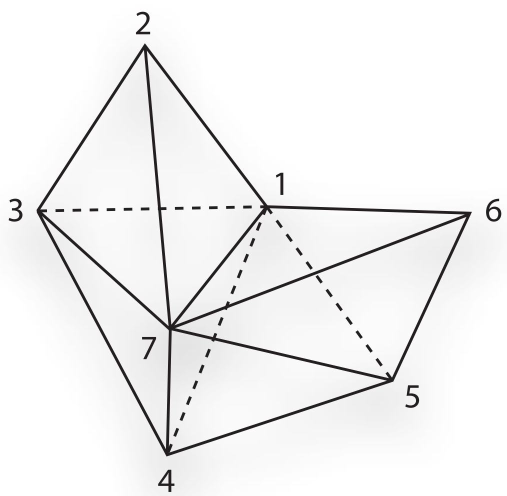
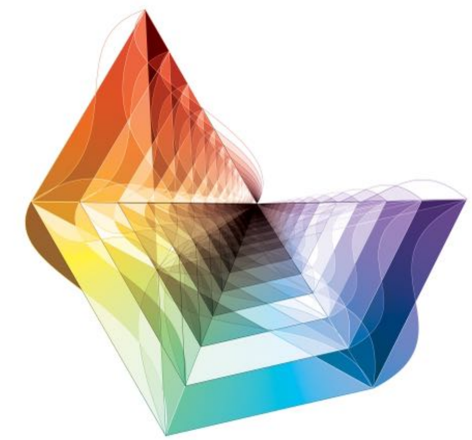


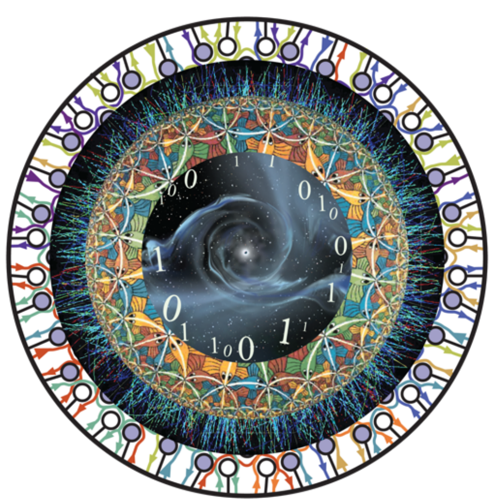


Scattering amplitudes

(not only for cosmologists)

Jaroslav Trnka

Center for Quantum Mathematics and Physics,
University of California, Davis
& IPNP, Charles University in Prague



CAS-JSPS, December 2022

Plan

- Motivation to study scattering amplitudes (history)
- New methods: amplitudes and geometry
- Amplitudes in EFTs (potentially relevant for cosmology)

Hidden simplicity in scattering amplitudes

Holy grail of theoretical physics

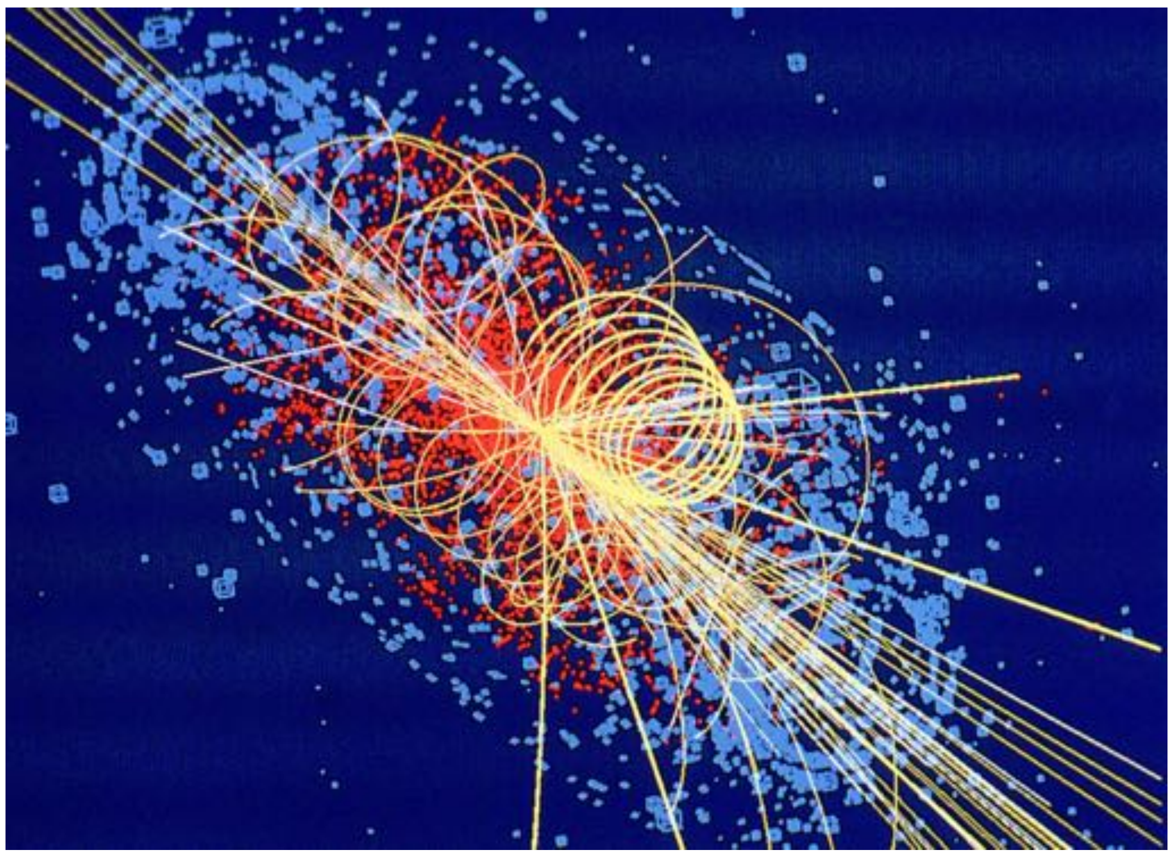
What are the Fundamental Laws of Nature?

What are the elementary physical forces?

Why is the Universe big?

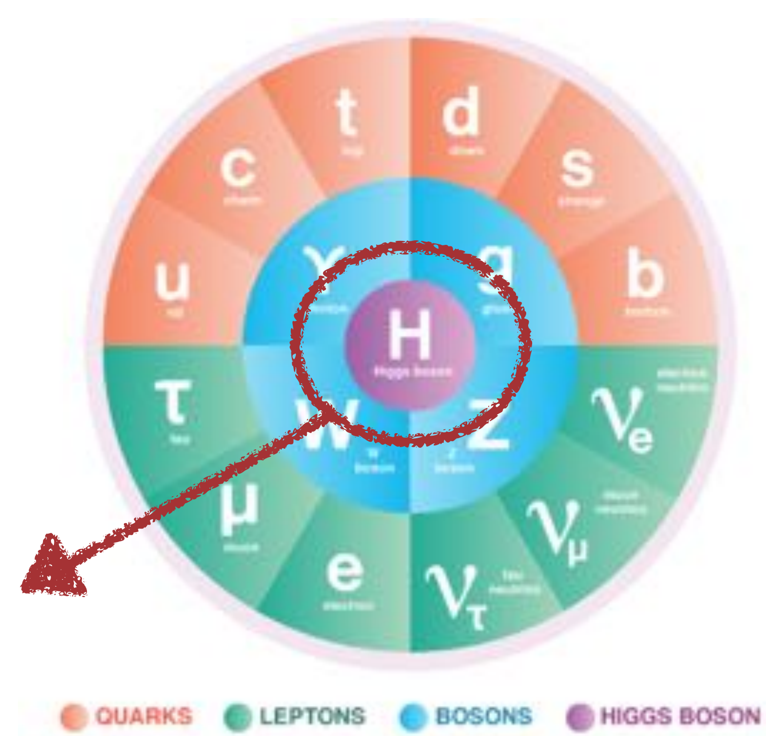
What is the theory of everything?

Probing fundamental laws



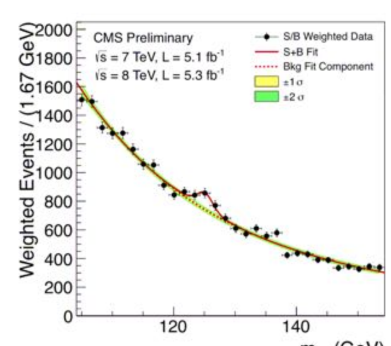
What is the menu
of elementary particles?

How do they interact?



Standard model of elementary particles

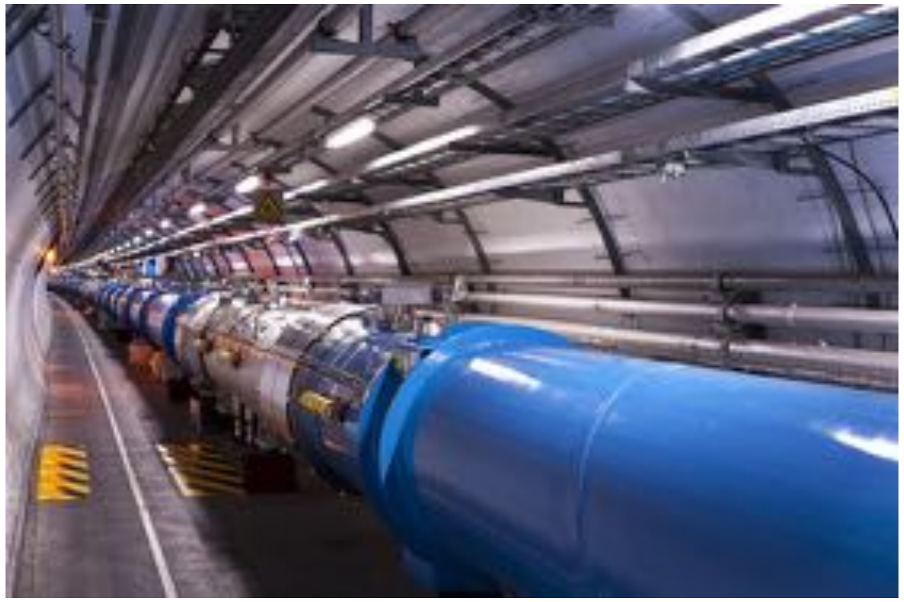
Higgs discovery
2012



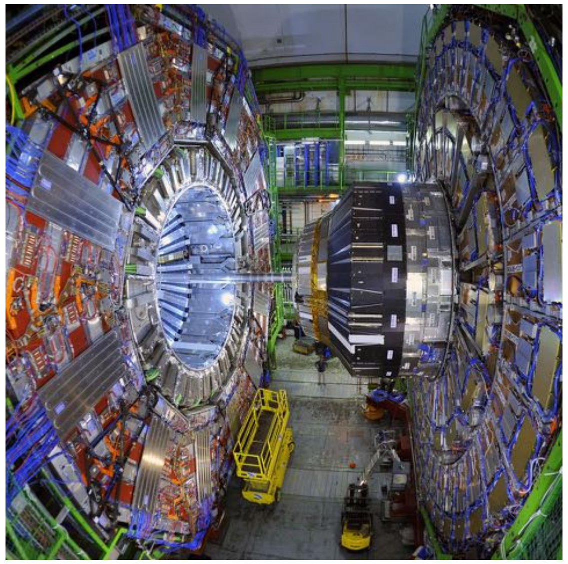
Search for new physics

Beyond Standard model

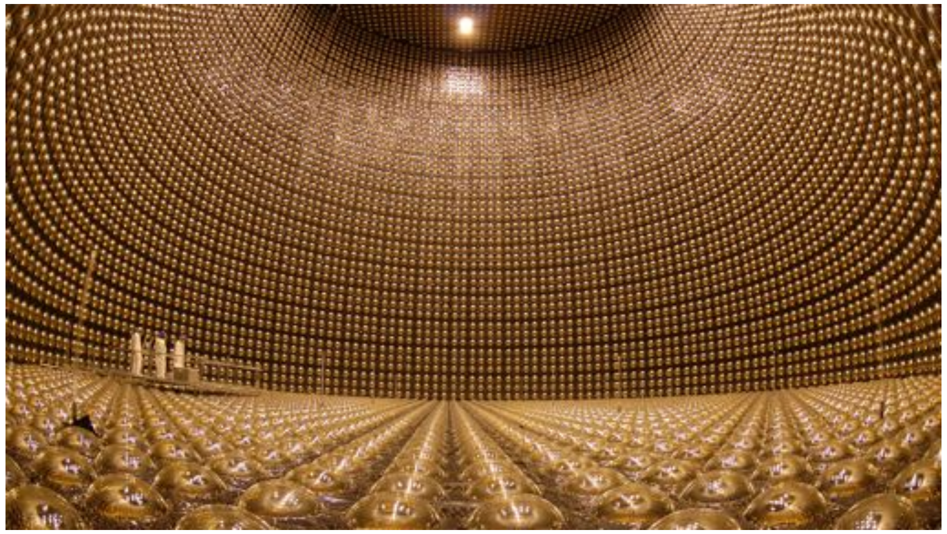
Higgs potential, proton decay,
WIMPs, search for SUSY,
neutrino mass, anomalies,…



LHC, Fermilab, future colliders



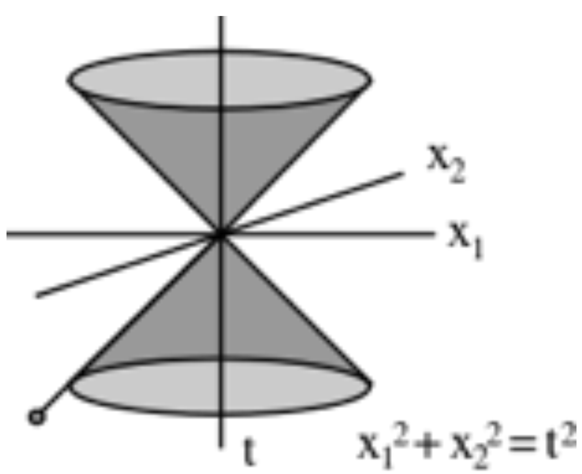
neutrino experiments



Theorist's perspective

- Theoretical framework to describe physical systems
- Compatible with two principles

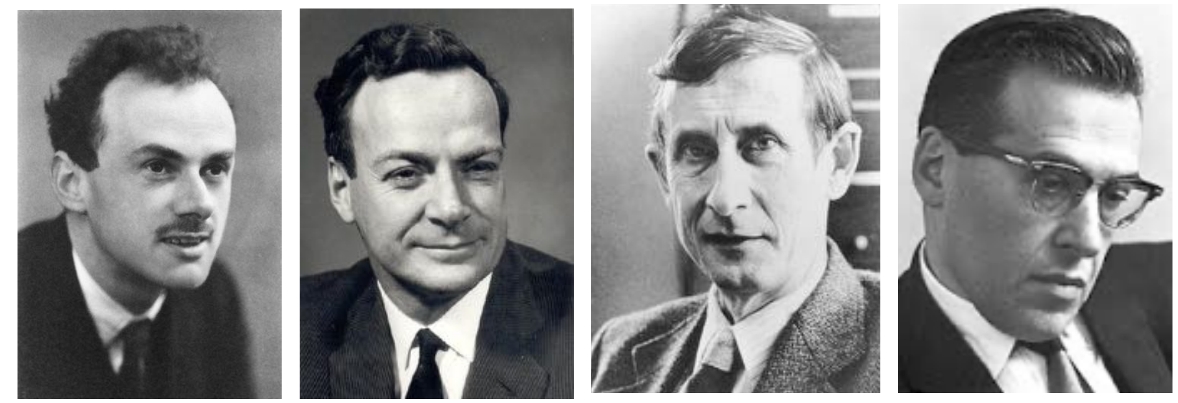
Special relativity	Quantum mechanics



$$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

Quantum Field Theory (QFT)

Quantum Field Theory



Dirac, Feynman, Dyson, Schwinger (1926-1950s)

- Elementary particles described by fields, their interactions (physical forces) by Lagrangian.

