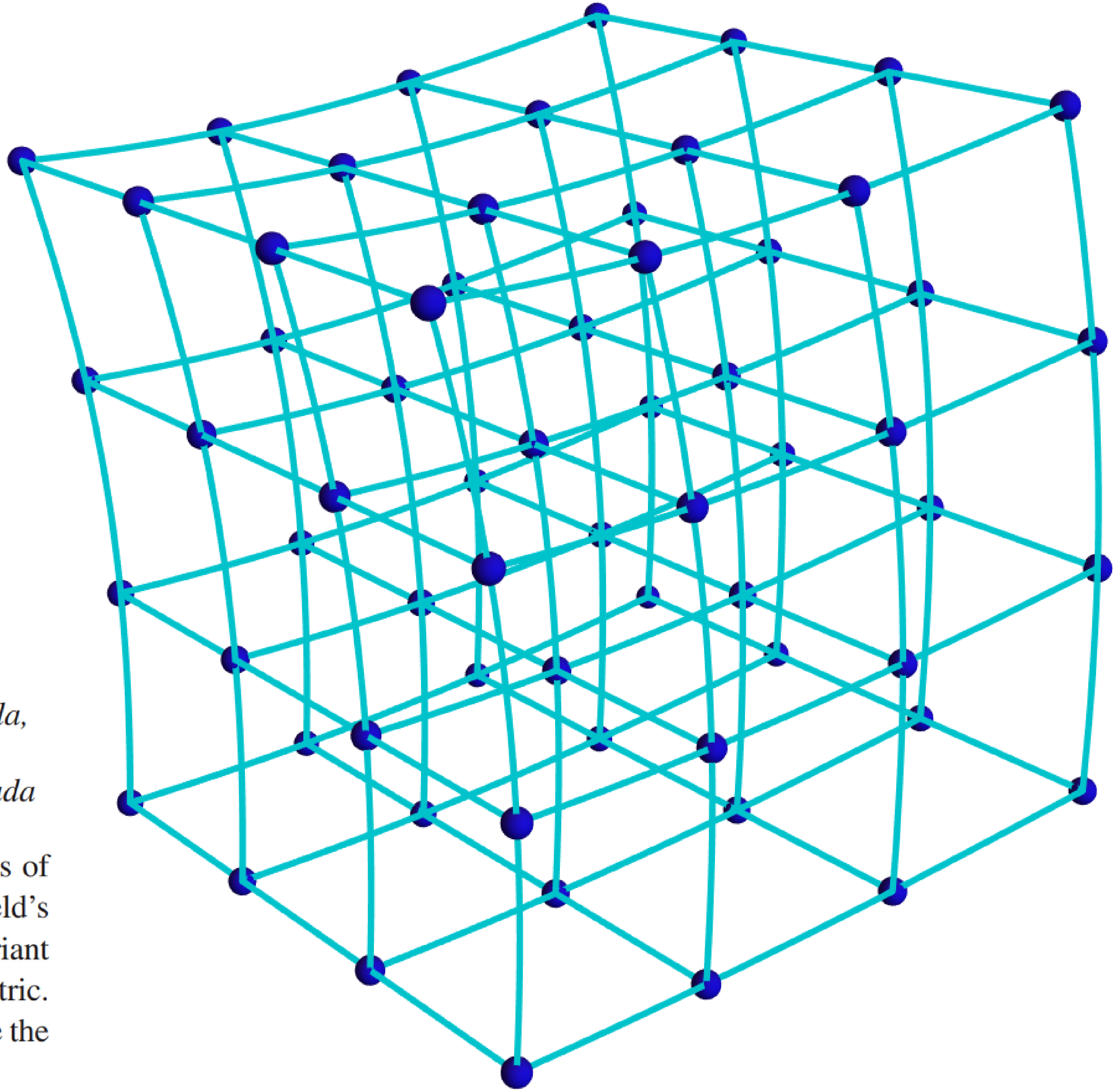
- There is potential to relate this with other relevant fields of physics such as AdS/CFT, Holography and Causal Set Theory.
- It might be possible to solve the cosmological constant problem using this idea.
- These ideas might give rise a theory of quantum gravity, where the geometry of spacetime is emergent from quantum fields.

Thank you!



Geometry of spacetime from quantum measurements

T. Rick Perche* and Eduardo Martín-Martínez†

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada,
Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada
and Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

We provide a setup by which one can recover the geometry of spacetime from local measurements of quantum particle detectors coupled to a quantum field. Concretely, we show how one can recover the field's correlation function from measurements on the detectors. Then, we are able to recover the invariant spacetime interval from the measurement outcomes, and hence reconstruct a notion of spacetime metric. This suggest that quantum particle detectors are the experimentally accessible devices that could replace the classical "rulers" and "clocks" of general relativity.

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