- Example: Quantum Electrodynamics - QFT for EM

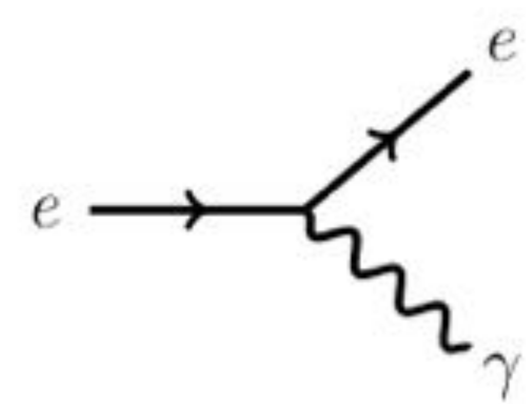
e Electron ψ

γ Photon A ← force carrier

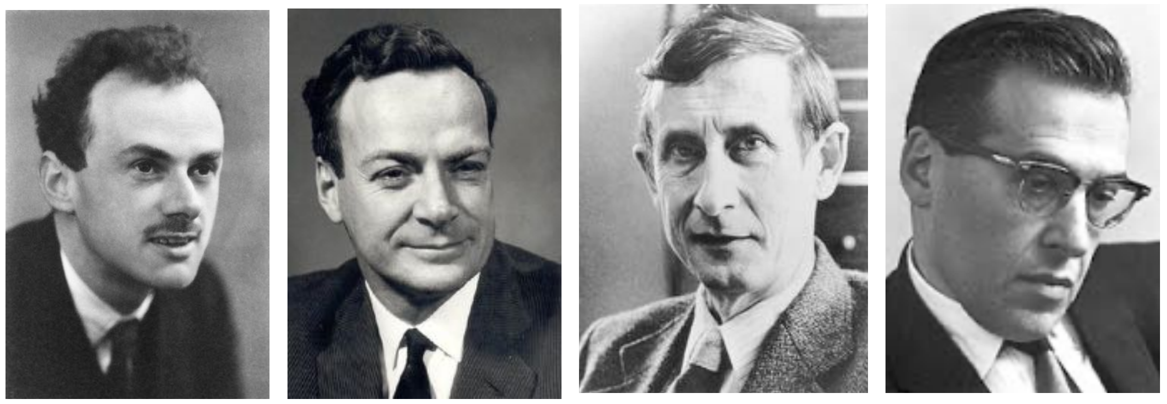
$$\mathcal{L} = q \cdot \psi \, \psi \, A$$

Lagrangian ↙ ↘ strength of interaction

We associate a picture



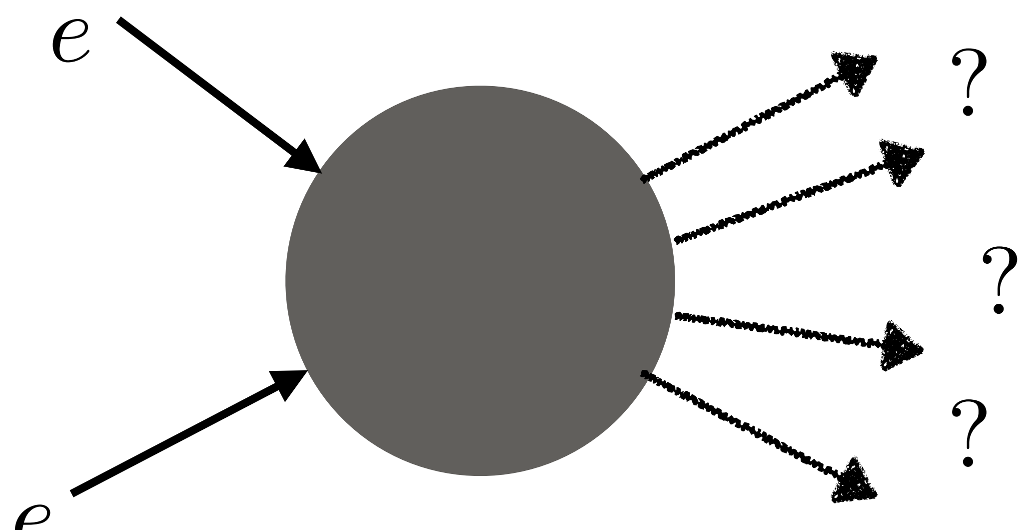
Quantum Field Theory



Dirac, Feynman, Dyson, Schwinger (1926-1950s)

- Use QFT to predict outcomes of particle experiments

initial state



final state

scattering process

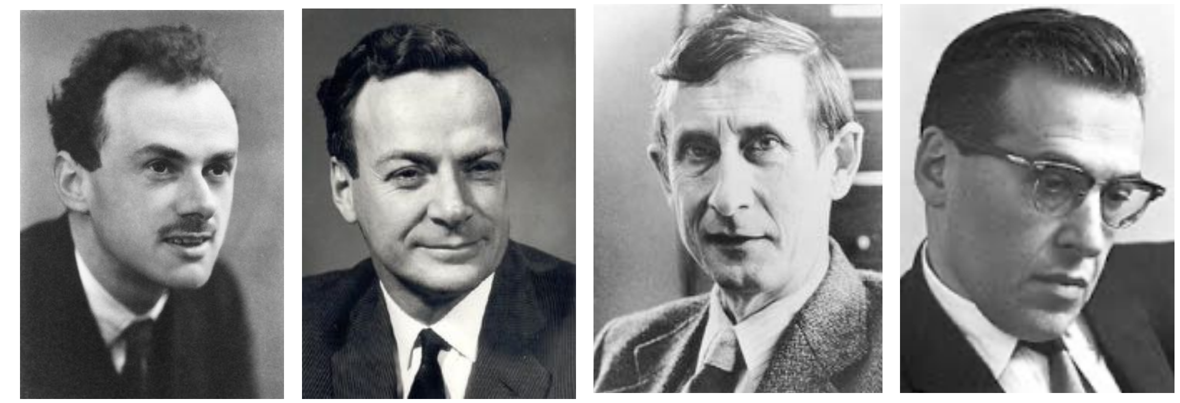
Quantum mechanics: all final states possible

Probabilities given by

Scattering amplitudes

$A(in, out)$

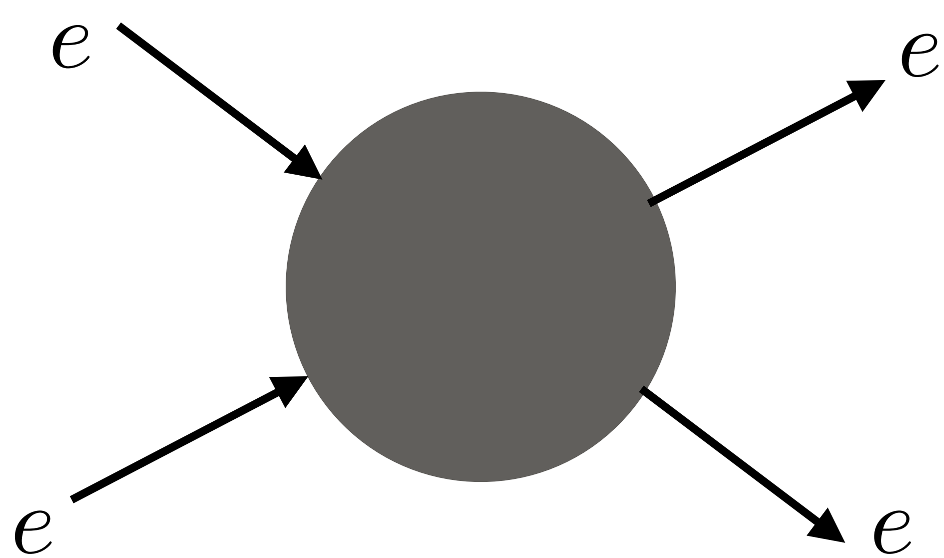
Quantum Field Theory



Dirac, Feynman, Dyson, Schwinger (1926-1950s)

- Use QFT to predict outcomes of particle experiments

initial state

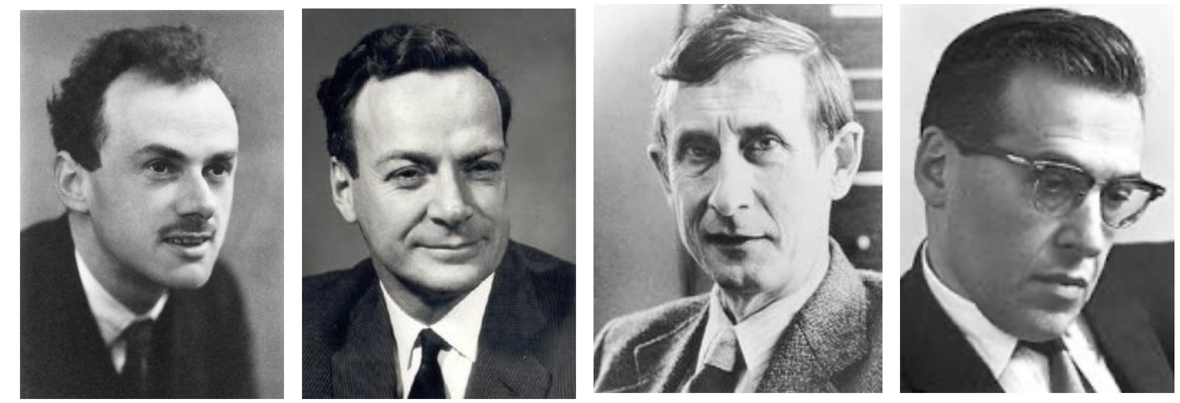


scattering process

$$A(ee \to ee)$$

Function of energies and angles of particles

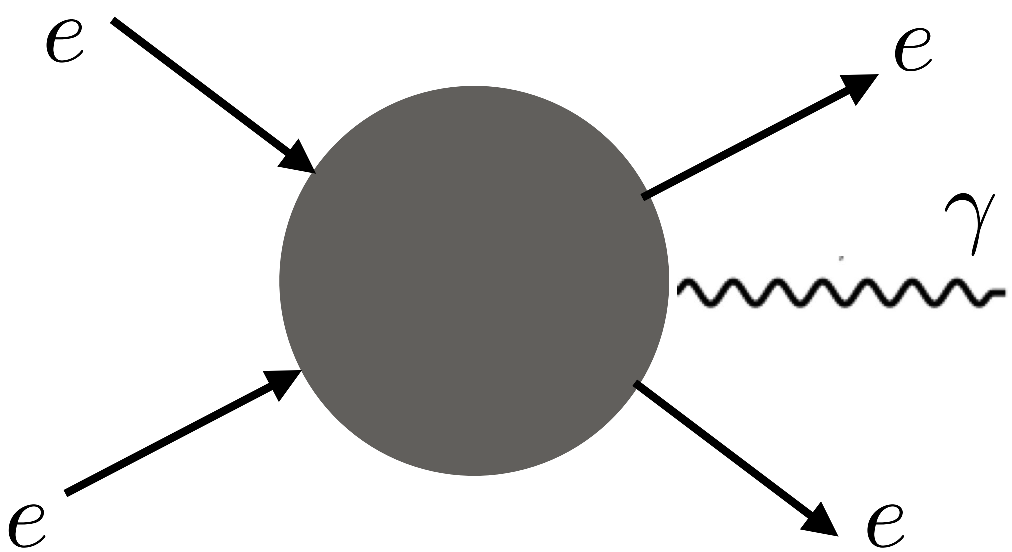
Quantum Field Theory



Dirac, Feynman, Dyson, Schwinger (1926-1950s)

- Use QFT to predict outcomes of particle experiments

initial state

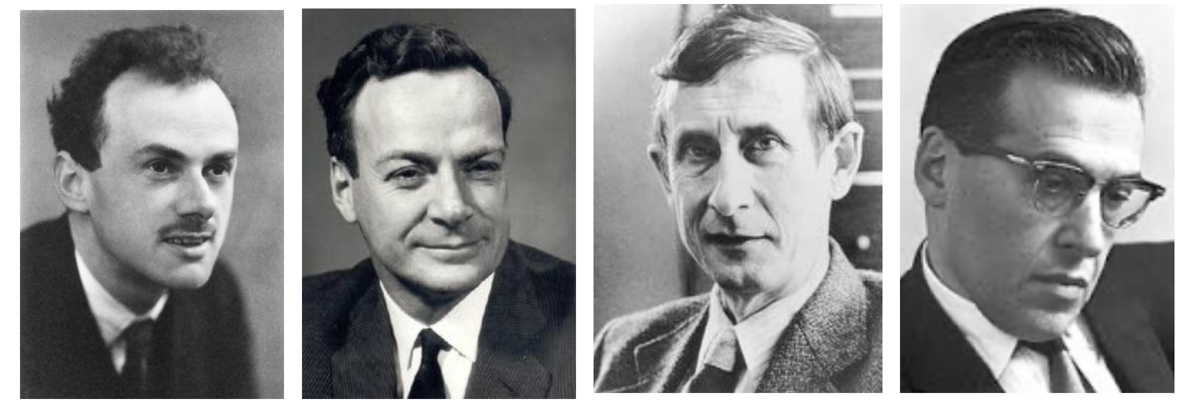


scattering process

$$A(ee \to ee\gamma)$$

Function of energies and angles of particles

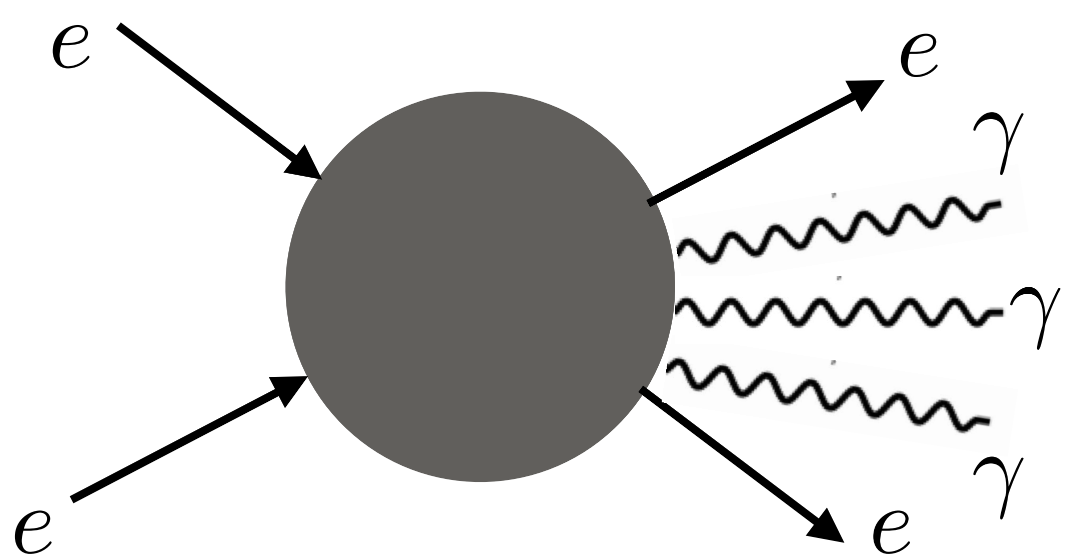
Quantum Field Theory



Dirac, Feynman, Dyson, Schwinger (1926-1950s)

- Use QFT to predict outcomes of particle experiments

initial state



scattering process

$$A(ee \to ee\gamma\gamma\gamma)$$

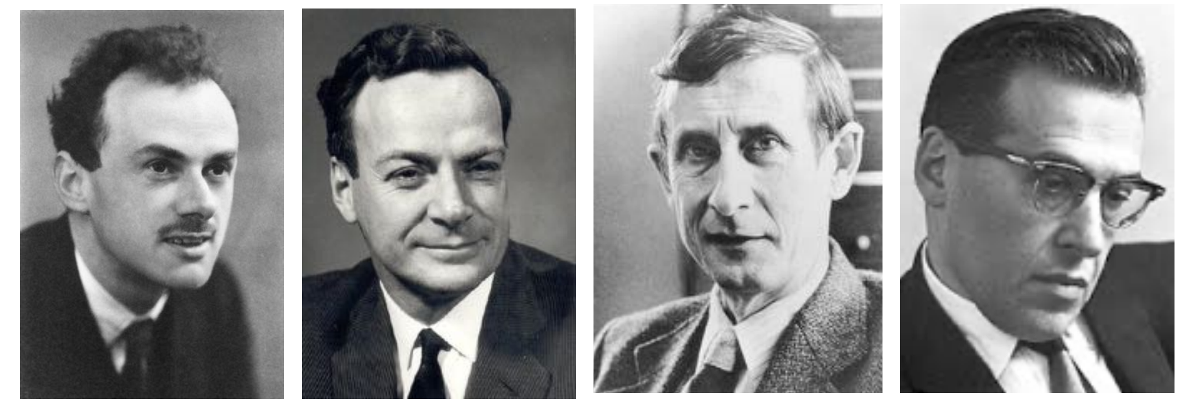
Function of energies and angles of particles

probability: square of amplitude

$$p_i \sim |A|^2$$

Unitarity: some of probabilities $\sum_i p_i = 1$

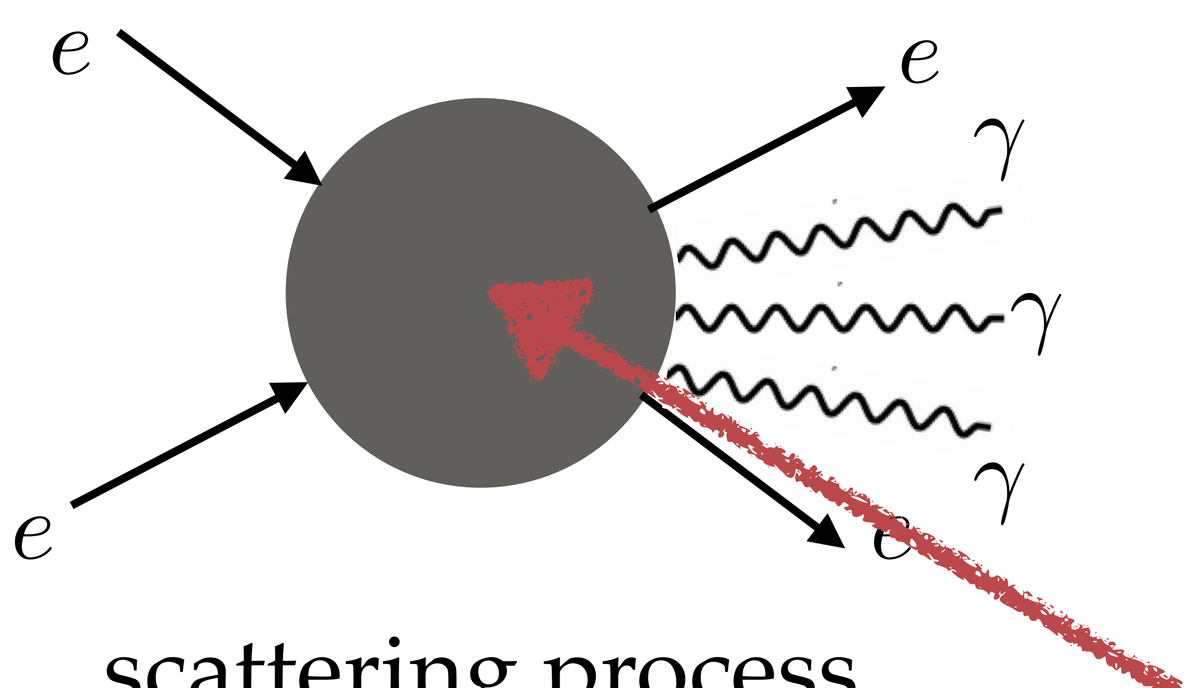
Quantum Field Theory



Dirac, Feynman, Dyson, Schwinger (1926-1950s)

- Use QFT to predict outcomes of particle experiments

initial state



scattering process

$$A(ee \to ee\gamma\gamma\gamma)$$

Function of energies and angles of particles

Big question:

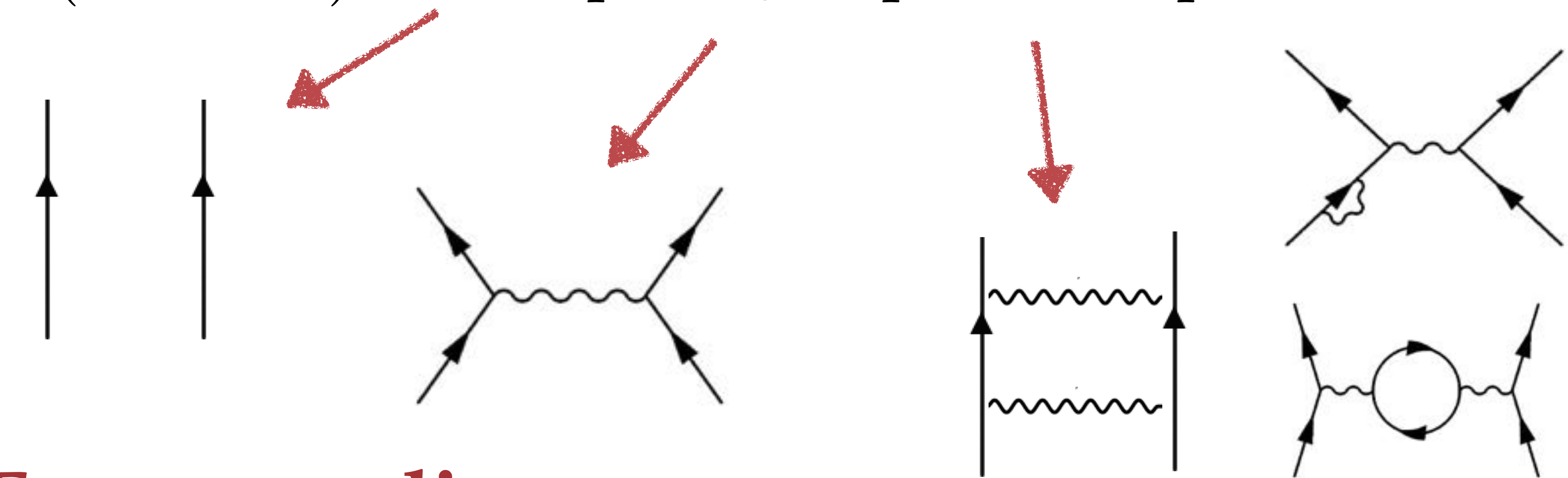
What happens inside?

Perturbative QFT

Feynman, Dyson, Schwinger (1940s-1950s)

- Expansion of the amplitude for small q: weak coupling

$$A(ee \to ee) = 1 + q^2 \cdot A_0 + q^4 \cdot A_1 + q^6 \cdot A_2 + \dots$$



and many others

Feynman diagrams

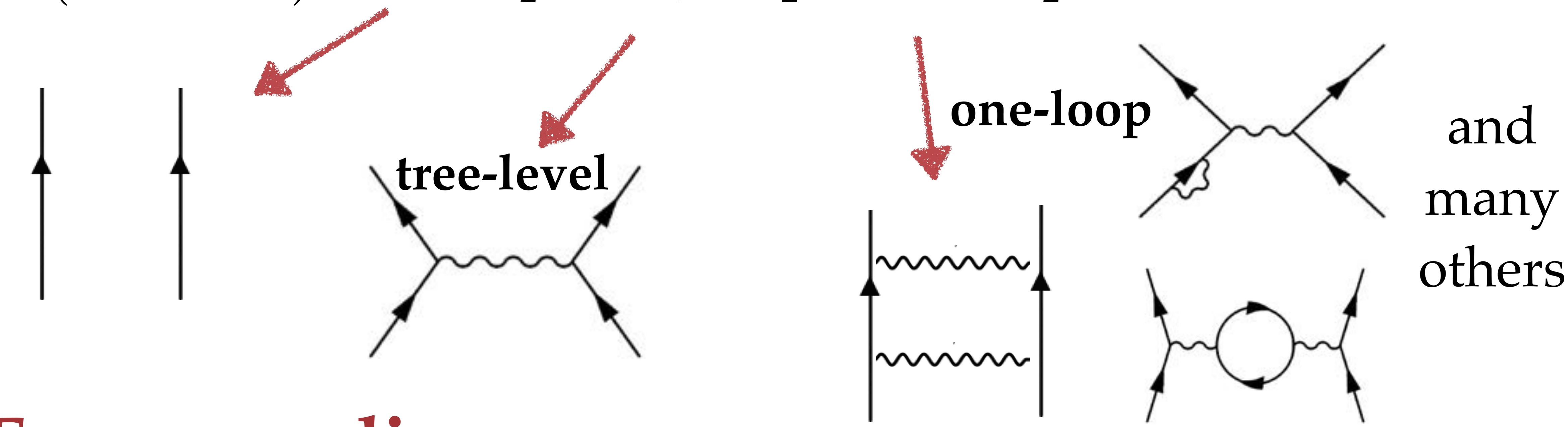
$$\left(\begin{array}{c} \text{Expansion of the path integral} \\ Z = \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}A \, e^{iS} \end{array} \right)$$

Perturbative QFT

Feynman, Dyson, Schwinger (1940s-1950s)

- Expansion of the amplitude for small q: weak coupling

$$A(ee \to ee) = 1 + q^2 \cdot A_0 + q^4 \cdot A_1 + q^6 \cdot A_2 + \dots$$



Feynman diagrams

Loop expansion

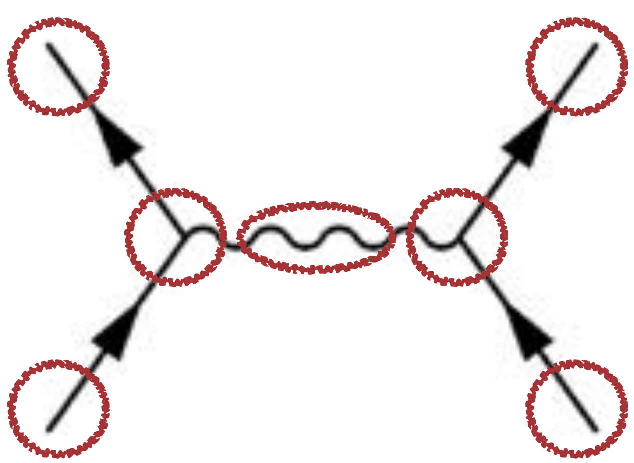
(Expansion of the path integral $Z = \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}A \, e^{iS}$)

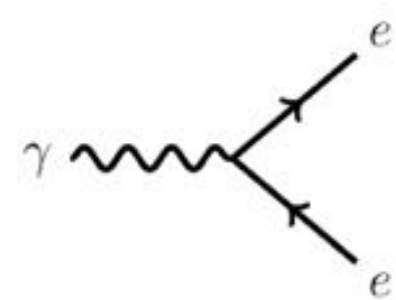
Quantum Field Theory

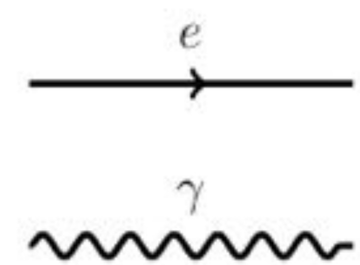
Feynman, Dyson, Schwinger (1940s-1950s)

- Simple diagrammatics: draw all Feynman diagrams
- Each diagram: contribution to amplitude

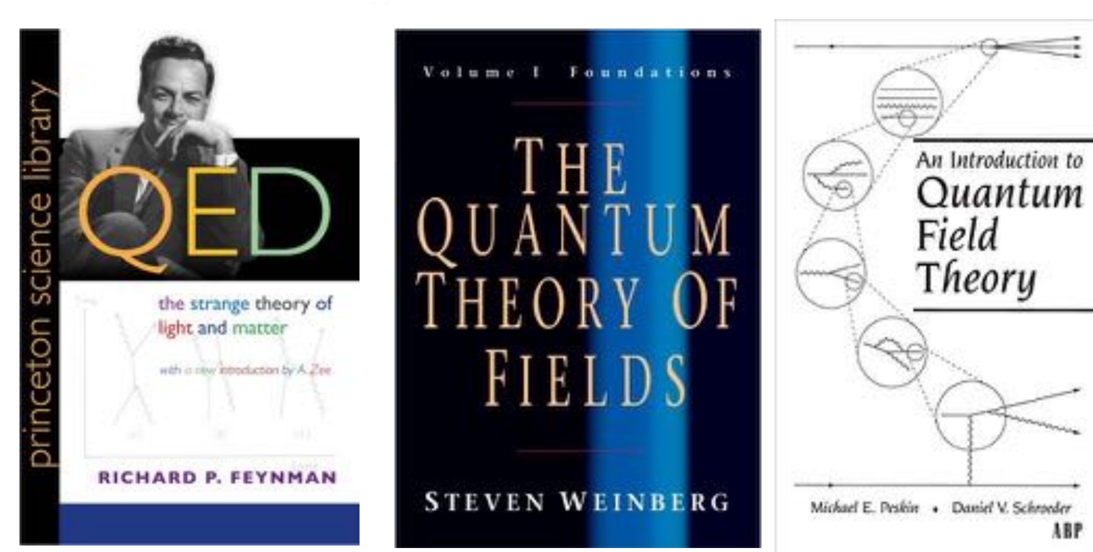
gluing these pictures







Feynman rules: prescription how to convert diagram into formula



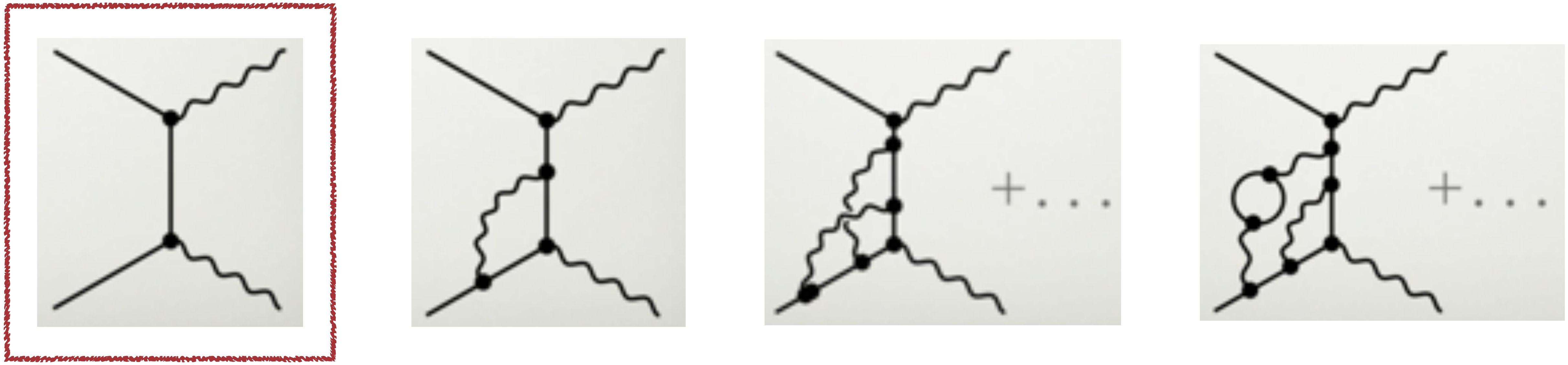
Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1928

Theory: $g_e = 2$

Experiment: $g_e \sim 2$



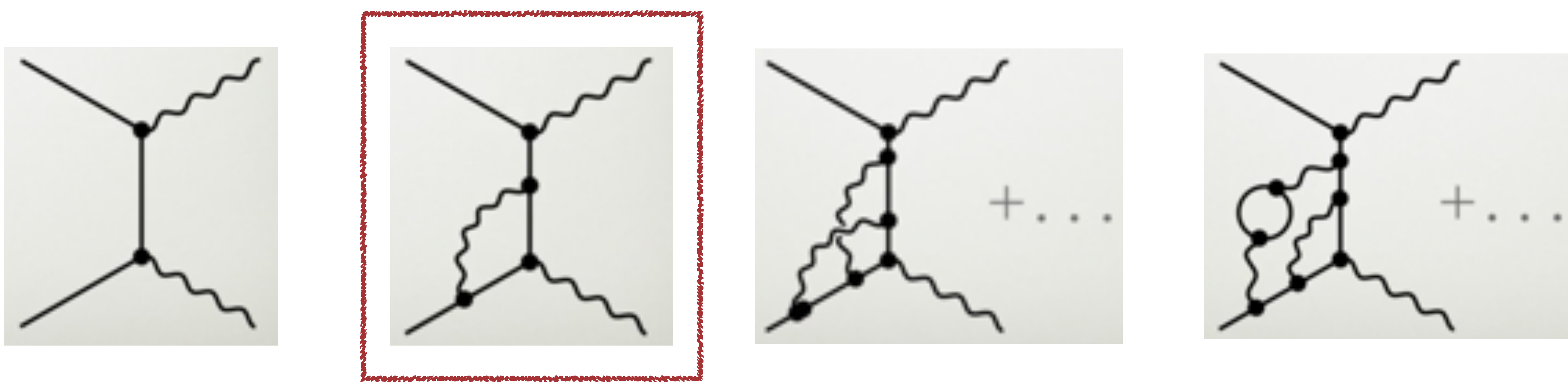
Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1947

Theory: $g_e = 2.00232$

Experiment: $g_e = 2.0023$

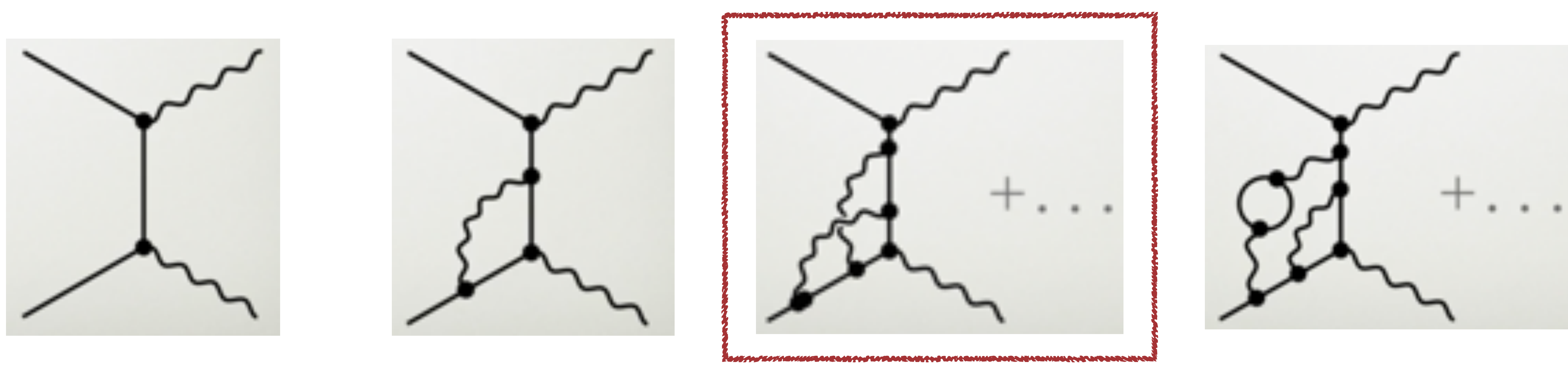


Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1957 Theory: $g_e = 2.0023193$

1972 Experiment: $g_e = 2.00231931$



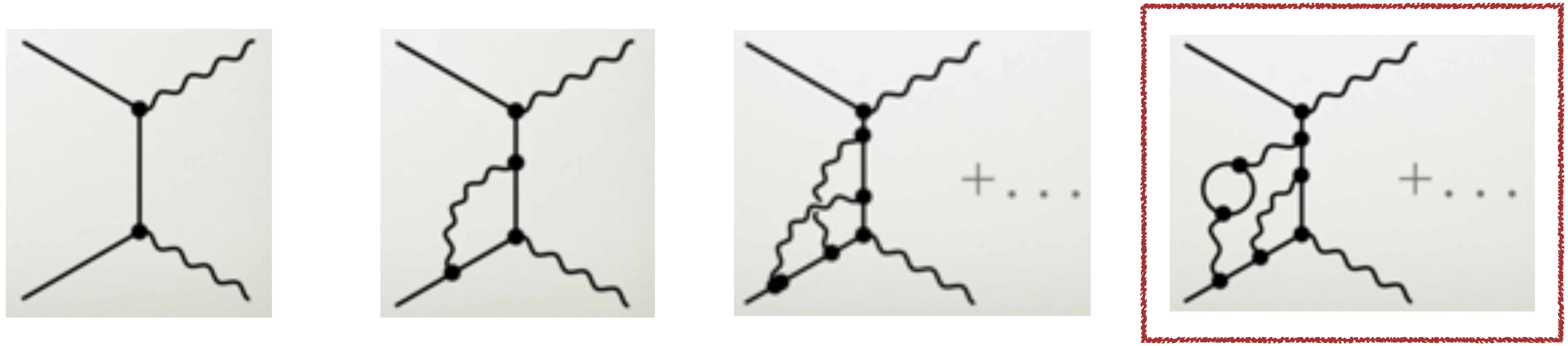
Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1990

Theory: $g_e = 2.0023193044$

Experiment: $g_e = 2.00231930438$

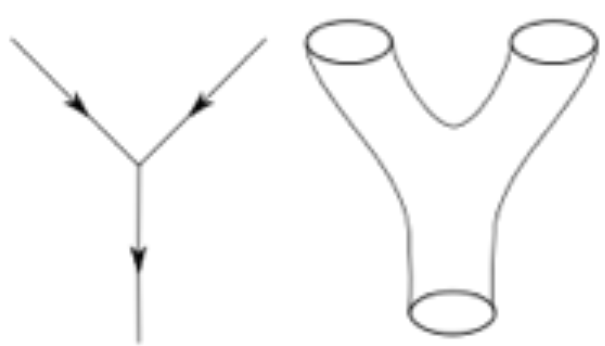


Problems with gravity

More conceptual problem: tension between QFT and gravity

QFT: **local observables** - interactions happen at a point, gravity forbids them - what is quantum gravity?

Our best attempt: **string theory**



concept: consistent unification of QFT and gravity

AdS/CFT correspondence: QFT is "dual" to string theory

strongly coupled, no gravity

weakly coupled, with gravity

Maldacena (1997)

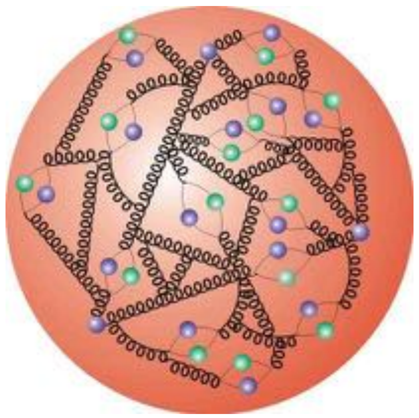
QFT picture incomplete

Despite all successes our understanding of QFT is incomplete

If there is a new formulation of QFT, we should see footprint in the weak coupling regime: structure of **scattering amplitudes**

QCD background and new physics

- Distinguish new physics from Standard model
  - Accurate theoretical predictions of background needed
- Colliders: protons at high energies
  - Main component is scattering of gluons



- Standard procedure: Feynman diagrams

Status of the art: early 1980s

- Most complicated process: $gg \to ggg$ at leading order

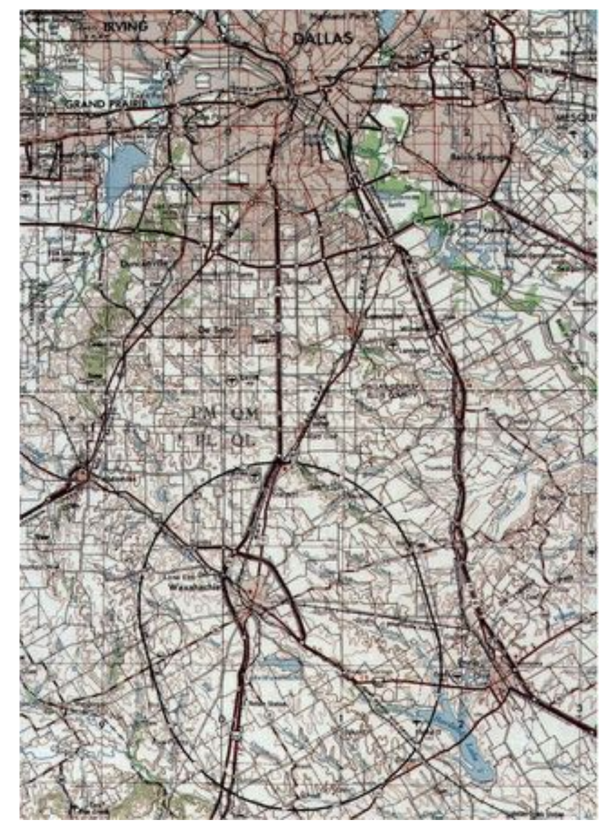
Brute force calculation
24 pages of result

$(k_1 \cdot k_4)(\epsilon_2 \cdot k_1)(\epsilon_1 \cdot \epsilon_3)(\epsilon_4 \cdot \epsilon_5)$

- Perhaps not every question has a simple answer…..

New collider

- 1983: Superconducting Super Collider approved
- Energy 40 TeV: many gluons!

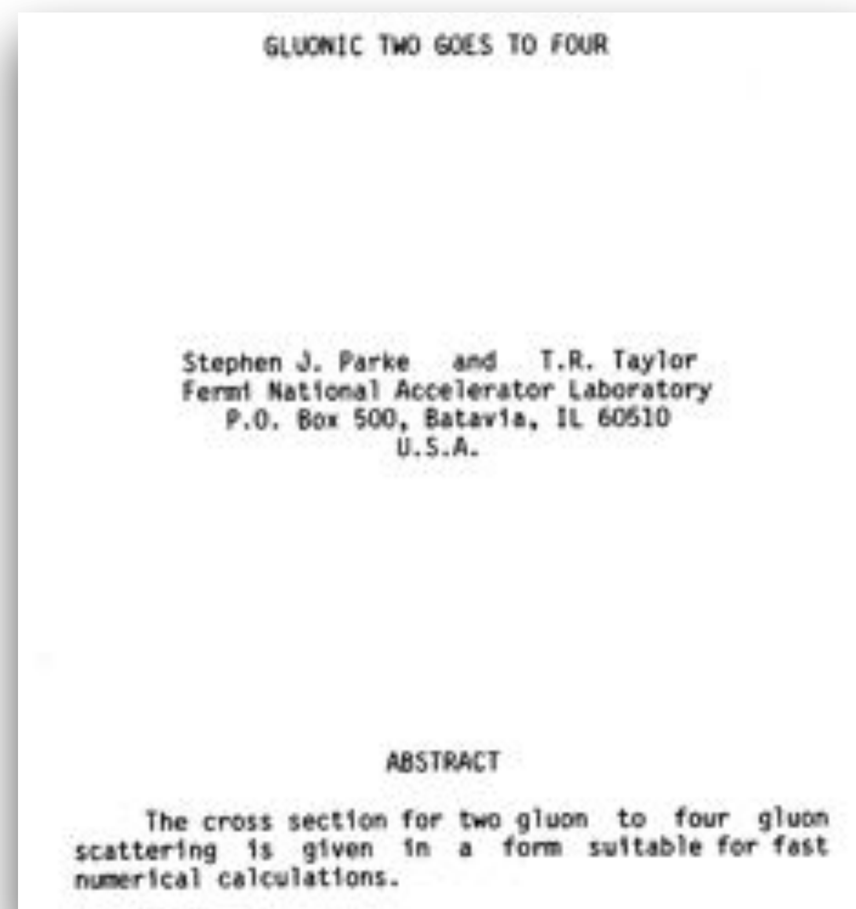


- Demand for calculations, next on the list: $gg \to gggg$

Hidden simplicity in scattering amplitudes


Parke, Taylor (1985)

- Process $gg \to gggg$
- 220 Feynman diagrams, $\sim$100 pages of calculations
- Paper with 14 pages of result



GLUONIC TWO GOES TO FOUR

Stephen J. Parke and T.R. Taylor
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510
U.S.A.

ABSTRACT

The cross section for two gluon to four gluon scattering is given in a form suitable for fast numerical calculations.

Hidden simplicity in scattering amplitudes


Parke, Taylor (1985)

- Process $gg \to gggg$

- 220 Feynman diagrams, $\sim$100 pages of calculations

Hidden simplicity in scattering amplitudes

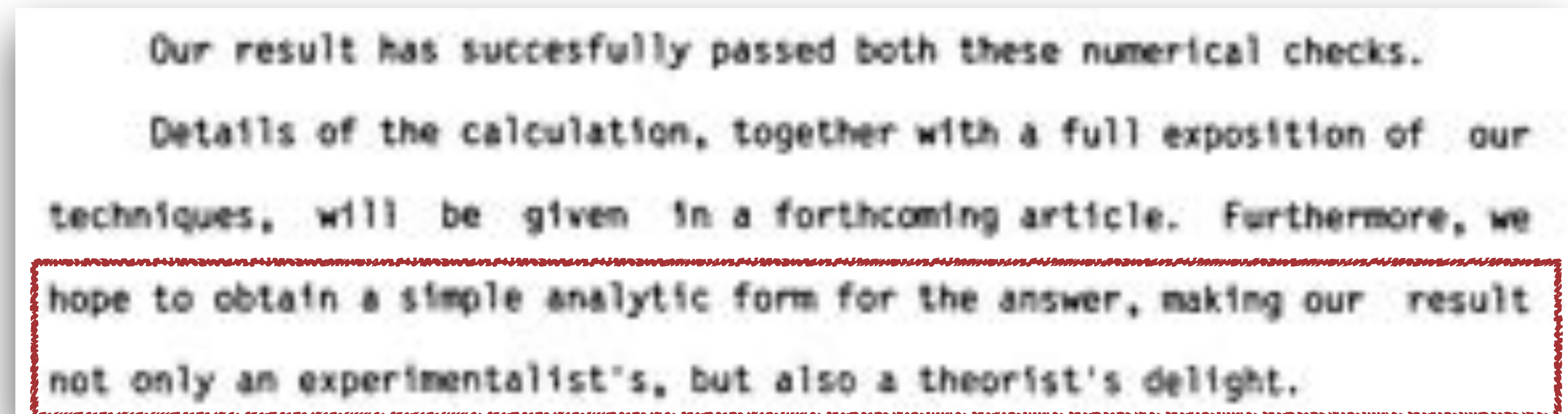

Parke, Taylor (1985)

Our result has succesfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

Hidden simplicity in scattering amplitudes


Parke, Taylor (1985)



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- Within a year they realized

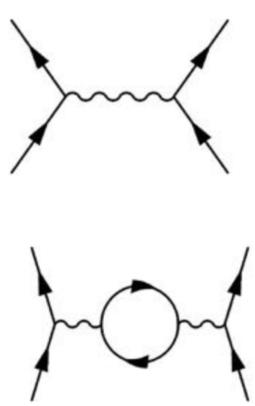
$$|A_6|^2 \sim \frac{(p_1 \cdot p_2)^3}{(p_2 \cdot p_3)(p_3 \cdot p_4)(p_4 \cdot p_5)(p_5 \cdot p_6)(p_6 \cdot p_1)}$$

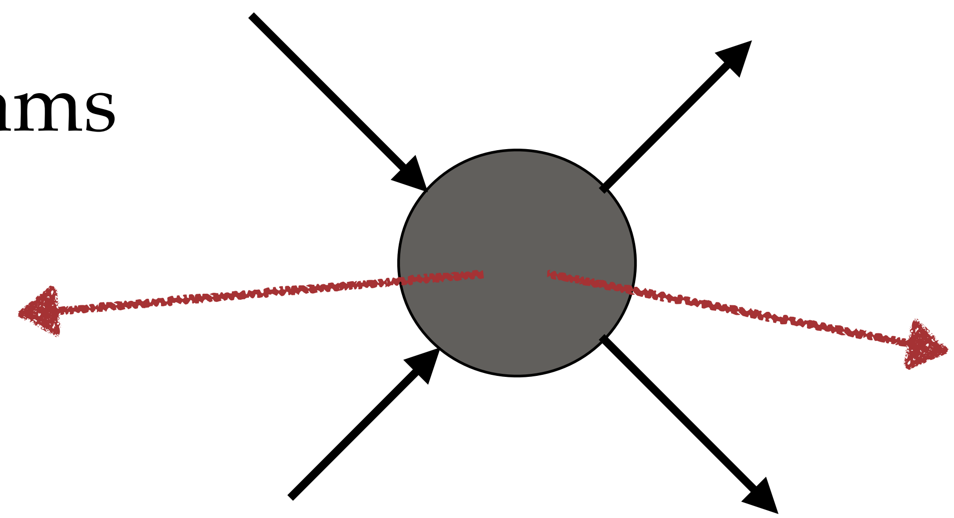
- Final result is much simpler than individual diagrams!

Change of strategy

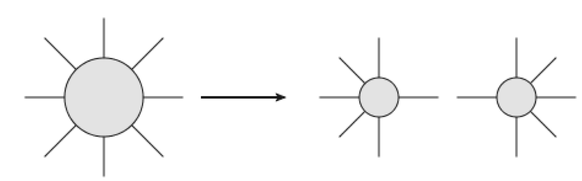
What is the scattering amplitude?

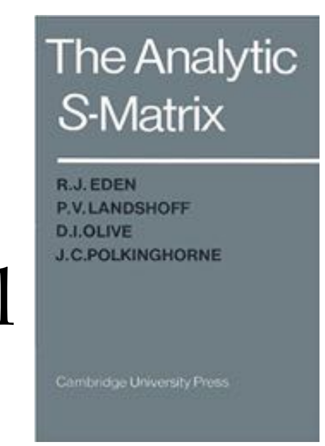
Feynman diagrams





Unique object fixed
by physical properties





Was not successful

(1960s)

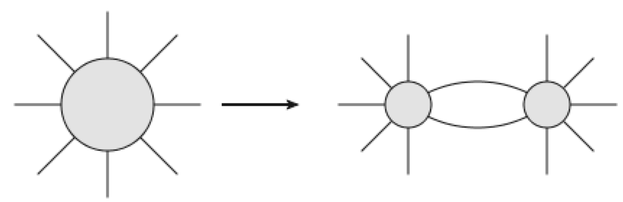
Modern methods use both:

- Calculate the amplitude directly
- Use perturbation theory

Life without Feynman diagrams

- New efficient methods of calculations
  - Unitarity methods

Bern, Dixon, Kosower (1990s)



BlackHat collaboration
QCD background for LHC

- Recursion relations

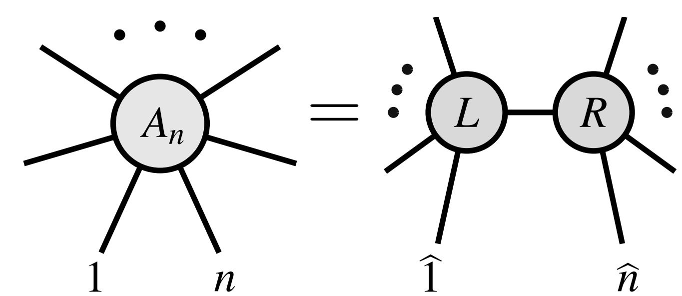
Britto, Cachazo, Feng, Witten (2005)

Cohen, Elvang, Kiermaier (2010)

Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka (2010)

Cheung, Kampf, Novotny, Shen, Trnka (2015)

Build amplitude recursively from simpler amplitudes



	$gg \to 4g$	$gg \to 5g$	$gg \to 6g$
Feynman diagrams	220	2485	34300
Terms in recursion	3	6	20

Breadth of Amplitudes field

Very broad field, many (often orthogonal) research interests

S-matrix is a tool to study many different things

- New efficient methods to calculate higher-loop amplitudes, numerics, special functions
- Use amplitudes as a tool to probe QFT (new principles, symmetries, unexpected connections, integrability)
- Applications of methods to other fields: gravitational waves, study of cosmological correlators

Amplitudes conferences

- Many meetings, conferences and workshops

- Annual Amplitudes conference
  - 2009 Durham
  - 2010 London
  - 2011 Michigan
  - 2012 Hamburg
  - 2013 Munich
  - 2014 Paris
  - 2015 Zurich
  - 2016 Stockholm
  - 2017 Edinburgh
  - 2019 Dublin
  - 2020 Michigan
  - 2021 Copenhagen
  - **2022 Prague**

Geometric picture for scattering amplitudes

with Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Henn, Herrmann,
Postnikov, Thomas and many others

Amplitude as a volume

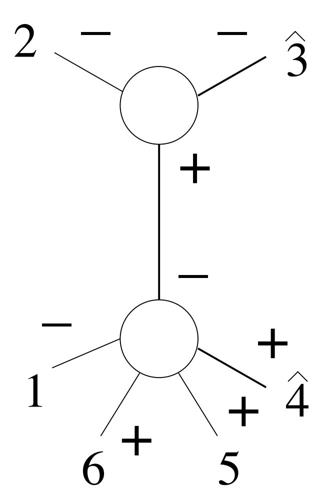
Hodges (2009)

In 2009 Hodges studied recursion relations for gluon amplitudes

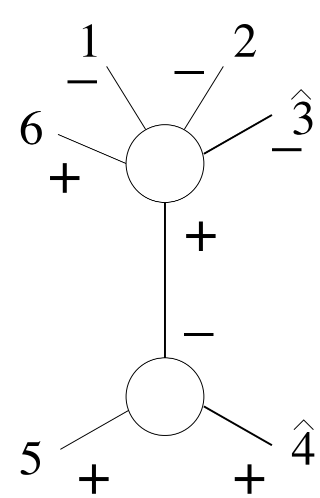
He wanted to use twistor
variables introduced by Penrose in 1970s
in his own attempt for quantum gravity

For particular six-gluon amplitude at tree-level he took the result

$A_6 =$



$+$



Amplitude as a volume

Hodges (2009)

In 2009 Hodges studied recursion relations for gluon amplitudes

He wanted to use twistor variables introduced by Penrose in 1970s in his own attempt for quantum gravity

For particular six-gluon amplitude at tree-level he took the result and rewrote using momentum twistor variables

$$A_6 = \frac{\langle 1345\rangle^3}{\langle 1245\rangle\langle 2345\rangle\langle 1234\rangle\langle 1235\rangle} - \frac{\langle 1356\rangle^3}{\langle 1256\rangle\langle 6123\rangle\langle 2356\rangle\langle 1235\rangle}$$

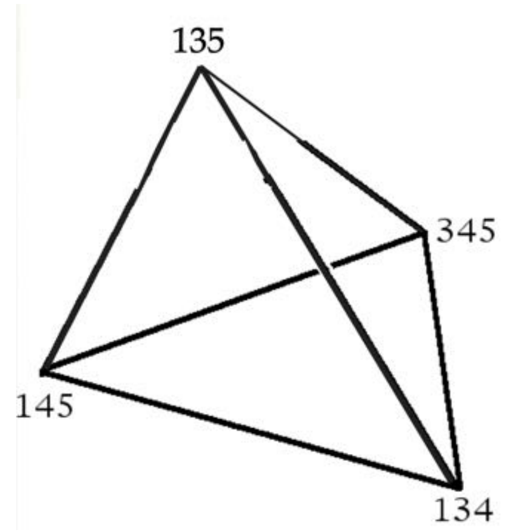
And the expressions look familiar to him

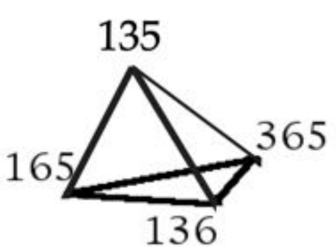
Amplitude as a volume

Hodges (2009)

They are **volumes** of tetrahedra in momentum twistor space!

$$\frac{\langle 1345\rangle^3}{\langle 1245\rangle\langle 2345\rangle\langle 1234\rangle\langle 1235\rangle} - \frac{\langle 1356\rangle^3}{\langle 1256\rangle\langle 6123\rangle\langle 2356\rangle\langle 1235\rangle}$$





Amplitude as a volume

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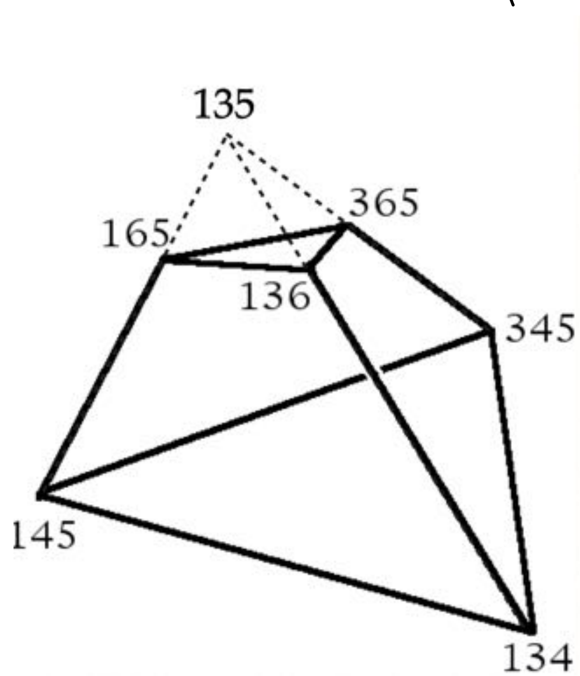
$$\frac{\langle 1345\rangle^3}{\langle 1245\rangle\langle 2345\rangle\langle 1234\rangle\langle 1235\rangle} - \frac{\langle 1356\rangle^3}{\langle 1256\rangle\langle 6123\rangle\langle 2356\rangle\langle 1235\rangle}$$



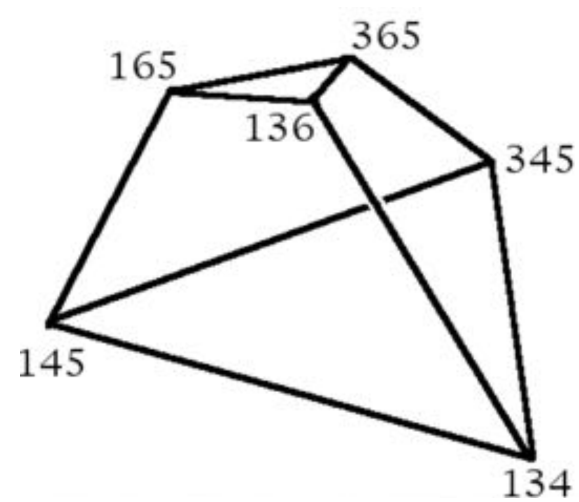
These two pieces subtract, we are triangulating polyhedron

Amplitude as a volume

Hodges (2009)

They are **volumes** of tetrahedra in momentum twistor space!

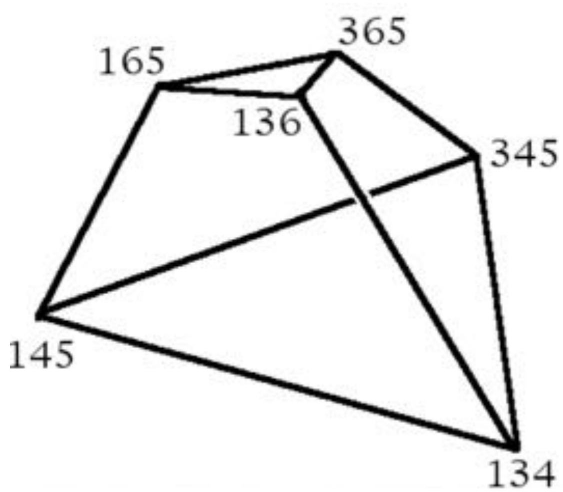
$$\frac{\langle 1345\rangle^3}{\langle 1245\rangle\langle 2345\rangle\langle 1234\rangle\langle 1235\rangle} - \frac{\langle 1356\rangle^3}{\langle 1256\rangle\langle 6123\rangle\langle 2356\rangle\langle 1235\rangle}$$



Amplitude is a volume of polyhedron!

Amplitude as a volume

Hodges (2009)

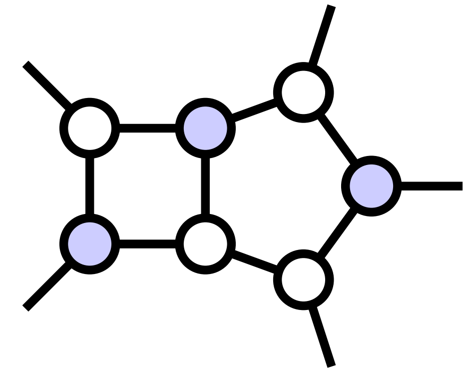


There is some triangulation in terms
220 pieces = Feynman diagrams

This was true for a simplest six-gluon amplitude, but did not seem to work for all tree-level amplitudes, neither loops.

We need "bigger space" to fit all amplitudes there.

On-shell diagrams and Positive Grassmannian

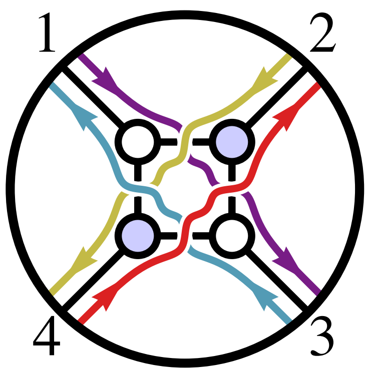


Postnikov, Goncharov (2005-2010)

Around the same time: **plabic graphs**

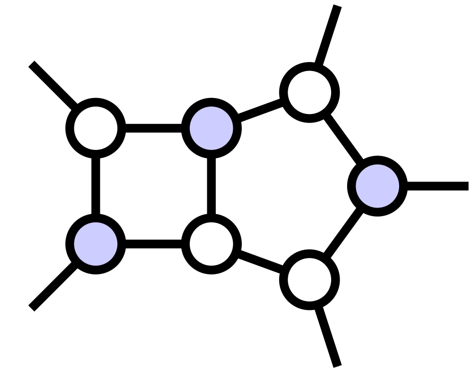
- represent permutations
- correspond to positive matrices

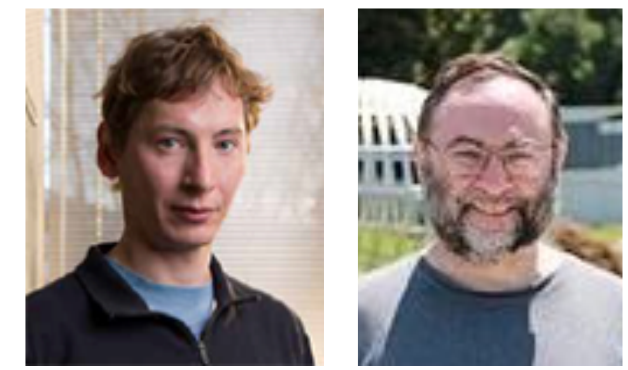
$$\begin{pmatrix} * & * & * & * \\ * & * & * & * \end{pmatrix} \leftrightarrow$$



$$\leftrightarrow (3, 4, 1, 2)$$

On-shell diagrams and Positive Grassmannian





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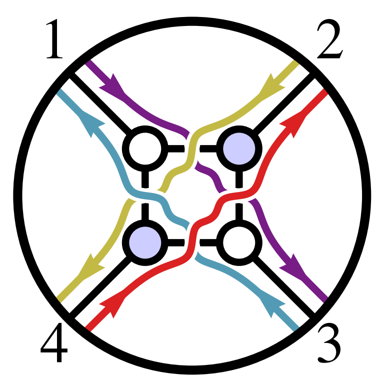

$$\leftrightarrow (3, 4, 1, 2)$$

Same graphs appear in amplitudes - **on-shell diagrams**
Terms in recursion relations, “cuts” of loop amplitudes

arXiv:1212.5605 [pdf, other]
Scattering Amplitudes and the Positive Grassmannian
Nima Arkani-Hamed, Jacob L. Bourjaily, Freddy Cachazo, Alexander B. Goncharov, Alexander Postnikov, Jaroslav Trnka
Comments: a handful of minor corrections and citations added/updated; 158 pages, 264 figures
Subjects: High Energy Physics - Theory (hep-th); Algebraic Geometry (math.AG); Combinatorics (math.CO)

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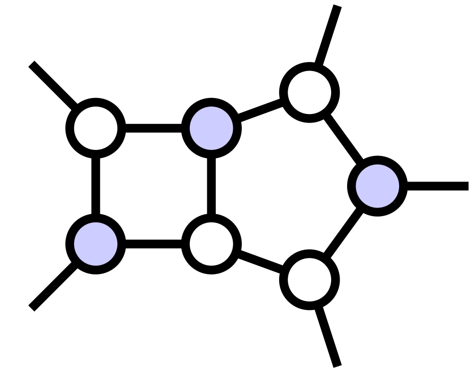

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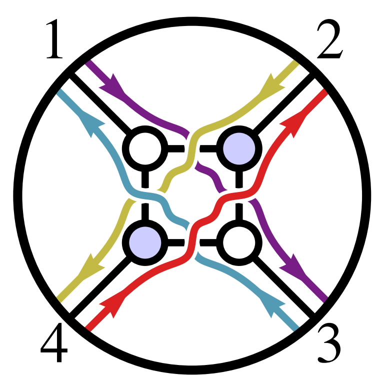

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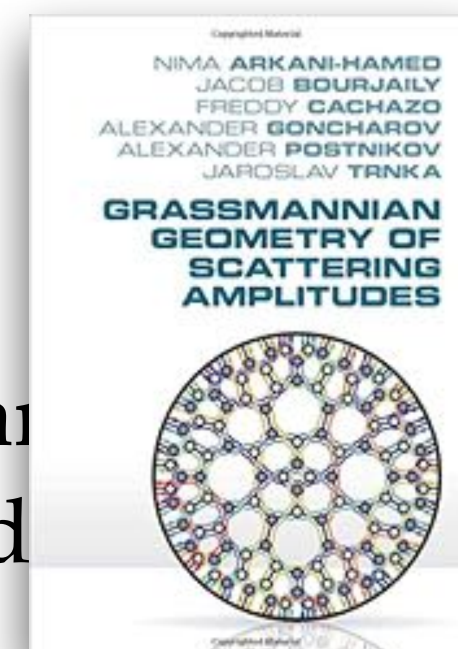

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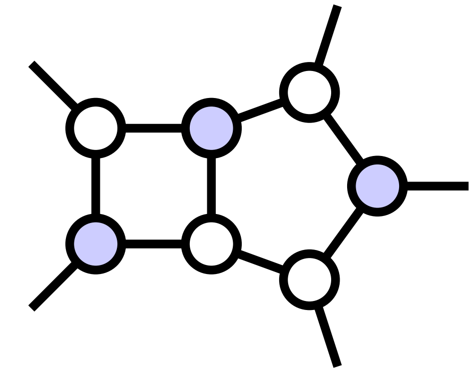
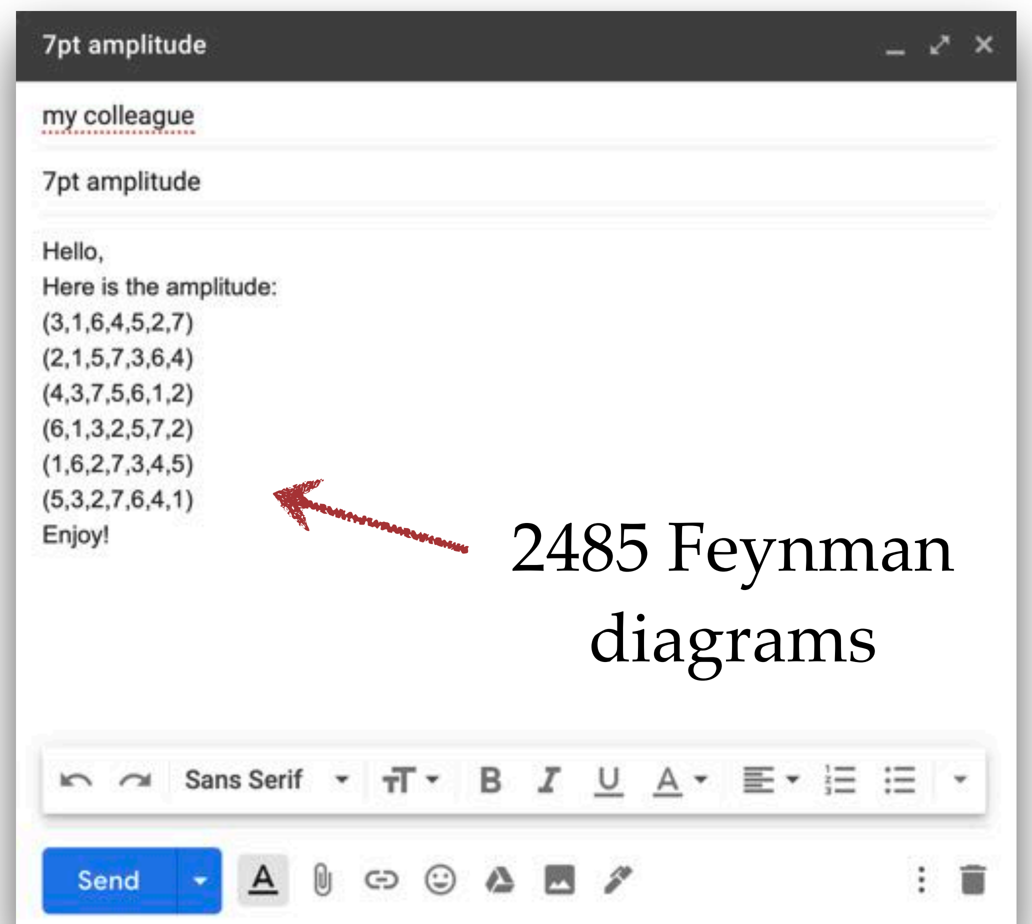
Record on arxiv?

On-shell diagrams and Positive Grassmannian

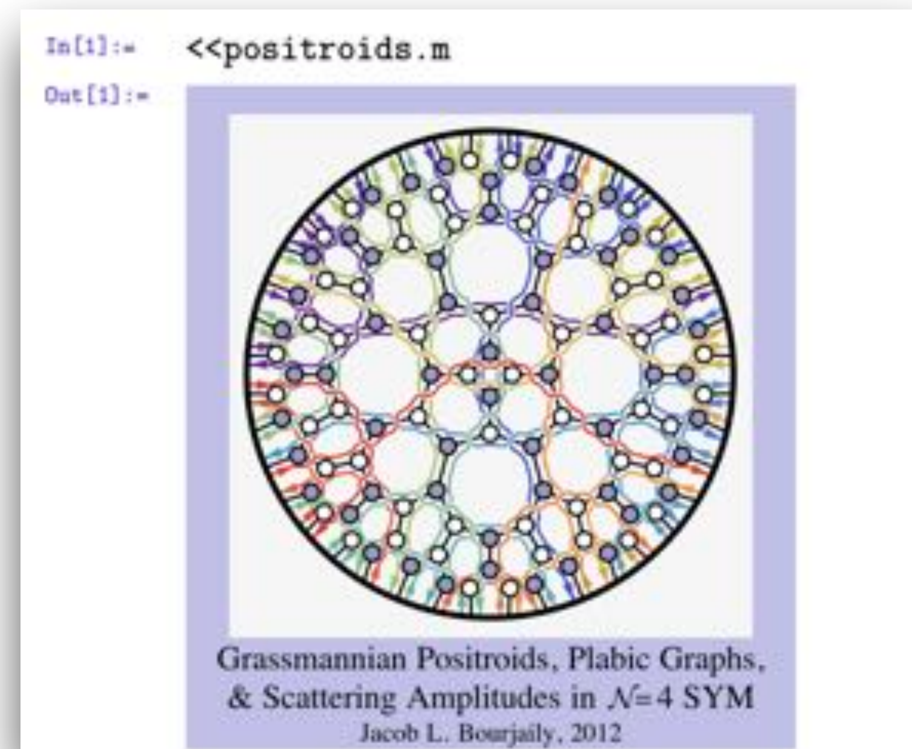


Very efficient way to calculate (tree-level) amplitudes



2485 Feynman diagrams

Shift-enter in Matematica gives the formula



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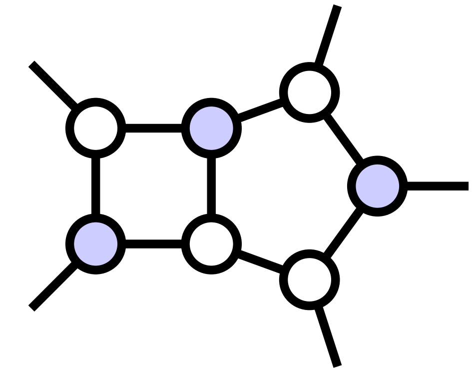

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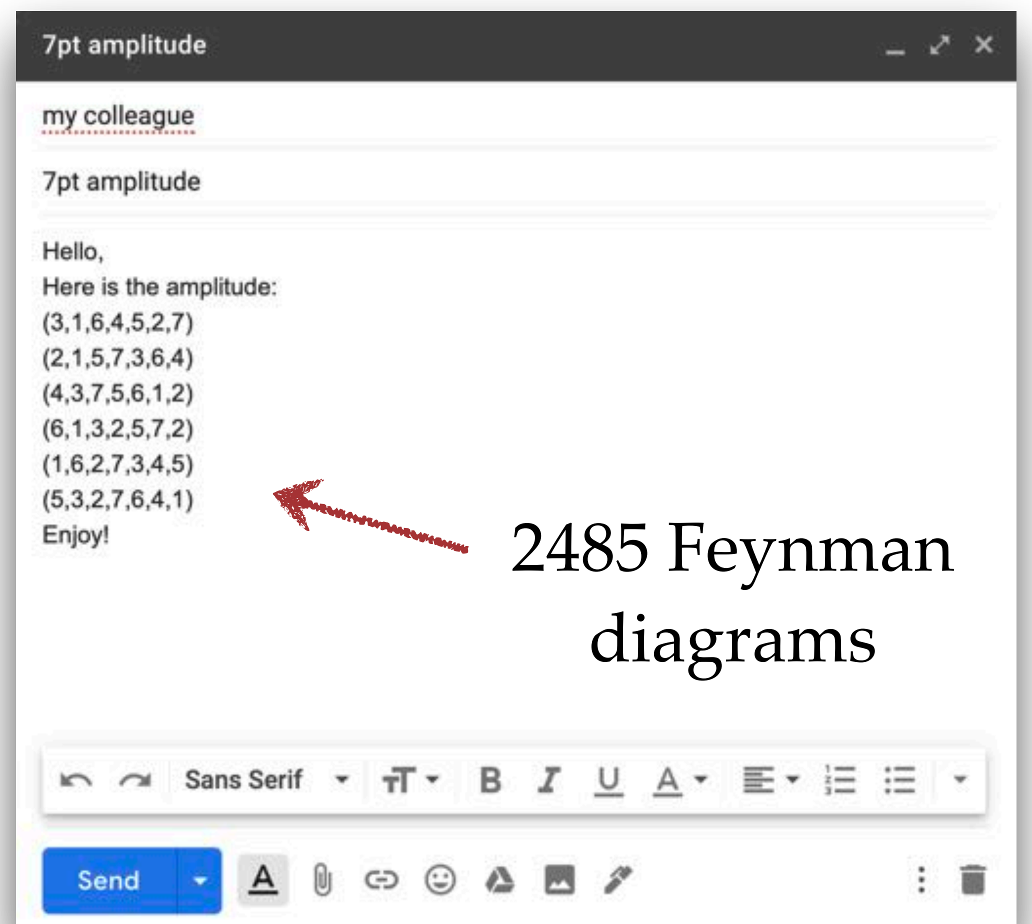


2485 Feynman diagrams

Shift-enter in Matematica gives the formula

What is the scattering amplitude as a single object?

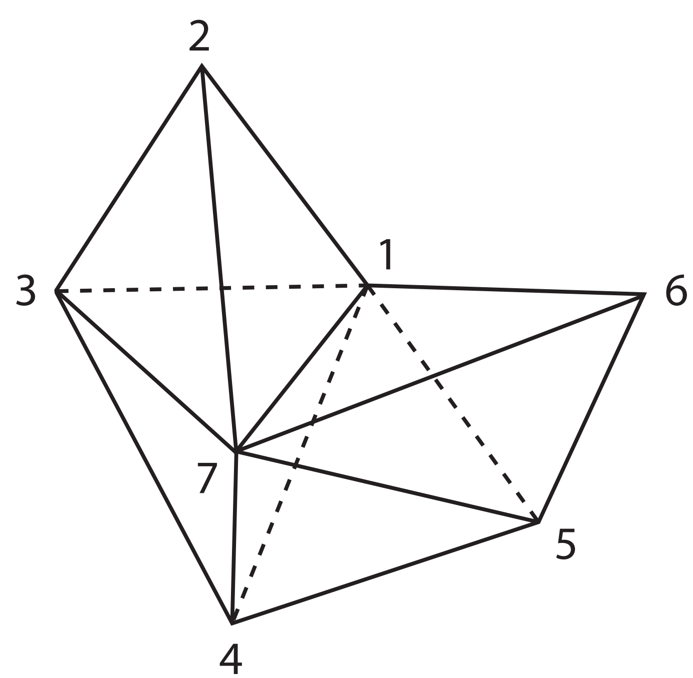
The Amplituhedron

Arkani-Hamed, Trnka (2013), Arkani-Hamed, Thomas, Trnka (2017)

Hodges' observation was not accidental

Special cases the amplitudes correspond to polyhedra, but the general space is the **Amplituhedron**

Seven-gluon scattering at tree-level



The Amplituhedron

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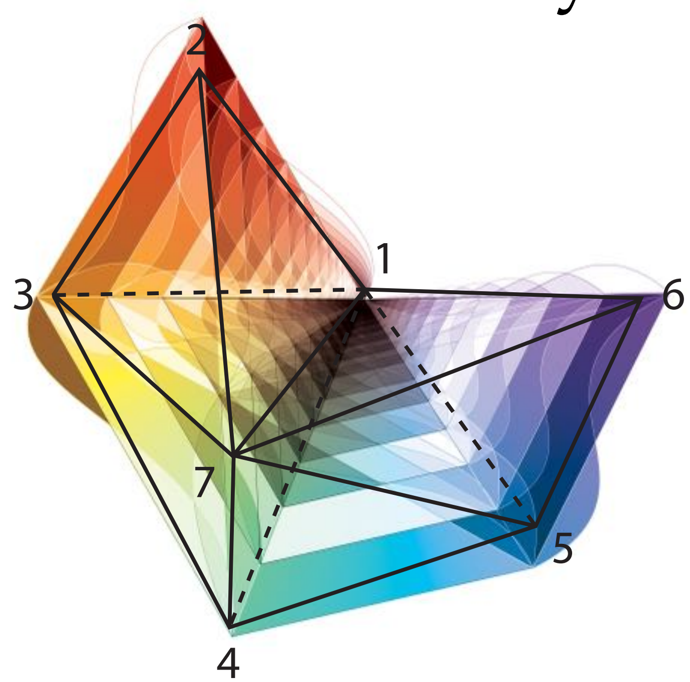
Special cases the amplitudes correspond to polyhedra, but the general space is the **Amplituhedron**

Higher-point amplitudes, loops

Multi-dimensional "curvy" spaces

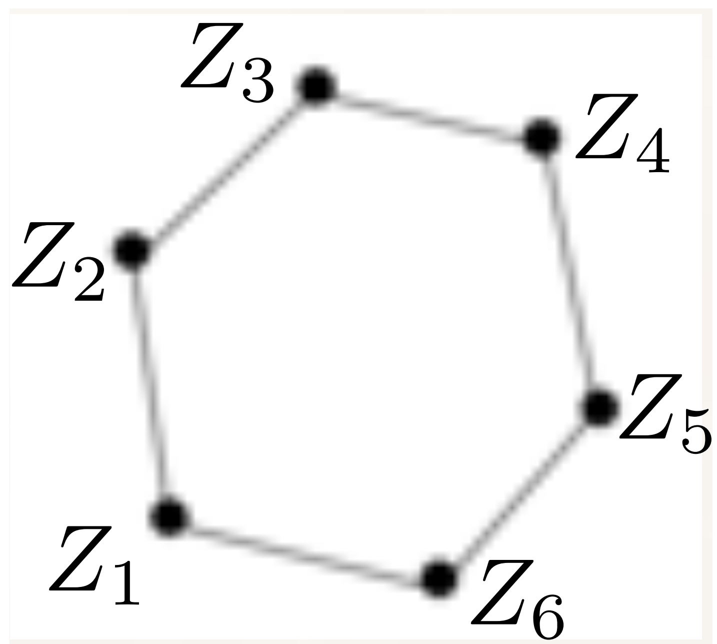


Gluon amplitudes are volumes of the Amplituhedron

The Amplituhedron

Arkani-Hamed, Trnka (2013), Arkani-Hamed, Thomas, Trnka (2017)

Toy example: polygon in the plane - points Z kinematical data



The polygon is a proxy for the Amplituhedron

The Amplituhedron

Arkani-Hamed, Trnka (2013), Arkani-Hamed, Thomas, Trnka (2017)

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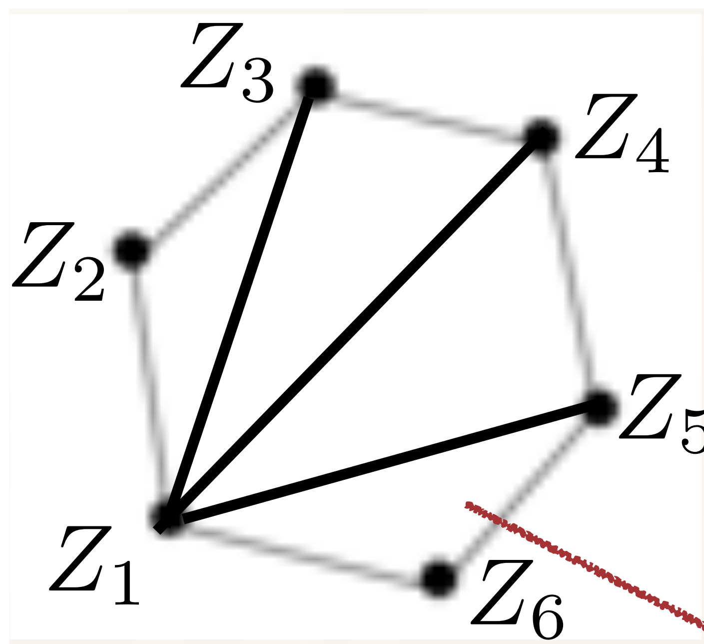
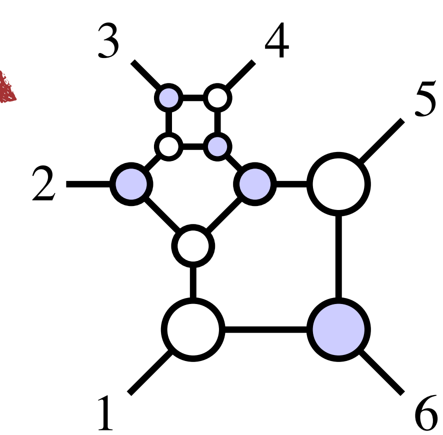


The polygon is a proxy for the Amplituhedron

Certain triangulations correspond to the **on-shell diagram representation**



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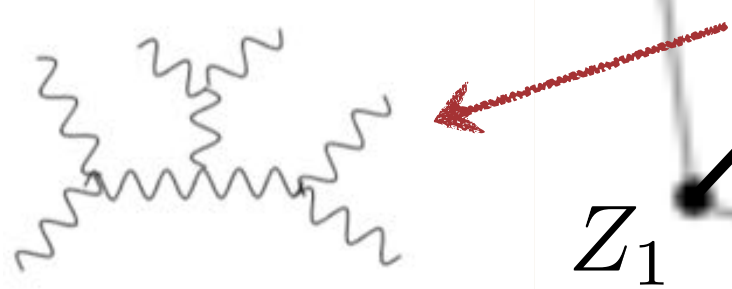
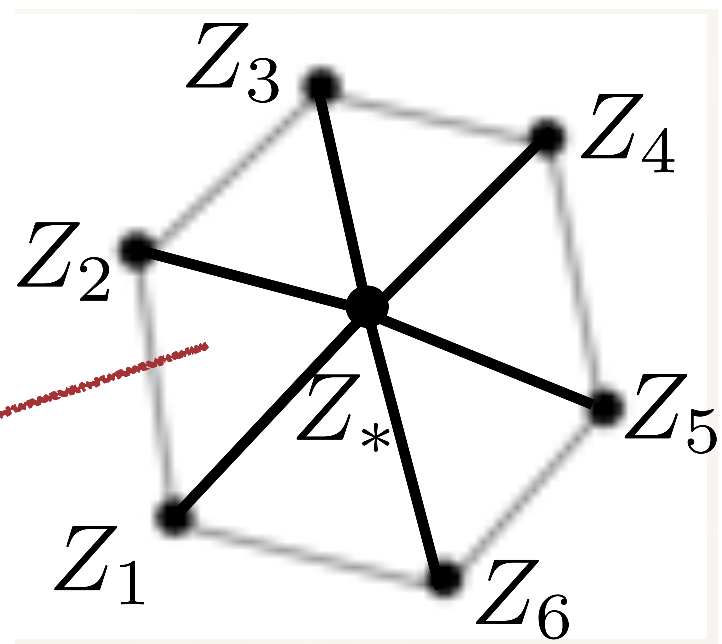




The polygon is a proxy for the Amplituhedron

Other triangulations correspond to **Feynman diagrams**

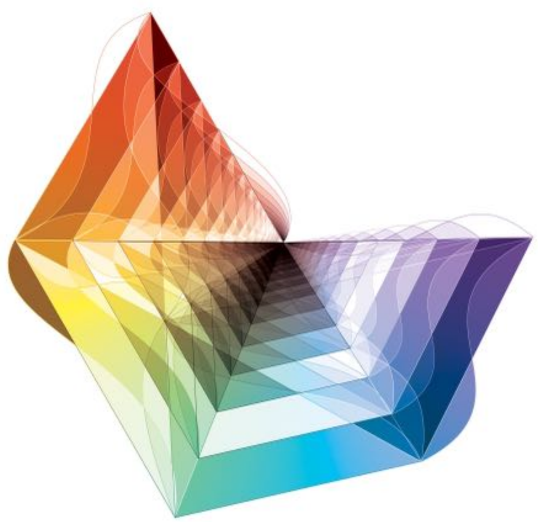
The reference point Z_* is related to the gauge choice

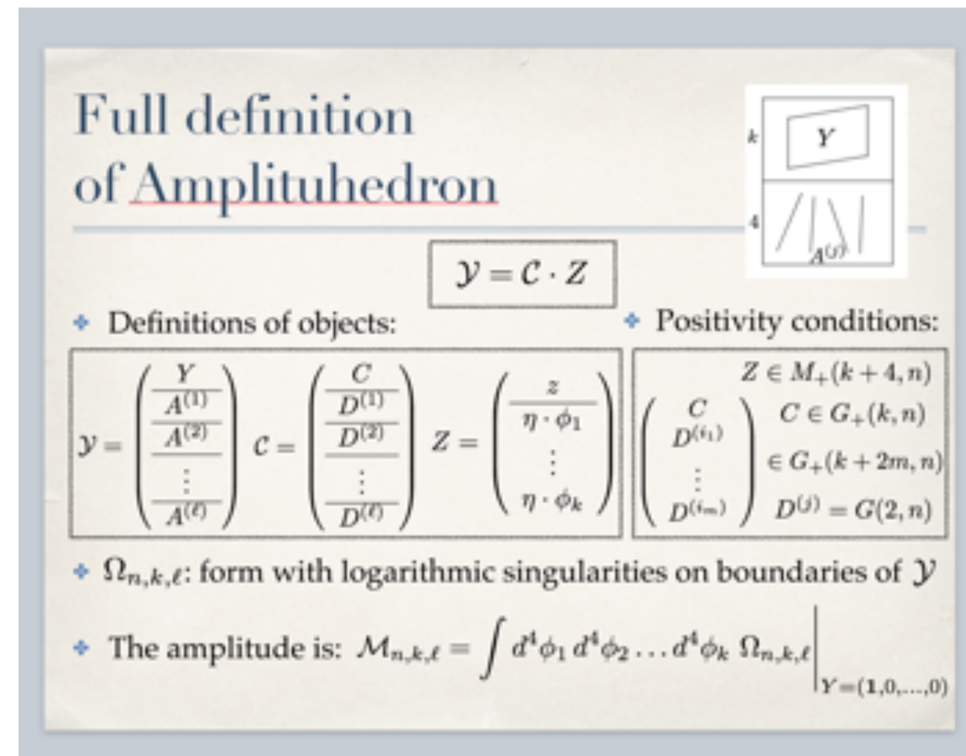
Invariant definition of the "amplitude": **area** of the polygon

The Amplituhedron

Arkani-Hamed, Trnka (2013), Arkani-Hamed, Thomas, Trnka (2017)

Full definition of the Amplituhedron:





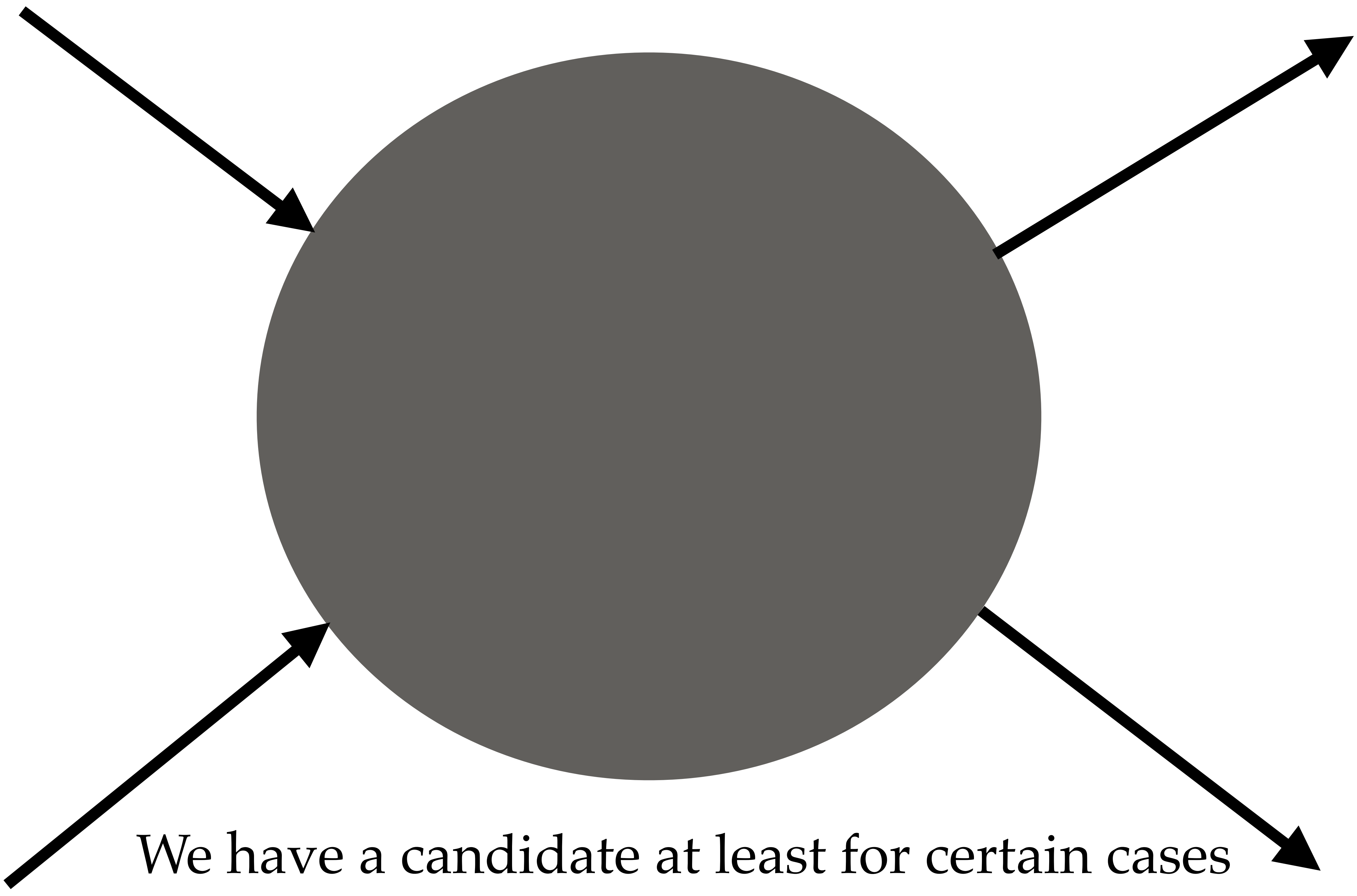
geometric region
specified by a set of
inequalities
geometry labeled by n, k, ℓ

differential volume form on this space:
tree-level amplitudes and loop integrands
in planar maximally supersymmetric Yang-Mills theory

tree-level = QCD
loops = simpler

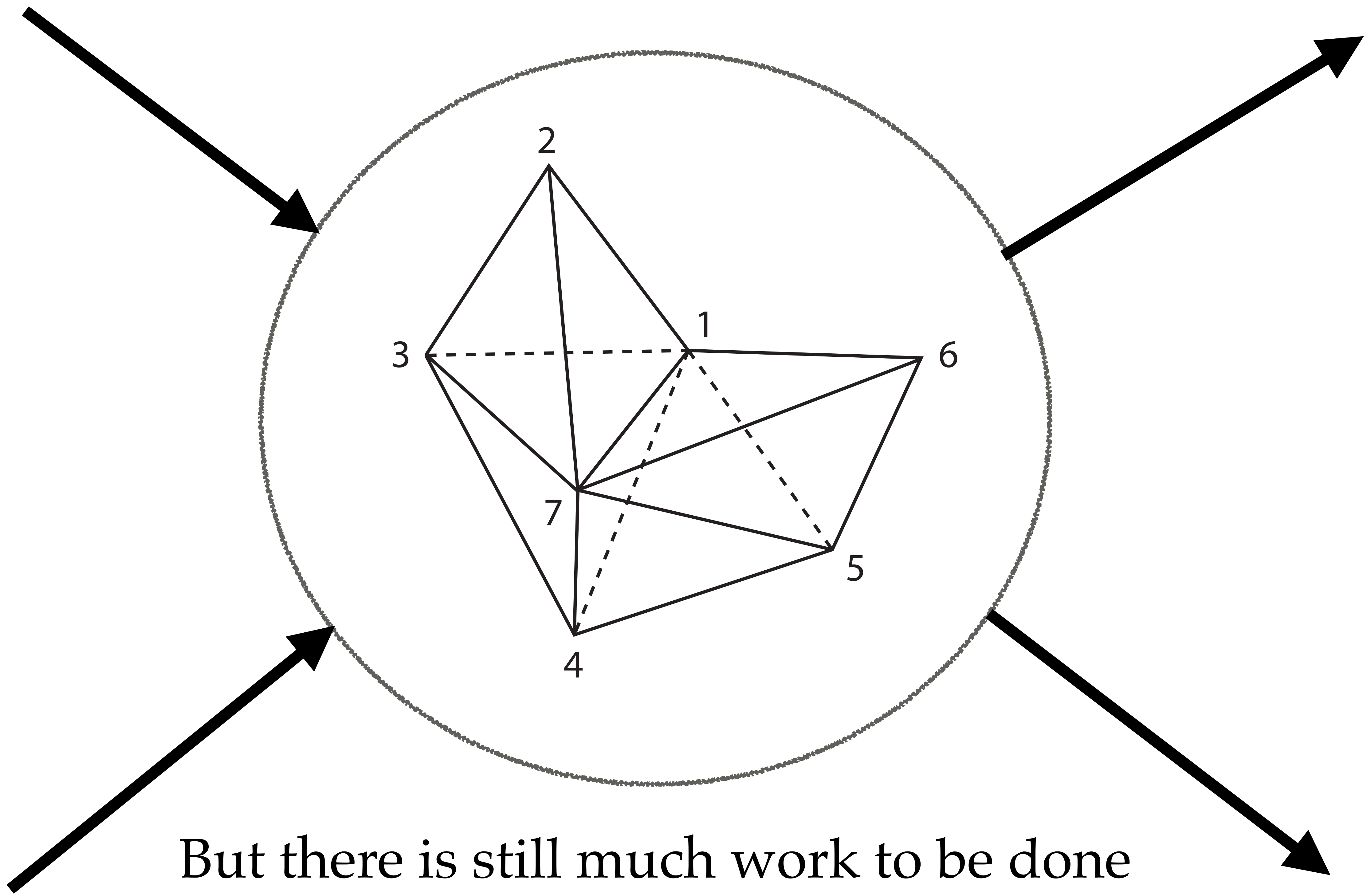
n number of particles $\quad \ell$ number of loops $\quad k$ helicity number

What is scattering amplitude?



We have a candidate at least for certain cases

What is scattering amplitude?



But there is still much work to be done

Fantasy

Classical determinism

Newton's equations

Fantasy

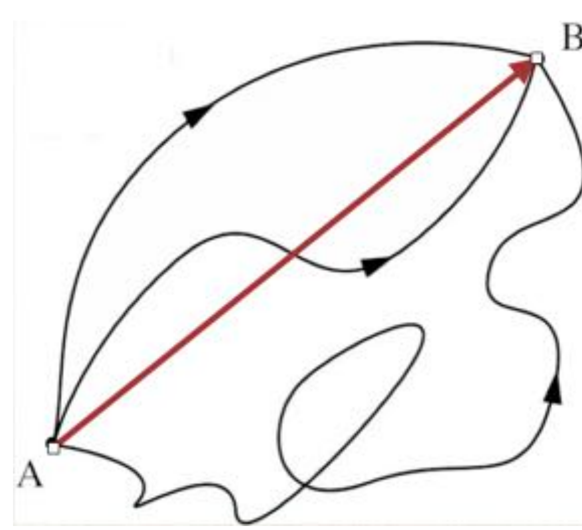
Classical determinism

Newton's equations

equivalent

Action principle

Determinism not manifest



Fantasy

Determinism gone
Probabilities

Classical determinism

QFT

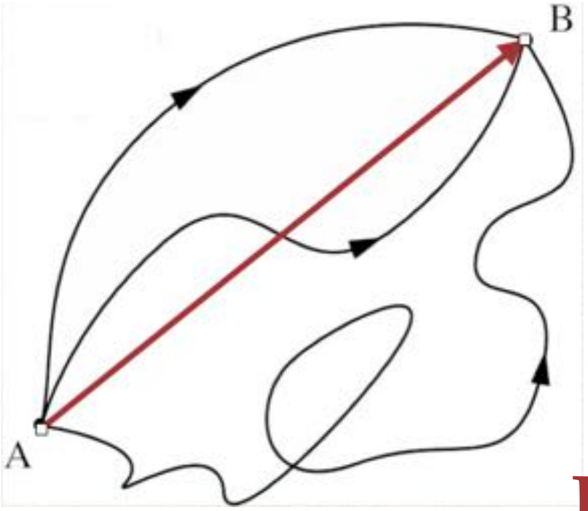
$\hbar \to 0$

Newton's equations

$\hbar \to 0$

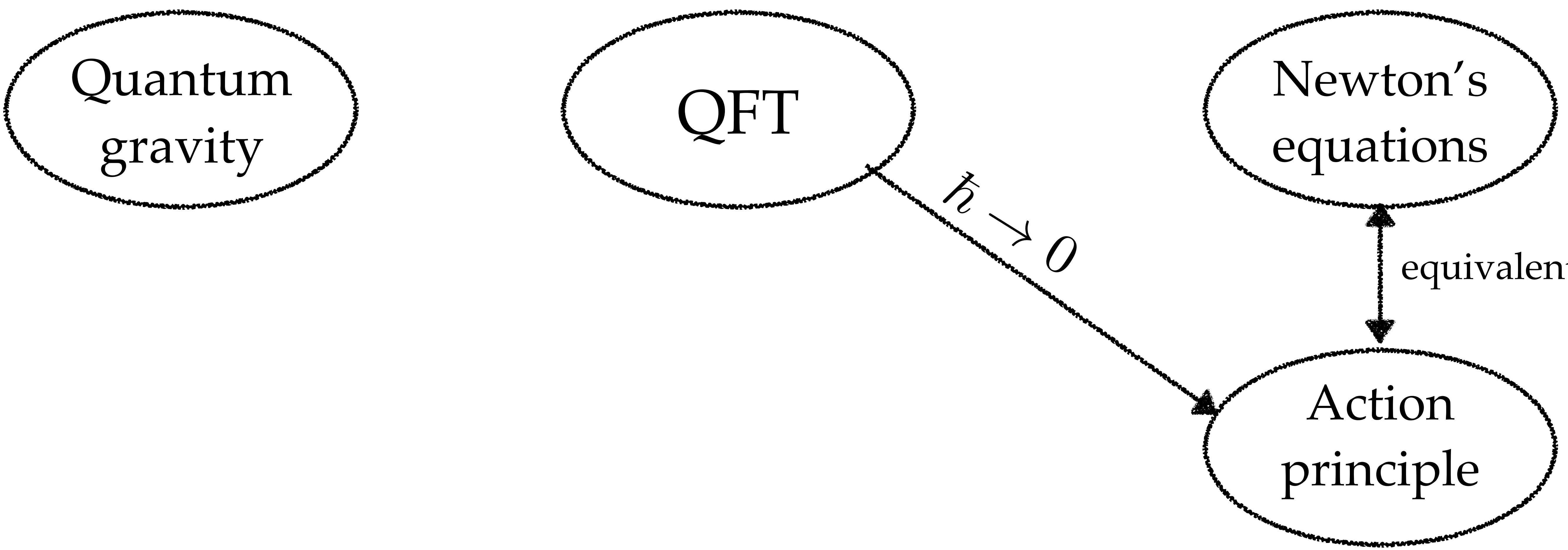
equivalent

Action principle

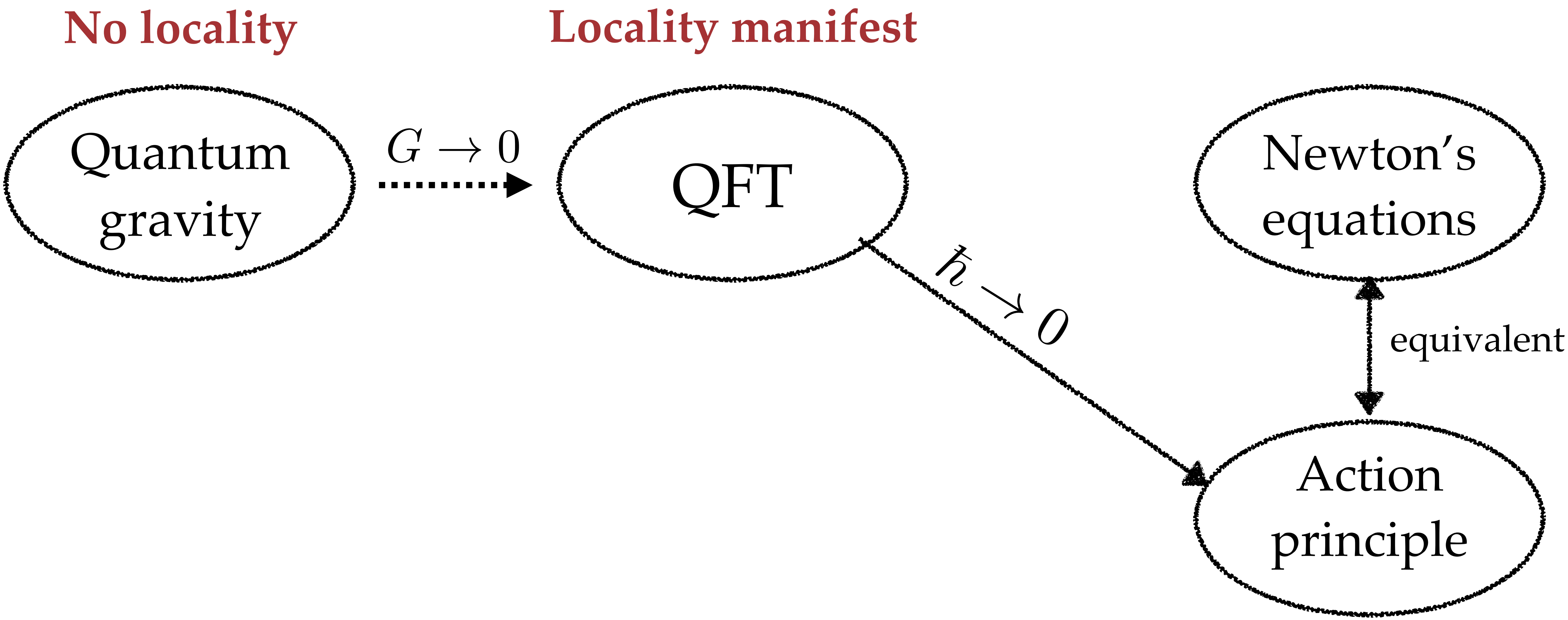


Determinism not manifest

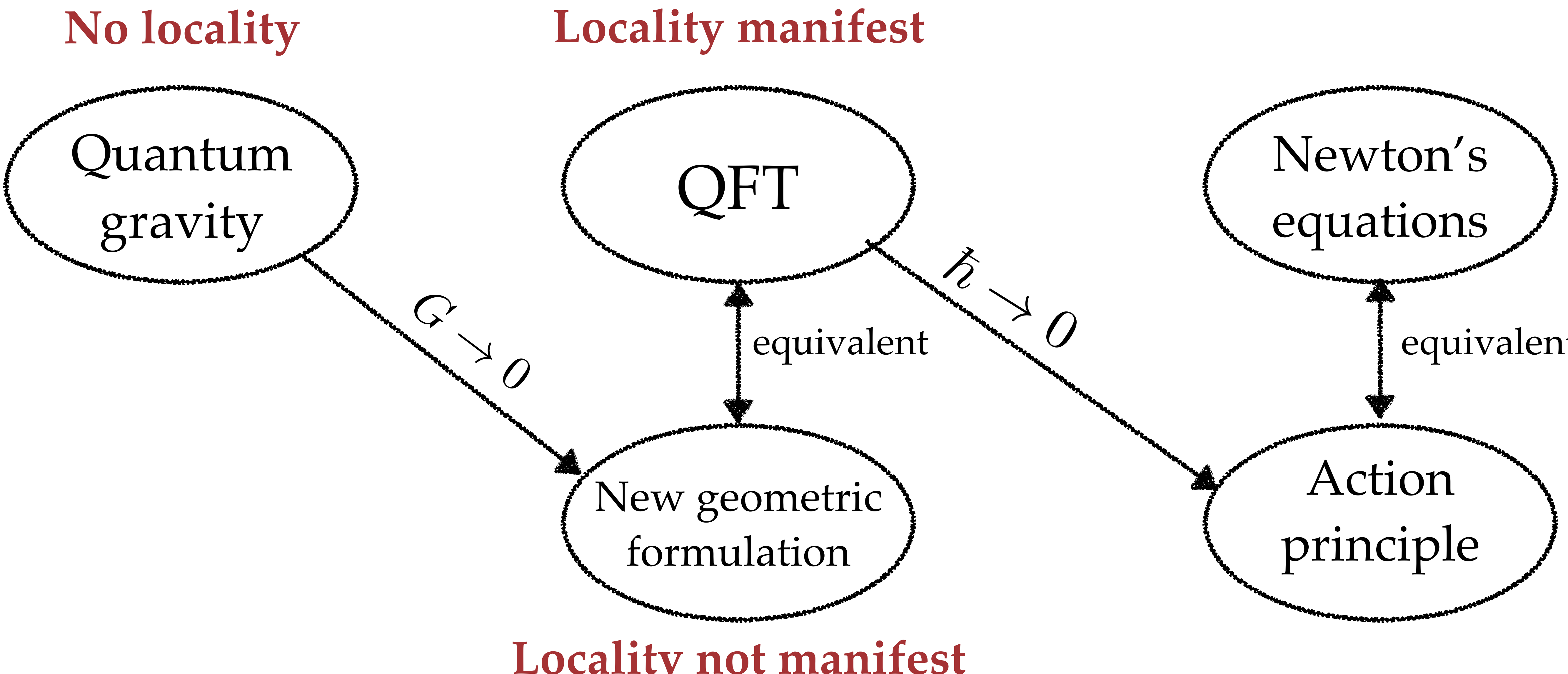
Fantasy


Quantum gravity
QFT
Newton's equations
$\hbar \to 0$
equivalent
Action principle

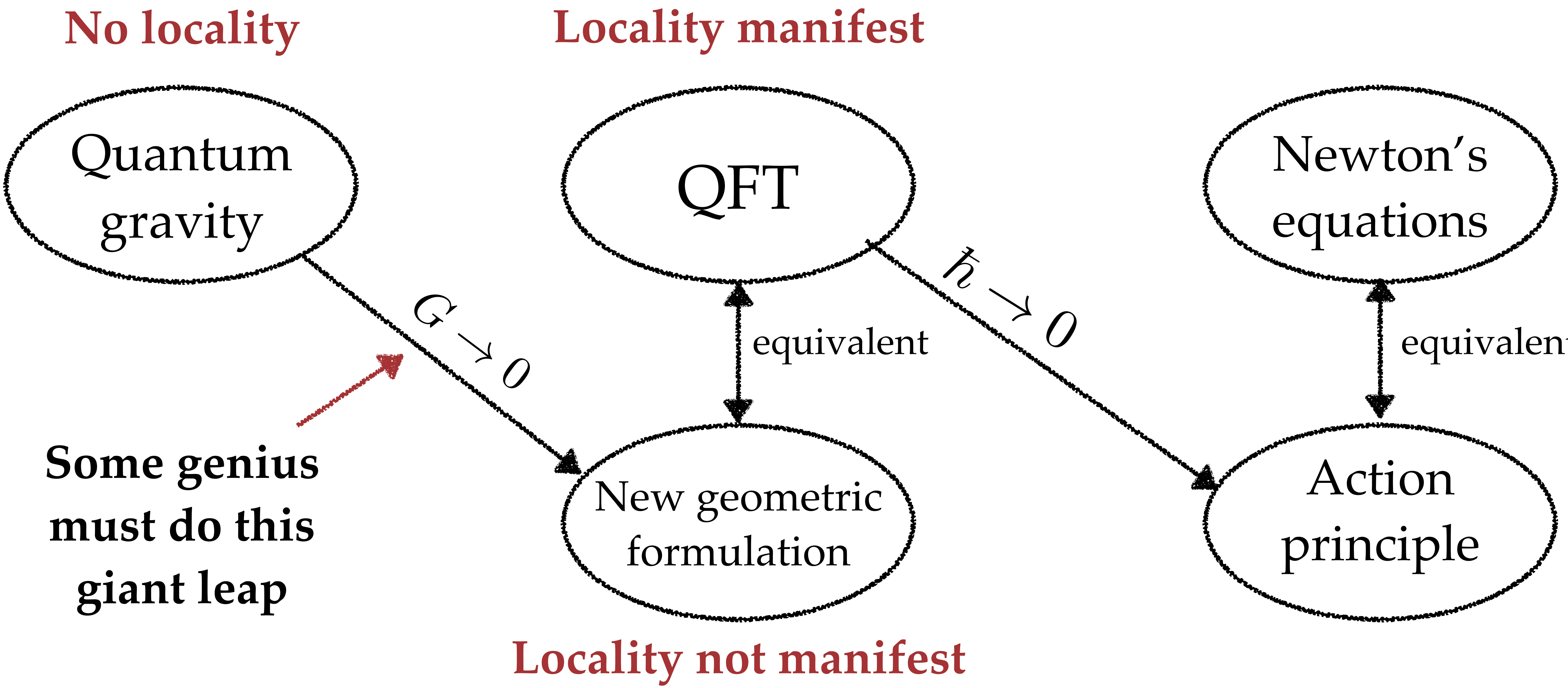
Fantasy


No locality
Locality manifest
Quantum gravity
$G \to 0$
QFT
$\hbar \to 0$
Newton's equations
equivalent
Action principle

Fantasy


No locality
Locality manifest
Quantum gravity
QFT
Newton's equations
$G \to 0$
equivalent
$\hbar \to 0$
equivalent
New geometric formulation
Action principle
Locality not manifest

Fantasy


No locality
Locality manifest
Quantum gravity
QFT
Newton's equations
$G \to 0$
$\hbar \to 0$
equivalent
equivalent
Some genius must do this giant leap
New geometric formulation
Action principle
Locality not manifest

Amplitudes in effective field theories

with Kampf, Novotny, Cheung, Shen, Shifman, Bartsch and others

Amplitudes in EFT

- Tree-level amplitudes of massless particles in EFTs
- Not considered: bad powercounting, problems with loops, on the opposite side to the spectrum of interesting theories than N=4 SYM theory
- Standard procedure: Lagrangian

↓

Symmetry

↓

Properties of amplitudes

Amplitudes in EFT

- Here we consider a completely different perspective
  - Start with generic Lagrangian with free couplings = free parameters in the amplitude
  - Impose kinematical constraints on scattering amplitudes: fix all parameters - find unique theory
- Classify interesting EFTs, perhaps find some new ones
- It is easier to impose kinematical constraints on amplitudes than to search in space of all symmetries

On-shell amplitudes

- Massless scalars in D-dimensions
- On-shell amplitudes $p^2 = 0$
- Tree-level, no renormalization
- Low energies: derivative expansion

Three point interactions

- Consider a single scalar field theory given by

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \mathcal{L}_{int}(\phi, \partial\phi, \dots)$$

- Simplest interaction is 3pt but there are no 3pt amplitudes except for $\mathcal{L}_{int} = \lambda\phi^3$

- Any derivatively coupled term can be written as

$$\mathcal{L}_{int} = (\Box\phi)(\dots) \quad \text{and removed by EOM}$$

Fundamental interaction

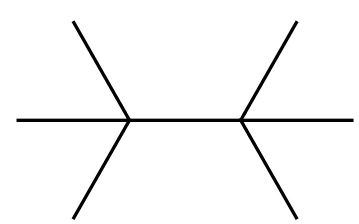
- Let us start with a 4pt interaction term

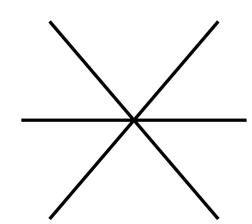
$$\mathcal{L}_{int} = \lambda_4(\partial^m \phi^4) \longrightarrow \text{many terms}$$

- Four point amplitude: special kinematics

- Six point amplitude: presence of contact terms

$$\frac{\partial^m \partial^m}{\partial^2} = \partial^{2m-2} \longrightarrow$$





Powercounting

$$\mathcal{L}_6 = \partial^{2m-2}\phi^6$$

- For $\mathcal{L}_{int} = \lambda_4 \phi^4$ no contact terms possible

Infinite tower

- We consider the infinite tower of terms

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial^m\phi^4) + \lambda_6(\partial^{2m-4}\phi^6) + \dots$$

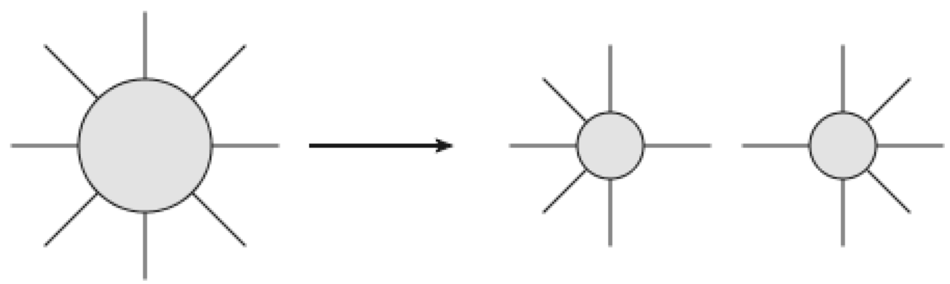
- Even if we start with the 4pt term we can do field redefinitions and generate infinite tower

- We get a generic amplitude $A_n(\lambda_4, \lambda_6, \dots)$

- Find constraints which uniquely specifies all couplings

On-shell constructibility

- On the pole the amplitude must factorize



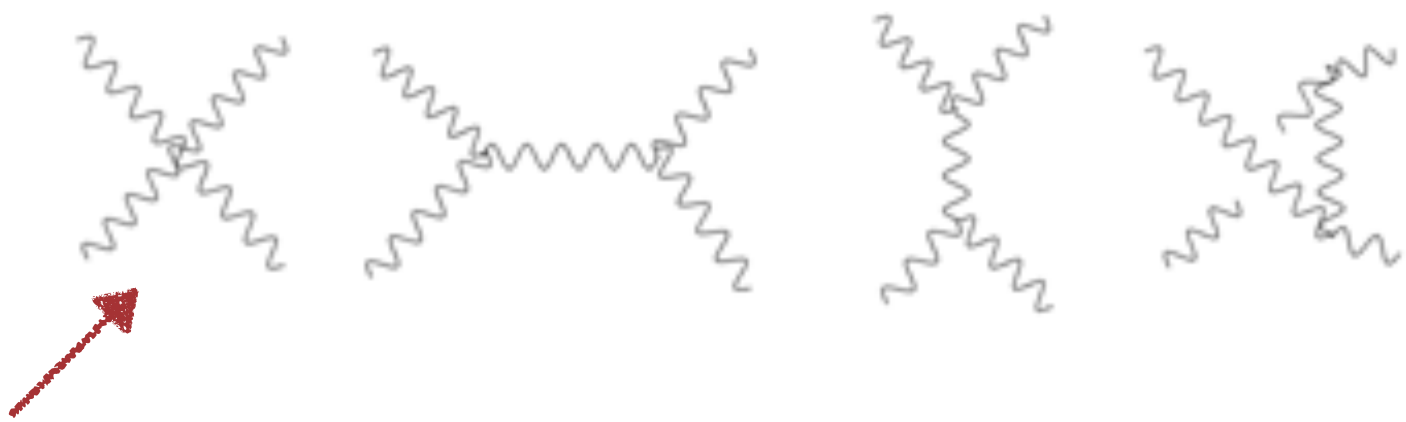
- Contact terms vanish on all poles: not detectable

- Therefore, EFT amplitudes are not specified only by factorization - unfixed kinematical terms

$$\frac{s_{12}s_{56}}{s_{123}} \sim \frac{(s_{12}+s_{123})s_{56}}{s_{123}} \quad \text{on the pole}$$

On-shell constructibility

- Naively, this problem arises also in YM theory
- In fact, the contact terms there is completely fixed



Contact term

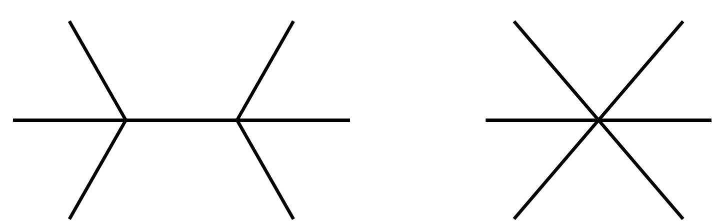
Imposing gauge invariance fixes it

- In our case, contact terms are unfixed with free parameters, there is no gauge invariance

Extra constraints

- If we want to fix the amplitude completely we have to impose additional constraints!

- It must link the contact terms to factorization terms



None of them individually satisfy condition X

- Natural condition for EFTs at low energies

Soft limit $p \to 0$

Single scalar

- The first non-trivial is the original example

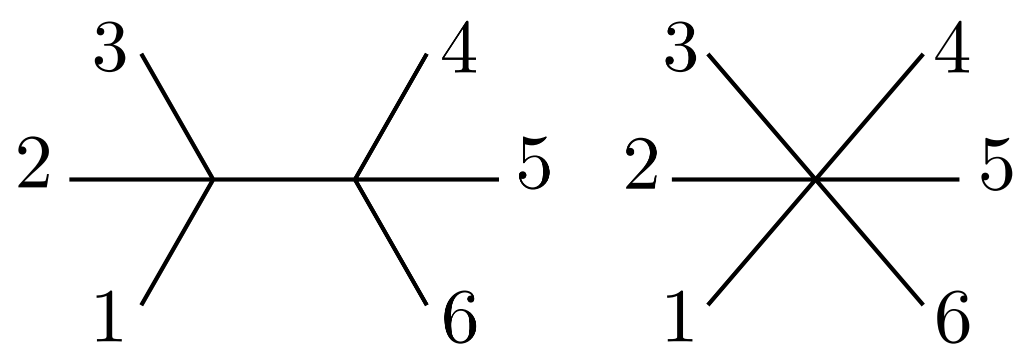
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial\phi)^6 + c_8(\partial\phi)^8 + \ldots$$

$$(\partial\phi)^{2n} = [(\partial_\mu\phi)(\partial^\mu\phi)]^n$$

- Calculate 6pt amplitude

$$s_{ij} = (p_i + p_j)^2$$

$$A_6 = \sum_\sigma$$



$$= 4\sum_\sigma c_4^2 \frac{(s_{12}s_{23} + s_{23}s_{13} + s_{12}s_{13})(s_{45}s_{56} + s_{45}s_{46} + s_{46}s_{56})}{s_{123}} + c_6\, s_{12}s_{34}s_{56}$$

trivial soft-limit vanishing $p_i \to 0 \quad \leftrightarrow$ Lagrangian trivially invariant

$$\phi \to \phi + a$$

Single scalar

- The first non-trivial is the original example

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial\phi)^6 + c_8(\partial\phi)^8 + \dots$$

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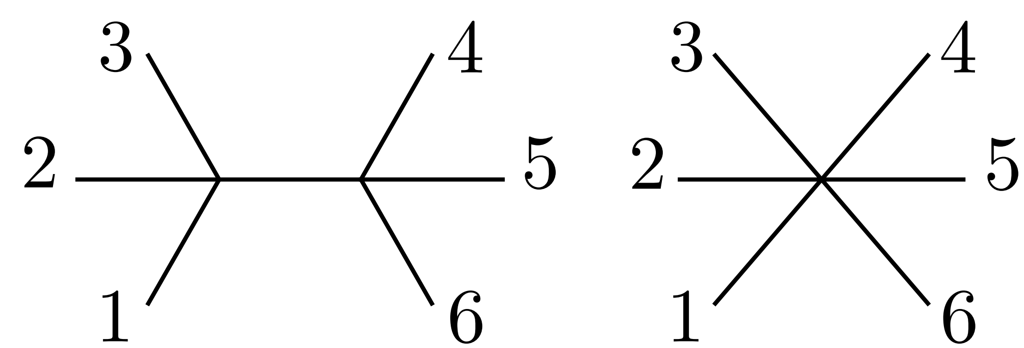

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Impose quadratic vanishing

$$A_6 \to \mathcal{O}(t^2)$$

$$p_i \to tp_i$$
$$t \to 0$$

Single scalar

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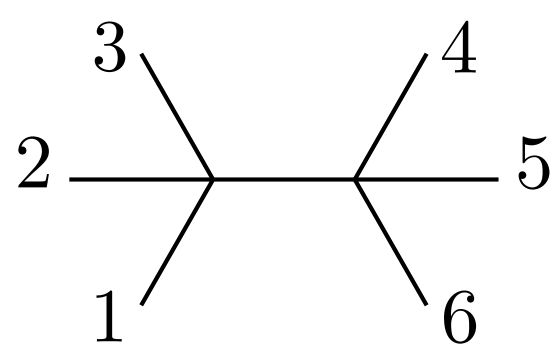

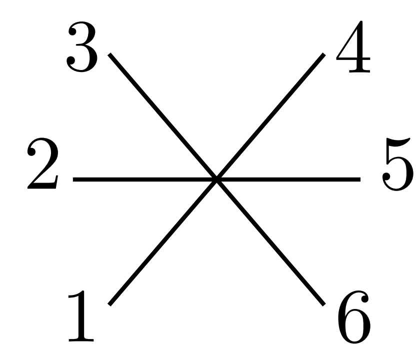

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There is a single solution and it fixes: $c_6 = 4c_4^2$

Single scalar

- The first non-trivial is the original example

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial\phi)^6 + c_8(\partial\phi)^8 + \ldots$$

- Apply to higher point amplitudes

$$(\partial\phi)^{2n} = [(\partial_\mu\phi)(\partial^\mu\phi)]^n$$

$$s_{ij} = (p_i + p_j)^2$$

$$\boxed{A_n = \mathcal{O}(t^2)} \qquad \text{for} \quad \begin{array}{c} p_i \to t p_i \\ t \to 0 \end{array}$$

- Cancelations between diagrams required, a unique solutions exists and relates $c_{2n} \sim c_4^{\#}$

Single scalar

- The Lagrangian becomes

$$\mathcal{L} = -\frac{1}{g}\sqrt{1 - g(\partial\phi)^2} \quad \text{where} \quad g = 8c_4$$

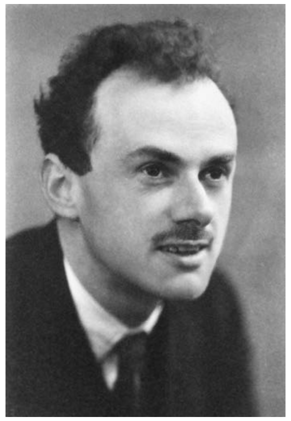
- Apply to higher point amplitudes

$$\boxed{A_n = \mathcal{O}(t^2)} \qquad \text{for} \quad \begin{matrix} p_i \to t p_i \\ t \to 0 \end{matrix}$$

- Cancelations between diagrams required, a unique solutions exists and relates $c_{2n} \sim c_4^{\#}$

Result: DBI action

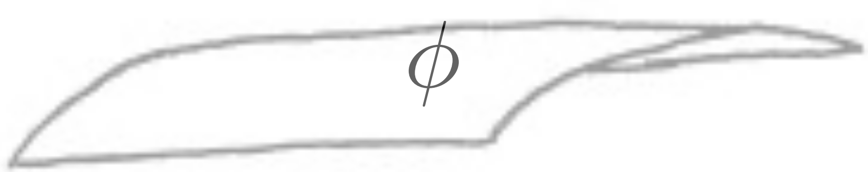
(Dirac, Born, Infeld 1934)



- The Lagrangian becomes

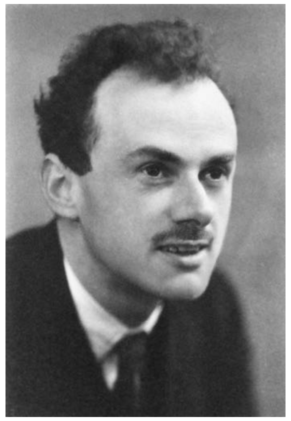
$$\mathcal{L} = -\frac{1}{g}\sqrt{1 - g(\partial\phi)^2} \quad \text{where} \quad g = 8c_4$$

- It describes the fluctuation of D-dimensional brane in (D+1) dimensions



- What is the symmetry principle behind this?

Result: DBI action

(Dirac, Born, Infeld 1934)

- Symmetry of the action: (D+1) Lorentz symmetry

$$\phi \to \phi + (b \cdot x) + (b \cdot \phi\partial\phi)$$

- It can be shown that this implies the soft limit behavior

- But we can also derive the action based on the soft limit

$$2\mathcal{L}'(X)/g = 2X\mathcal{L}'(X) - \mathcal{L}(X) \quad \to \quad \mathcal{L}(X) \sim \sqrt{1-gX}$$

$$\text{where} \quad X = (\partial\phi)^2$$

Next case

- Let us consider the next Lagrangian

$$\mathcal{L}_2 = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial^6\phi^4) + \lambda_6(\partial^{10}\phi^6) + \ldots$$

- Calculate amplitudes: impose again $A_n = \mathcal{O}(t^2)$

Galileon

- Let us consider the next Lagrangian

$$\mathcal{L}_2 = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial^6\phi^4) + \lambda_6(\partial^{10}\phi^6) + \ldots$$

- Calculate amplitudes: impose again $A_n = \mathcal{O}(t^2)$

  Fully specifies a family of solutions

  Galilean symmetry $\phi \to \phi + a + (b \cdot x)$

  Galileons

  Relevant for cosmological models

- There are (d-2) Lagrangians:

$$\mathcal{L}_n = \phi \det[\partial^{\mu_j}\partial_{\nu_k}\phi]^n_{j,k=1} \qquad n \leq d$$

Special Galileon

- Not enough for us: not minimal, not unique

- We impose even stronger condition

$$\boxed{A_n = \mathcal{O}(t^3)} \qquad \text{for} \quad \begin{matrix} p_i \to t p_i \\ t \to 0 \end{matrix}$$

- And there exists an unique solution, linear combination of Galileon Lagrangians: we called it special Galileon

Effective Field Theories from Soft Limits
Clifford Cheung, Karol Kampf, Jiri Novotny, Jaroslav Trnka
(Submitted on 12 Dec 2014)

A Hidden Symmetry of the Galileon
Kurt Hinterbichler, Austin Joyce
(Submitted on 29 Jan 2015)

$$\phi \to s_{\mu\nu} x^\mu x^\nu + \frac{\lambda_4}{12} s^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

Classification

- Use soft-limit as classification tool

  (Cheung, Kampf, Novotny, Shen, JT 2016) (Elvang, Hadjiantonis, Jones, Paranjape 2018)

- Extension to vector fields: Born-Infeld theory

  (Cheung, Kampf, Novotny, Shen, JT 2018)

$$\mathcal{L} = \sqrt{(-1)^{D-1}\det\left(\eta_{\mu\nu} + F_{\mu\nu}\right)}$$

- Non-trivial soft theorems

  (Kampf, Novotny, Shifman, JT 2019)

- Close connection to color-kinematics duality, worldsheet integrals (scattering equations)

  (Bern, Carrasco, Johansson 2009) (Cachazo, He, Yuan 2014)

Conclusion

Summary

- Amplitudes: probe to learn new things about QFTs
- Many different research directions, I mentioned some of them, but different people care about different things
- Amplituhedron & positive geometry: completely new approach (physics -> math), only special theories now
- Recent connections to cosmology: EFTs for inflation, calculation of cosmological correlators (related to S-matrix)

Thank you for your attention